Quantum Well Gain

QW Material Gain:

\[ g(\hbar \omega) = C_0 \left| \hat{e} \cdot \hat{P}_{cv} \right|^2 \rho^{2d}(\hbar \omega) \frac{f_g(\hbar \omega)}{f_e(\hbar \omega)} \]

\[ C_0 = \frac{\pi e^2}{n_\text{e} c \epsilon_0 m_0^2 \omega} \]

\[ \left| \hat{e} \cdot \hat{P}_{cv} \right|^2 \approx \frac{m_0}{6} E_p \]

\[ \rho^{2d}(E) = \frac{m^*}{\pi \hbar^2 L_z} \sum_{n=1}^{\infty} \delta(E - E_{cn}) \]
Advantages of Quantum Well Lasers

(1) Low threshold current density:
Compare fundamental material property
\[ J_{th}^{bulk} = \frac{qN_{tr}^{bulk}}{\tau} d_{active} \]
\[ J_{th}^{QW} = \frac{qN_{tr}^{QW}}{\tau} L_z \]
Since \( N_{tr}^{bulk} \approx N_{tr}^{QW} \) \[ \frac{J_{th}^{QW}}{J_{th}^{bulk}} = \frac{L_z}{d_{active}} \approx 10 \text{ nm} \]
\[ 100 \text{ nm} \approx 10\% \]

(2) Higher differential gain \( \rightarrow \) Larger bandwidth:
Resonance frequency:
\[ \omega_r = \frac{\sqrt{\gamma a S}}{\tau_p} \approx \sqrt{a} = \sqrt{\frac{\partial g}{\partial N}} \]

(3) Lower chirp:
Smaller wavelength shift when the laser is directly modulated

Transparency Carrier Concentration in Bulk

At transparency: \( F_C - F_v = E_C - E_v \)
or \( F_C - E_C = F_v - E_v \)
Let \( \Delta = \frac{F_C - E_C}{k_B T} = \frac{F_v - E_v}{k_B T} \)

Electron concentration: \( \therefore F_C > E_C \)
\[ N = 2 \left( \frac{\pi m_e^* k_B T}{2\pi^2 h^2} \right)^{3/2} \frac{4}{3\sqrt{\pi}} \left( \frac{F_C - E_C}{k_B T} \right)^{3/2} = N_C \cdot \frac{4}{3\sqrt{\pi}} \Delta^{3/2} \]

Hole concentration: \( \therefore F_v > E_{h1} \)
\[ P = 2 \left( \frac{\pi m_h^* k_B T}{2\pi^2 h^2} \right)^{3/2} e^{\frac{F_v - E_v}{k_B T}} = N_v e^{-\Delta} \]
\[ N = P \Rightarrow \frac{4}{3\sqrt{\pi}} \Delta^{3/2} = \left( \frac{m_h^*}{m_e^*} \right)^{3/2} e^{-\Delta} \Rightarrow \text{Solve } \Delta \]

For GaAs \( (m_e^* = 0.067m_0, m_h^* = 0.5m_0) \)
\( \Delta = 2.15, \quad N = 9 \times 10^{17} \text{ cm}^{-3} \)
**Transparency Carrier Concentration in QW**

At transparency: \[ F_C - F_V = E_{e_1} - E_{h_1} \]

or \[ F_C - E_{e_1} = F_V - E_{h_1} \]

Let \[ \Delta = \frac{F_C - E_{e_1}}{k_B T} = \frac{F_V - E_{h_1}}{k_B T} \]

Electron concentration: \[ \therefore F_C > E_{e_1} \]

\[ N = \frac{m_e^* k_B T}{\pi h^2 L_z} \left( \frac{F_C - E_{e_1}}{k_B T} \right) = N_C^{2d} \cdot \Delta \]

Hole concentration: \[ \therefore F_V > E_{h_1} \]

\[ P = \frac{m_h^* k_B T}{\pi h^2 L_z} e^{\frac{F_V - E_{h_1}}{k_B T}} = N_V^{2d} e^{-\Delta} \]

\[ N = P \Rightarrow \Delta = \frac{m_h^*}{m_e^*} e^{-\Delta} \Rightarrow \text{Solve } \Delta \]

For GaAs \((m_e^* = 0.067 m_0, m_h^* = 0.5 m_0)\)

\[ \Delta = 1.56, \quad N = N_C^{2d} \cdot \Delta = 10^{18} \text{ cm}^{-3} \]

Note: \(N\) is independent of \(L_z\)

---

**Reduction of Lasing Threshold Current Density by Lowering Valence Band Effective Mass**

Bernard-Duraffourg Condition:

\[ F_C - F_V \geq h \omega \geq E_{e_1} - E_{h_1} \]

Ordinary Semiconductor

High transparency carrier concentration

\[ m_h^* \approx 6m_e^* \]

Ideal Semiconductor

Low transparency carrier concentration

\[ m_h^* \approx m_e^* \]

Bernard-Duraffourg Condition in Quantum Well

Bernard-Duraffourg Condition:

\[ F_C - F_V = E - E_{h1} \]

(a) \( m_h^* > m_e^* \) (as in most semiconductors)

\[ F_V > E_{h1} \]

\[ F_C \gg E_{e1} \]

\[ N_{tr} = \rho_c^{2d} (F_C - E_{e1}) = \frac{m_e^*}{\pi \hbar^2 L_z} (F_C - E_{e1}) \]

Large \( N_{tr} \rightarrow \) High threshold current

(b) \( m_h^* = m_e^* \) (Ideal semiconductor)

\[ F_V = E_{h1} \]

\[ F_C = E_{e1} \]

\[ N_{tr} = \frac{m_e^*}{\pi \hbar^2 L_z} \int_{E_{e1}}^{\infty} f_c(E)dE \quad \text{is low} \]

Transparency Carrier Concentration for Ordinary Semiconductor

(b) Ideal Semiconductor

\[ m_h^* = m_e^* \quad \Rightarrow \quad F_V = E_{h1} \quad F_C = E_{e1} \]

\[ N_{tr} = \frac{m_e^*}{\pi \hbar^2 L_z} \int_{E_{e1}}^{\infty} \frac{1}{E-E_{e1}}dE \]

\[ = \frac{k_B T m_e^*}{\pi \hbar^2 L_z} \int_{0}^{\infty} \frac{1}{1 + e^x}dx \]

\[ = \frac{k_B T m_e^*}{\pi \hbar^2 L_z} \left( -\ln(1 + e^{-x}) \right)_{0}^{\infty} \]

\[ = \frac{k_B T m_e^*}{\pi \hbar^2 L_z} \ln 2 \]

For \( m_e^* = 0.067 m_0 \)

\[ N_{tr} \approx 4.6 \times 10^{17} \text{ cm}^{-3} \]
Transparency Carrier Concentration for Ordinary Semiconductor

(a) Ordinary Semiconductor

\[ N_v = \rho_e^{2d} (F_C - E_{e1}) = \frac{m_e^*}{\pi \hbar^2 L_z} \Delta \]

To estimate \( \Delta \), note that \( N = P \)

\[ P = N_v^{2d} e^{\frac{-\Delta}{k_B T}} = \frac{k_B T m_h^*}{\pi \hbar^2 L_z} e^{\frac{-\Delta}{k_B T}} \]

\[ N = P \Rightarrow e^{\frac{-\Delta}{k_B T}} = \frac{\Delta}{k_B T} \frac{m_e^*}{m_h^*} \]

Transparency Condition:

\[ F_C - F_v = E_{e1} - E_{h1} \]

For \( m_h^* \approx 6m_e^* \) (in 1.55 \( \mu m \) laser),
\[ \Delta = 1.43 k_B T \]

\[ N_v = 1.43 \frac{k_B T m_e^*}{\pi \hbar^2 L_z} \]

Effective Mass Asymmetry Penalty

\[ \frac{N_{tr}^{\text{Ordinary}}}{N_{tr}^{\text{Ideal}}} = \frac{1.43}{\ln 2} = 2 \]

Threshold current density reduction is more than a factor of 2:

\[ J_{th} = J_{\text{nonrad}} + J_{\text{rad}} + J_{\text{Auger}} \]

\[ \frac{J_{th}}{q d} = AN + BN^2 + CN^3 = \frac{N}{\tau} + BN^2 + CN^3 \]

\( \tau \): Shockley-Read-Hall nonradiative recombination lifetime

\( J_{\text{Auger}} \) is greatly reduced when \( N \) is lowered

(1) \( N^3 \) is reduced by 8x

(2) \( C \) is also reduced due to band structure change by strain
**Bandgap-vs-Lattice Constant of Common III-V Semiconductors**

![Graph showing the relationship between bandgap energy and lattice constant for various III-V semiconductors.](image)

Fig. 7.6. Bandgap energy and lattice constant of various III-V semiconductors at room temperature (adopted from Tien, 1988).

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**Qualitative Band Energy Shifts Under Strain**

- **Biaxial Strain**
  - C
  - LH, HH
  - Tensile Strain
  - Compressive Strain

- **Hydrostatic Strain**
  - C
  - $\delta E_C$

- **Hydrostatic Strain**
  - LH, HH, SO

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EE232 Lecture 14-11

Prof. Ming Wu
**Strain and Stress**

\[ \varepsilon = \varepsilon_{xx} = \varepsilon_{yy} = \frac{a_0 - a(x)}{a_0} \]

- \(a_0\): lattice constant of InP
- \(\varepsilon < 0\): compressive strain
- \(\varepsilon > 0\): tensile strain

\[ \varepsilon_{\perp} = \varepsilon_{zz} = -2 \frac{C_{12}}{C_{11}} \varepsilon \]

- \(C_y\): Compliance Tensor
- \(C_{12} \approx 0.5C_{11}\)

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz}
\end{bmatrix}
= \begin{bmatrix}
C_{11} & C_{12} & C_{12} \\
C_{12} & C_{11} & C_{12} \\
C_{12} & C_{12} & C_{11}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz}
\end{bmatrix}
\]

Biaxial stress:
\[ \sigma_{xx} = \sigma_{yy} = \sigma \]
\[ \sigma_{zz} = 0 \]
\[ \Rightarrow C_{12} \varepsilon_{xx} + C_{12} \varepsilon_{yy} + C_{11} \varepsilon_{zz} = 0 \]
\[ \varepsilon_{zz} = -2 \frac{C_{12}}{C_{11}} \varepsilon \]

---

**Band Edge Shift**

\[
E_C = E_g(x) + \delta E_C
\]
\[
E_{HH} = -P_\varepsilon - Q_\varepsilon
\]
\[
E_{LH} = -P_\varepsilon + Q_\varepsilon
\]

\[
\delta E_C = a_C (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = 2a_C \left(1 - \frac{C_{12}}{C_{11}}\right) \varepsilon
\]

\[
P_\varepsilon = -a_\nu (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = -2a_\nu \left(1 - \frac{C_{12}}{C_{11}}\right) \varepsilon
\]

\[
Q_\varepsilon = -b \left(\frac{\varepsilon_{xx} + \varepsilon_{yy} - \varepsilon_{zz}}{2}\right) = -b \left(1 + 2 \frac{C_{12}}{C_{11}}\right) \varepsilon
\]

- \(a = a_C - a_\nu\): hydrostatic potential
- \(b\): shear potential
## Strain Parameters in III-V Semiconductors

<table>
<thead>
<tr>
<th>Material</th>
<th>Lattice Constant $a(\text{Å})$</th>
<th>Deformation Potentials $(\text{eV})$</th>
<th>Elastic Moduli $(10^{11} \text{ dyn/cm}^2)$</th>
<th>$(10^{-6} \text{ eV/bar})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaAs</td>
<td>5.6533</td>
<td>$-8.68$ $-1.7$ $-4.55$</td>
<td>11.88 5.38 5.94</td>
<td>11.5 0.34</td>
</tr>
<tr>
<td>InAs</td>
<td>6.0583</td>
<td>$-5.79$ $-1.8$ $-3.6$</td>
<td>8.329 4.526 3.959</td>
<td>10.0 0.371</td>
</tr>
<tr>
<td>AlAs*</td>
<td>5.6611</td>
<td>$-7.96$ $-1.5$ $-3.4$</td>
<td>12.02 5.70 5.89</td>
<td>10.2 0.30</td>
</tr>
<tr>
<td>GaP*</td>
<td>5.4512</td>
<td>$-9.76$ $-1.5$ $-4.6$</td>
<td>14.12 6.253 7.047</td>
<td>11.0 0.10</td>
</tr>
<tr>
<td>InP</td>
<td>5.8688</td>
<td>$-6.16$ $-2.0$ $-5.0$</td>
<td>10.22 5.76 4.60</td>
<td>8.5 0.10</td>
</tr>
<tr>
<td>AlP*</td>
<td>5.4635</td>
<td>$-8.38$ $-1.75$ $-4.8$</td>
<td>13.2 6.3 6.15</td>
<td>9.75 0.10</td>
</tr>
<tr>
<td>GaSb</td>
<td>6.0959</td>
<td>$-8.28$ $-1.8$ $-4.6$</td>
<td>8.842 4.026 4.322</td>
<td>14.7 0.8</td>
</tr>
<tr>
<td>InSb</td>
<td>6.4794</td>
<td>$-7.57$ $-2.0$ $-4.8$</td>
<td>6.47 3.65 3.02</td>
<td>16.5 0.98</td>
</tr>
<tr>
<td>AlSb*</td>
<td>6.1355</td>
<td>$2.04$ $-1.35$ $-4.3$</td>
<td>8.769 4.341 4.076</td>
<td>$-3.5$ 0.75</td>
</tr>
</tbody>
</table>

* Indirect gap.

## Band-Edge Profile and Subband Dispersion

(a) A quantum well under a compressive strain

(b) An unstrained quantum well

(c) A quantum well under a tensile strain