Discussion Notes 7: Frequency Response:

1) Review and practice dB calculations:

Ratio of voltages in decibels: \(20 \log |V_{out}/V_{in}| \) dB

<table>
<thead>
<tr>
<th>(V_{out}/V_{in})</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>-6dB</td>
</tr>
<tr>
<td>2</td>
<td>6dB</td>
</tr>
<tr>
<td>10</td>
<td>20dB</td>
</tr>
<tr>
<td>40</td>
<td>32dB</td>
</tr>
</tbody>
</table>

Explain how to easily calculate dB of 40 knowing dB of 10 and 2.

2) Finding Voltage Transfer function:

In the s-domain analysis, the impedance of a capacitor is defined as: \(1/sC\) (with \(s = jw\) for physical frequencies) and \(sL\) for an inductor. Usual circuit analysis techniques are then used to derive the voltage transfer function \(T(s) = V_{o}(s)/V_{i}(s)\).

Example:
Find the voltage transfer function \(T(s) = V_{o}(s)/V_{i}(s)\) for the circuit below:

Use voltage division to express \(V_{out}\) in terms of \(V_{in}\) and get:
\(T(s) = [1/(CR1)] / [s + 1/C(R1//R2)]\)

3) Poles and Zeros/ Bode Plots:

The transfer function can be expressed in the form:
\[H(jw) = \frac{A(jw\tau_0)(1 + jw\tau_2)(1 + jw\tau_4)...(1 + jw\tau_{2n})}{(1 + jw\tau_1)(1 + jw\tau_3)...(1 + jw\tau_{2n-1})}\]
where the \( \tau_i \) correspond to the breakpoints in the Bode plot, called zeros in the numerator and poles in the denominator. The breakpoint frequencies can be found from taking the reciprocal of \( \tau_i \).

Note: The transfer function has a zero at \( jw = \infty \).

We are interested in the magnitude and phase of the transfer function:
For \( V_{out}/V_{in} = 1/(1+jwRC) \)
Magnitude: \( |V_{out}/V_{in}| = \left[ 1/(1+(wRC)^2) \right]^{1/2} \)
Phase = \( \tan^{-1}(-wRC) \)

Example:

An amplifier has the voltage transfer function:

\[
H(jw) = 10jw/ \left( (1+jw/10^2)*(1+ jw/10^5) \right)
\]

Question:

a) Find the poles and zeros and sketch the magnitude of the gain versus frequency.

b) Find the Bode plot for the phase

Explain the following when giving the solution:

For zeros

<table>
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<tr>
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<th>Phase</th>
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<tr>
<td>( w \ll wo )</td>
<td>Little effect on magnitude</td>
<td>0°</td>
</tr>
<tr>
<td>( w = wo )</td>
<td>breakpoint in plot</td>
<td>+45°</td>
</tr>
<tr>
<td>( w \gg wo )</td>
<td>Slope = +20dB/decade</td>
<td>+90° after 1 decade</td>
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The term \( jw\tau_o \) contributes also +90 to the phase.

For poles

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<td>-45°</td>
</tr>
<tr>
<td>( w \gg wo )</td>
<td>Slope = -20dB/decade</td>
<td>-90° after 1 decade</td>
</tr>
</tbody>
</table>

Solution:

a) The zeros: \( w = 0 \) and \( w = \infty \)
The poles: \( w = 10^2 \) rad/s and \( w = 10^5 \) rad/s

b) The zero at \( w = 0 \) gives rise to a constant +90 degree phase function. The pole at \( w = 10^2 \) rad/s gives rise to the phase function \( -\tan^{-1}(w/10^2) \) and \( w = 10^5 \) rad/s to the function \( -\tan^{-1}(w/10^2) \)

4) 3dB error/ pole frequency:
For $|V_{out}/V_{in}| = |1/(1+jwRC)|$. When $w = 1/RC$, $|V_{out}/V_{in}| = |1/(1+j)| = 1/\sqrt{2} = 0.707$ → -3dB.

The Bode plot has a -3dB difference from the horizontal line approximation.

Draw sample Bode plot on the board to show the 3dB difference.