Today:

- Priority queues (Data Structures §6.4, §6.5)
- Range queries (§6.2)
- Java utilities: SortedSet, Map, etc.

Next topic: Hashing (Data Structures Chapter 7).
Priority Queues, Heaps

- Priority queue: defined by operations “add,” “find largest,” “remove largest.”
- Examples: scheduling long streams of actions to occur at various future times.
- Also useful for sorting (keep removing largest).
- Heap is common implementation.
- Enforces heap property: all labels in both children of node are less (or greater) than node’s label.
- So node at top has largest (or smallest) label.
- Are free to add smaller value to less bushy subtree, thus maintaining bushiness (keeping tree balanced).
- Insertion and deletion always proportional to $\lg N$ in worst case.
Example: Inserting into a simple heap

Data:

1 17 4 5 9 0 -1 20

Initial Heap:

Add 8: Dashed boxes show where heap property violated

re-heapify up
Heap insertion continued

Now insert 18:

```
20
  17
   8
   4
  1 5 18
```

```
20
  17
   8
   4
  1 5 18
```

\[\Rightarrow\]

```
20
  18
   8
   17
  1 5 4
```

\[\Rightarrow\]

```
20
  18
   8
   17
  1 5 4
```

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Removing Largest from Heap

To remove largest: Move bottommost, rightmost node to top, then re-heapify down as needed (swap offending node with larger child) to re-establish heap property.

Initial

Final
Heaps in Arrays

• Since heaps are complete (missing items only at bottom level), can use arrays for compact representation.

• Example of removal from last slide (dashed arrows show children):

Nodes stored in level order. Children of node at index $K$ are in $2K$ and $2K + 1$.
Ranges

- So far, have looked for specific items
- But for BSTs, need an ordering anyway, and can also support looking for ranges of values.
- Example: perform some action on all values in a BST that are within some range (in natural order):

```java
/** Apply WHATTODO to all labels in T that are 
 *  >= L and < U, in ascending natural order. */
static void visitRange (BST T, Comparable<Key> L, Comparable<Key> U, 
                      Action whatToDo)
if (T != null) {
    int compLeft = L.compareTo (T.label ()),
    compRight = U.compareTo (T.label ());
    if (compLeft < 0) /* L < label */
        visitRange (T.left (), L, U, whatToDo);
    if (compLeft <= 0 && compRight > 0) /* L <= label < U */
        whatToDo.action (T);
    if (compRight > 0) /* label < U */
        visitRange (T.right (), L, U, whatToDo);
}
```
Time for Range Queries

• Time for range query $\in O(h + M)$, where $h$ is height of tree, and $M$ is number of data items that turn out to be in the range.

• Consider searching the tree below for all values, $x$, such that $25 \leq x < 40$.

• In this example, the $h$ comes from the starred nodes; the $M$ comes from other non-dashed nodes. Dashed nodes are never looked at.
Ordered Sets and Range Queries in Java

- **Class** `SortedSet` supports range queries with *views* of set:
  - `S.headSet(U)`: subset of `S` that is `< U`.
  - `S.tailSet(L)`: subset that is `≥ L`.
  - `S.subSet(L,U)`: subset that is `≥ L, < U`.

- **Changes to views modify** `S`.

- **Attempts to**, e.g., **add to a headSet beyond** `U` are disallowed.

- **Can iterate through a view to process a range**:

  ```java
  SortedSet<String> fauna = new TreeSet<String>(Arrays.asList("axolotl", "elk", "dog", "hartebeest", "duck"));
  for (String item : fauna.subSet("bison", "gnu"))
    System.out.printf("%s, ", item);
  ```

  would print "dog, duck, elk,"

- **Java library type** `TreeSet<T>` **requires either that** `T` **be Comparable,**
  **or that you provide a Comparator**:

  ```java
  SortedSet<String> rev_fauna = new TreeSet<String>(Collections.reverseOrder());
  ```
Example of Representation: BSTSet

- Use binary search tree to represent set. Can use same representation for both BSTSet and its subsets.
- Each set has pointer to BST, plus bounds (if any).
- In this representation, size is rather expensive!

```java
class SortedSet<String> {
    fauna = new BSTSet<String> (collection of stuff);
    subset = fauna.subSet ("bison","gnu");
    Iterator<String> i = subset.iterator ();
```

Diagram:

- Sentinel fauna:
- BSTSet.this:
- I:
- Last:
- Next:
- Subset:
- Hartebeest
- Dog
- Axolotl
- Elk
- Duck
- Bison
- Gnu