Associated Reference Convention:

A current direction is chosen entering each positively-referenced terminal.

Device Graph: DIGRAPH (Directed Graph)
Associated Reference Convention:

2-port Device

Device Graph

n-port Device
KVL around closed node sequence:

\[ \text{KVL around closed node sequence:} \]

\[ (v_2 + v_3 - v_1) + (-v_3 + v_5 - v_4) = 0 \]

These 3 KVL equations are not linearly-independent because the 3rd equation can be obtained by adding the first 2 equations:
• Circuits containing \( n \)-terminal devices can have many distinct digraphs, due to different (arbitrary) choices of the datum terminal for each \( n \)-terminal device.

• Although the KCL and KVL equations associated with 2 different digraphs of a given circuit are different, they contain the same information because each set of equations can be derived from the other.
A Circuit with 3 different digraphs

1. Choose ③ as datum for $\mathcal{D}$

2. Choose ② as datum for $\mathcal{D}$

3. Choose ① as datum for $\mathcal{D}$
KCL at \( \textcircled{2} \) : \( i_3 + i_4 = 0 \)

KCL at \( \textcircled{4} \) : \( i_5 + i_6 = 0 \)

KVL around \( \textcircled{2} - \textcircled{3} - \textcircled{2} \) : \( v_4 - v_3 = 0 \)

KVL around \( \textcircled{4} - \textcircled{5} - \textcircled{4} \) : \( v_6 - v_5 = 0 \)
Adding a wire connecting one node from each separate component does not change KVL or KCL equations.

\[ \{7\} \text{ is a cut set} \implies i_7 = 0 \]
Adding a wire connecting one node from each separate component does not change KVL or KCL equations.
Since nodes 3 and 5 are now the same node, they can be combined into one node, and the redrawn digraph is called a **hinged** graph.
A \mathbf{i} = 0 \quad \Rightarrow \quad \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & -1 \\
-1 & 0 & -1 & 1 & 0 & 0 \\
0 & -1 & 1 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4 \\
i_5 \\
i_6 \\
\end{bmatrix} = \begin{bmatrix}0 \\
0 \\
0 \\
\end{bmatrix}

A is called the **reduced Incidence Matrix** of the diagraph $G$ relative to datum node 4.
Choose node 4 as datum node for digraph $G$.

KCL Equations:

1. $i_1 + i_2 - i_6 = 0$
2. $-i_1 - i_3 + i_4 = 0$
3. $-i_2 + i_3 + i_5 = 0$

Independent KCL Equations

$$A \mathbf{i} = 0 \implies$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Independent KVL Equations

$$\mathbf{v} = A^T \mathbf{e}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

$$v_1 = e_1 - e_2$$
$$v_2 = e_1 - e_3$$
$$v_3 = -e_2 + e_3$$
$$v_4 = e_2$$
$$v_5 = e_3$$
$$v_6 = -e_1$$
choose node 3 as datum and let \( \hat{e}_1, \hat{e}_2, \hat{e}_4 \) be new node-to-datum voltages.

**KCL Equations:**

\[
\begin{align*}
1 & \quad i_1 + i_2 - i_6 = 0 \\
2 & \quad -i_1 - i_3 + i_4 = 0 \\
4 & \quad -i_4 - i_5 + i_6 = 0 \\
\end{align*}
\]

**Independent KCL Equations**

\[
\hat{A} \mathbf{i} = \mathbf{0} \quad \Rightarrow 
\]

\[
\begin{bmatrix}
1 & -1 & 0 \\
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 1 & -1 \\
0 & 0 & -1 \\
-1 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4 \\
i_5 \\
i_6 \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

**Independent KVL Equations**

\[
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
v_5 \\
v_6 \\
\end{bmatrix}
= 
\begin{bmatrix}
1 & -1 & 0 \\
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 1 & -1 \\
0 & 0 & -1 \\
-1 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2 \\
\hat{e}_3 \\
\hat{e}_4 \\
\hat{e}_5 \\
\hat{e}_6 \\
\end{bmatrix}
\Rightarrow 
\begin{bmatrix}
v_1 = \hat{e}_1 - \hat{e}_2 \\
v_2 = \hat{e}_1 \\
v_3 = -\hat{e}_2 \\
v_4 = \hat{e}_2 - \hat{e}_4 \\
v_5 = -\hat{e}_4 \\
v_6 = -\hat{e}_1 + \hat{e}_4 \\
\end{bmatrix}
\]
Let $G$ be a connected **digraph** with "$n$" nodes and "$b$" branches. Pick any node as the **datum node** and label the remaining nodes arbitrarily from 1 to $n-1$. Label the branches arbitrarily from 1 to $b$.

The **reduced incidence matrix** $A$ of $G$ is an $(n-1) \times b$ matrix where each row $j$ corresponds to node $(j)$, and each column $k$, corresponds to branch $k$, and where the $jk$th element $a_{jk}$ of $A$ is constructed as follow:

$$a_{jk} = \begin{cases} 1 & \text{, if branch } k \text{ leaves node } (j) \\ -1 & \text{, if branch } k \text{ enters node } (j) \\ 0 & \text{, if branch } k \text{ is not connected to node } (j) \end{cases}$$
How to write An Independent System of KCL and KVL Equations

Let $\mathcal{N}$ be any connected circuit and let the digraph $\mathcal{G}$ associated with $\mathcal{N}$ contain “$n$” nodes and “$b$” branches. Choose an arbitrary datum node and define the associated node-to-datum voltage vector $\mathbf{e}$, the branch voltage vector $\mathbf{V}$, and the branch current vector $\mathbf{i}$. Then we have the following system of independent KCL and KVL equations.

$$(n-1) \text{ Independent KCL Equations :}$$

$$\mathbf{A} \mathbf{i} = \mathbf{0}$$

$b$ Independent KVL Equations :

$$\mathbf{v} = \mathbf{A}^T \mathbf{e}$$