Today: Backtracking searches, game trees (DSIJ, Section 6.5)
Searching by “Generate and Test”

• We’ve been considering the problem of searching a set of data stored in some kind of data structure: “Is \( x \in S \)?”

• But suppose we don’t have a set \( S \), but know how to recognize what we’re after if we find it: “Is there an \( x \) such that \( P(x) \)?”

• If we know how to enumerate all possible candidates, can use approach of Generate and Test: test all possibilities in turn.

• Can sometimes be more clever: avoid trying things that won’t work, for example.

• What happens if the set of possible candidates is infinite?
Backtracking Search

• Backtracking search is one way to enumerate all possibilities.

• Example: *Knight’s Tour*. Find all paths a knight can travel on a chessboard such that it touches every square exactly once and ends up one knight move from where it started.

• In the example below, the numbers indicate position numbers (knight starts at 0).

• Here, knight (N) is stuck; how to handle this?

```
   6   5
   4   7
10  2   8
  3  0   9
 N   1
```
General Recursive Algorithm

/** Append to PATH a sequence of knight moves starting at ROW, COL
* that avoids all squares that have been hit already and
* that ends up one square away from ENDROW, ENDCOL. B[i][j] is
* true iff row i and column j have been hit on PATH so far.
* Returns true if it succeeds, else false (with no change to PATH).
* Call initially with PATH containing the starting square, and
* the starting square (only) marked in B. */

boolean findPath (boolean[][] b, int row, int col,
                 int endRow, int endCol, List path) {
    if (path.size () == 64) return isKnightMove (row, col, endRow, endCol);
    for (r, c = all possible moves from (row, col)) {
        if (! b[r][c]) {
            b[r][c] = true; // Mark the square
            path.add (new Move (r, c));
            if (findPath (b, r, c, endRow, endCol, path)) return true;
            b[r][c] = false; // Backtrack out of the move.
            path.remove (path.size ()-1);
        }
    }
    return false;
}
Another Kind of Search: Best Move

- Consider the problem of finding the best move in a two-person game.
- One way: assign a value to each possible move and pick highest.
  - Example: number of our pieces - number of opponent's pieces.
- But this is misleading. A move might give us more pieces, but set up a devastating response from the opponent.
- So, for each move, look at opponent's possible moves, assume he picks the best one for him, and use that as the value.
- But what if you have a great response to his response?
- How do we organize this sensibly?
Game Trees

- Think of the space of possible continuations of the game as a tree.
- Each node is a position, each edge a move.

• Suppose numbers at the bottom are the values of those final positions to me. Smaller numbers are of more value to my opponent.

• What should I move? What value can I get if my opponent plays as well as possible?
Game Trees, Minimax

- Think of the space of possible continuations of the game as a tree.
- Each node is a position, each edge a move.

- Numbers are the values we guess for the positions (larger means better for me). Starred nodes would be chosen.
- I always choose child (next position) with maximum value; opponent chooses minimum value ("Minimax algorithm")
Alpha-Beta Pruning

- We can prune this tree as we search it.

At the ‘\( \geq 5 \)’ position, I know that the opponent will not choose to move here (since he already has a \(-5\) move).

At the ‘\( \leq -20 \)’ position, my opponent knows that I will never choose to move here (since I already have a \(-5\) move).
Cutting off the Search

• If you could traverse game tree to the bottom, you’d be able to force a win (if it’s possible).

• Sometimes possible near the end of a game.

• Unfortunately, game trees tend to be either infinite or impossibly large.

• So, we choose a maximum depth, and use a heuristic value computed on the position alone (called a static valuation) as the value at that depth.

• Or we might use iterative deepening (kind of breadth-first search), and repeat the search at increasing depths until time is up.

• Much more sophisticated searches are possible, however (take CS188).
Some Pseudocode for Searching

/** A legal move for WHO that either has an estimated value >= CUTOFF
 * or that has the best estimated value for player WHO, starting from
 * position START, and looking up to DEPTH moves ahead. */
Move findBestMove (Player who, Position start, int depth, double cutoff)
{
    if (start \textit{is a won position for} who) return WON_GAME; /* Value \infty */
    else if (start \textit{is a lost position for} who) return LOST_GAME; /* Value \text{-}\infty */
    else if (depth == 0) return guessBestMove (who, start, cutoff);

    Move bestSoFar = REALLY_BAD_MOVE;
    for (each legal move, M, \textit{for} who \textit{from position} start) {
        Position next = start.makeMove (M);
        Move response = findBestMove (who.opponent (), next,
            depth-1, -bestSoFar.value ());

        if (-response.value () > bestSoFar.value ()) {
            \textit{Set} M\text{'}s value to \text{-}response.value (); \textit{Value for who = - Value for opponent}
            bestSoFar = M;
            if (M.value () >= cutoff) break;
        }
    }
    return bestSoFar;
}
Static Evaluation

- This leaves static evaluation, which looks just at the next possible move:

```java
Move guessBestMove (Player who, Position start, double cutoff)
{
    Move bestSoFar;
    bestSoFar = Move.REALLY_BAD_MOVE;
    for (each legal move, M, for who from position start) {
        Position next = start.makeMove (M);
        // Set M's value to heuristic guess of value to who of next;
        if (M.value () > bestSoFar.value ()) {
            bestSoFar = M;
            if (M.value () >= cutoff)
                break;
        }
    }
    return bestSoFar;
}
```