1. (25 pts) Lag compensation.
For open loop plant \( G(s) \)
\[
G(s) = \frac{24(s + 4)}{(s + 2)(s + 6)(s + 8)}
\]
with unity gain feedback, a) Use MATLAB to find a lag compensator \( D(s) = \frac{k_{lag}}{\alpha \frac{s+1/T}{s+1/\alpha T}} \) with appropriate gain \( k_{lag} \) so as to obtain phase margin of 45° and static error less than 1% for a step input.
b) Use a MATLAB Bode plot to verify gain and phase margin for the lag compensated system.
c) Using MATLAB, plot the steady state error for a step input for the lag compensated system.
d) Using MATLAB, plot the steady state error for a step input for 99\( G(s) \) with unity gain feedback, and compare to the response in c).

2. (20 pts) State Space
For the following transfer function,
\[
\frac{Y(s)}{U(s)} = \frac{(s + 1)(s + 4)(s + 10)}{(s + 3)(s + 2)^2}
\]
give state space description (with state variable \( x \)) in the following forms (see Nise 12.4 and Fig 5.31):
a) phase variable form (Nise 12.4).
b) controllable canonical form (Nise p. 265)
c) cascade form (Nise p. 265)
d) parallel form (Nise p. 265)

3. (10 pts) State space
Convert the following differential equation to state space in controllable canonical form (with state variable \( x \)), where \( u(t) \) is the input and \( y(t) \) is the output.
\[
\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = u + 3\frac{du}{dt} + 2\frac{d^2u}{dt^2}
\]
4. (20 pts) State space to transfer function
Given
\[
\dot{x} = Ax + Bu = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -6 & -12 & -9 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \quad \text{and} \quad y = \begin{bmatrix} -3 & -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u(t)
\]
a) Draw a block diagram for this system with input \( u(t) \) and output \( y(t) \) using integrators, addition junctions, and static gains.
b) Starting from the block diagram, determine the transfer function \( \frac{Y(s)}{U(s)} \).

5. (25 pts) State space solutions
Given the following
\[
\dot{x} = Ax + Bu = \begin{bmatrix} -5 & -6 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ u(t) \end{bmatrix}, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \end{bmatrix}, \quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]
Solve for the state transition matrix \( e^{At} \), \( x(t) \), and \( y(t) \), where \( u(t) \) is the unit step. (Hint: use Laplace transform.)