EE 232 Lightwave Devices
Lecture 19: p-i-n Photodiodes and Photoconductors

Reading: Chuang, Chap. 15 (2nd Ed)

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P-i-n Photodiode

• Reverse-biased p-i-n junction
• Most of the voltage drop across the i-region, the main absorption region
• High field separates photogenerated electron and hole
• Large bandgap materials are used for P and N if possible
• Fast response
• Low noise
• No gain (quantum efficiency < 100%)
### I-V Curve

Dark current: \( I = I_0 (e^{\frac{qV}{kT}} - 1) \)

Photocurrent: \( I_{ph} = \frac{\eta q P_{opt}}{h\omega} \)

Quantum efficiency: \( \eta = \eta_0 (1 - R)(1 - e^{-\alpha d}) \)

Total current: \( I = I_0 (e^{\frac{qV}{kT}} - 1) + I_{ph} \)

### Absorption Coefficient

- Light intensity decays exponentially in semiconductor:
  \[ I(x) = I_0 e^{-\alpha x} \]
- Direct bandgap semiconductor has a sharp absorption edge
- Si absorbs photons with \( h\nu > E_g = 1.1 \text{ eV} \), but the absorption coefficient is small
  – Sufficient for CCD
- At higher energy (~ 3 eV), absorption coefficient of Si becomes large again, due to direct bandgap transition to higher CB
**Two Types of p-i-n Photodiodes**

Surface-Illuminated p-i-n
\[ \eta = \eta_i (1 - R)(1 - e^{-\alpha d}) \]
- \( \eta_i \): internal quantum efficiency
- \( R \): reflectivity
- \( d \): absorption layer thickness

Waveguide p-i-n
\[ \eta = \eta_i (1 - R)(1 - e^{-\Gamma \alpha L}) \]
- \( \eta_i \): internal quantum efficiency
- \( R \): reflectivity
- \( \Gamma \): confinement factor
- \( L \): length of waveguide PD

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**Ramo’s Theorem**

The current caused in external circuit by a moving charge \( q \) moving at a velocity \( v(t) \) in a parallel plate with a separation of \( d \) and a voltage bias of \( V \) is
\[ i(t) = \frac{q v(t)}{d} \]

Proof:
Work done on the charge:
\[ W = \text{Force} \times \text{Displacement} = q E dx = q \frac{V}{d} dx \]
Work provided by power supply:
\[ W = i(t) V dt \]
\[ \Rightarrow \]
\[ i(t) V dt = q \frac{V}{d} dx \]
\[ i(t) = \frac{q dx}{d} \frac{dx}{dt} = \frac{q v(t)}{d} \]
Response of One Photogenerated Electron-Hole Pair

Electron current ends when the last electron generated near P-side reaches N-electrode: \( t = \frac{d}{v_e} \)

Hole current ends when the last hole generated near N-side reaches P-electrode: \( t = \frac{d}{v_h} \)

Hole is usually slower \( \rightarrow \) A conservative estimate of the transit time: \( \tau = \frac{d}{v_h} \)

Total charge generated:
\[
Q = \int_0^t i_e(t)\,dt + \int_0^t i_h(t)\,dt
\]
\[
= \frac{q v_e}{d} (d - x) + \frac{q v_h}{d} x = q
\]
One absorbed photon \( \rightarrow \) one charge detected

Transit Time
Total Response Time of p-i-n

(1) RC time:
\[ \tau_{RC} = RC = R \frac{E_A}{d} \]  
(A: area of p-i-n)

(2) Transit time:
\[ \tau_t = \frac{d}{v_b} \]

Total response time:
\[ \tau = \tau_{RC} + \tau_t \]
\[ f_{3db} = \frac{1}{2\pi\tau} \]

Absorption layer thickness for optimum frequency response:
\[ \tau = \tau_{RC} + \tau_t = \frac{R E_A}{d} + \frac{d}{v_b} \]
\[ \tau \geq 2 \sqrt{\left(\frac{R E_A}{d}\right)\left(\frac{d}{v_b}\right)} \]

Optimum bandwidth occurs when
\[ \frac{R E A}{d} = \frac{d}{v_b} \]
\[ d_{\text{optimum}} = \sqrt{R E A v_b} \]

More Rigorous Analysis of p-i-n Response Time

Small-signal analysis: assume the input light is modulated at frequency \( \omega \),
the photocurrent is proportional to
\[ |i(t)| \propto \left| \frac{1}{1 + j \omega RC} \sin^2 \left( \frac{\omega \tau}{2} \right) \right| \]

The first term is single-pole response from RC,
while the second term is the phase delay due to transit time response.
Comparison of Numeric Examples

Example:
\[ \tau_{RC} = 14.4 \text{ ps} \]
\[ \tau_t = 20 \text{ ps} \]
\[ f_{3dB} = \frac{1}{2\pi} \frac{1}{\tau_{RC} + \tau_t} = 4.6 \text{ GHz} \]

\[ |i(t)| \propto \left[ \frac{1}{1 + j\omega RC} \sin^2 \left( \frac{\omega \tau_t}{2} \right) \right] = |H(\omega)| \]

Solving \[ |H(\omega)| = \frac{1}{\sqrt{2}} \cdot f_{3dB} = 9.7 \text{ GHz} \]

The discrepancy is smaller when RC dominates, and larger when transit time dominates.

(Transit time response has a sharp drop-off).

Bandwidth-Efficiency Product

(1) For surface-illuminated p-i-n (assume AR coating: R=0%), in the extreme of thin absorbing layer and transit-time-dominated response:
\[ \eta = \eta_t (1 - e^{-\alpha d}) \approx \eta_t (1 - (1 - \alpha d)) = \eta_t \alpha d \]
\[ f_{3dB} = \frac{1}{2\pi} \frac{v_h}{d} \]

Bandwidth-efficiency product: \[ f_{3dB} \times \eta = \left( \frac{1}{2\pi} \frac{v_h}{d} \right) (\eta_t \alpha d) = \frac{\eta_t \alpha v_h}{2\pi} \]

(2) On the other hand, the efficiency of waveguide p-i-n is
\[ \eta = \eta_t (1 - e^{-\alpha L}) \approx \eta_t \Gamma \alpha L \]

RC-limited bandwidth: \[ f_{3dB} = \frac{1}{2\pi} \frac{d}{R \varepsilon L_w} \]

Bandwidth-efficiency product: \[ f_{3dB} \times \eta = \left( \frac{1}{2\pi} \frac{d}{R \varepsilon L_w} \right) (\eta_t \Gamma \alpha L) = \frac{\eta_t \Gamma \alpha v_{h}}{2\pi R \varepsilon w} \]

⇒ In general, there is a bandwidth-efficiency trade-off
Photoconductors

Dark current:

\[ J_0 = \sigma_0 E = (n_0 q \mu_n + p_0 q \mu_p) E \]

Light illumination generates electron-hole pairs, increasing the conductivity:

\[ \frac{d\delta n}{dt} = G_0 - \frac{\delta n}{\tau_n} \]

Steady state: \( d/dt \rightarrow 0 \)

\[ \delta n = G_0 \tau_n \]

\[ \Delta J = \delta n \cdot (\mu_n + \mu_p) E \]

Photoconductor requires both contacts to be Ohmic and the semiconductor doping type to be the same.

Photocarrier Generation Rate

Light intensity \( \propto e^{-\alpha x} \)

\[ P_{\text{opt}}(1 - R) = \{1 - e^{-\alpha d}\} \]

\[ G_0 = \eta \frac{P_{\text{opt}}}{h \omega \hbar d} : \text{photocarrier generation rate} \left[ \frac{1}{cm^3 s} \right] \]

\[ \eta = \eta_t (1 - R)(1 - e^{-\alpha d}) \]

\( R \): reflectivity of photoconductor surface

\( \alpha \): absorption coefficient

\( d \): absorption length

\( e^{-\alpha d} \): fraction of light remains after absorption length \( d \)
Photoconductive Gain

\[
\Delta l = h \omega \Delta l = h \omega \left( G_{\text{opt}} \tau_a \eta \right) \left( \mu_n + \mu_p \right) E \\
\Delta l = \frac{1}{\hbar \omega} \left( \frac{P_{\text{opt}}}{l w d} \right) q \left( \mu_n + \mu_p \right) E \\
\Delta l = \eta \frac{P_{\text{opt}}}{\hbar \omega} \tau_a q \left( \mu_n E \right) = \eta \frac{q}{\hbar \omega} \frac{1}{\tau_a} \frac{1}{d} v_n \\
\tau_i = \frac{d}{v_n} : \text{transit time} \\

\Delta l = \left( \eta \frac{P_{\text{opt}}}{\hbar \omega} \frac{q}{\hbar \omega} \right) \left( \frac{\tau_a}{\tau_i} \right) \\
\text{Photocurrent} \quad \text{Photoconductive Gain}
\]

Analogy to Current Gain in Bipolar Transistor

Current gain in bipolar transistor:

\[
\beta = \frac{I_C}{I_B} \\
\text{The current gain can also be expressed as} \\
\beta = \frac{\tau_{\text{vb}}}{\tau_i} \\
\tau_i : \text{transit time} \\
\tau_{\text{vb}} : \text{carrier recombination lifetime in the base}
\]
**Frequency of Photoconductors**

\[
\frac{dN}{dt} = \eta \frac{P_{opt}}{h\omega lwd} - \frac{N}{\tau_s}
\]

Small signal response:

\[N = N_0 + N_0 e^{j\omega t}\]

\[j\omega N_i = \eta \frac{P_1}{h\omega lwd} - \frac{N_i}{\tau_n}\]

\[N_i = \eta \frac{P_1}{h\omega lwd} \frac{1}{j\omega + 1/\tau_n}\]

\[I_1 = J \times (N_0 g_{p1})/\omega\]

\[\frac{I_1}{P_1} = \left( \eta q \right) \left( \frac{\tau_n}{\tau_1} \right) \frac{1}{\frac{\omega}{\tau_n} + 1}\]

\[= \left( \text{DC Quantum Efficiency} \right) \times \left( \text{Photoconductive Gain} \right) \times \left( \text{Normalized Frequency Response} \right)\]