EE247
Lecture 7

- Automatic on-chip filter tuning (continued from last lecture)
  - Continuous tuning
    - Reference integrator locked to a reference frequency
    - DC tuning of resistive timing element
  - Periodic digitally assisted filter tuning
    - Systems where filter is followed by ADC & DSP, existing hardware can be used to periodically update filter freq. response
- Continuous-time filters
  - Highpass filters
  - Bandpass filters
    - Lowpass to bandpass transformation
    - Example: 6th order bandpass filter
    - Gm-C BP filter using simple diff. pair

Summary last lecture

- Continuous-time filters
  - Opamp MOSFET-RC filters
  - Gm-C filters

- Frequency tuning for continuous-time filters
  - Trimming via fuses or laser
  - Automatic on-chip filter tuning
    - Continuous tuning
      - Utilizing VCF built with replica integrators
      - Use of VCO built with replica integrators
      - Reference integrator locked to reference frequency
Summary
Reference Integrator Locked to Reference Frequency

Tuning error due to gm-cell offset voltage resolved

Advantage over previous schemes:

\[ f_{\text{clk}} \] can be chosen to be at much higher frequencies compared to filter bandwidth \((N > 1)\)

\[ \rightarrow \text{Feedthrough of clock attenuated by filter} \]

DC Tuning of Resistive Timing Element

Tuning circuit \(Gm\) \(\rightarrow\) replica of \(Gm\) used in filter

\(R_{\text{ext}}\) used to lock \(Gm\) to accurate off-chip \(R\)

Feedback forces \(Gm=1/R_{\text{ext}}\)

Issues with DC offset

Account for capacitor variations in this gm-C implementation by trimming

Digitally Assisted Frequency Tuning
Example: Wireless Receiver Baseband Filters

- Systems where filter is followed by ADC & DSP
  - Take advantage of existing digital signal processor capabilities to periodically update the filter critical frequency
  - Filter tuned only at the outset of each data transmission session (offline/periodic tuning) — can be fine tuned during times data is not transmitted

Example: Seventh Order Tunable Low-Pass OpAmp-RC Filter
Digitally Assisted Filter Tuning Concept

Assumptions:
- System allows a period of time for the filter to undergo tuning (e.g. for a wireless transceiver during idle periods)
- An AC (e.g. a sinusoid) signal can be generated on-chip whose amplitude is a function of an on-chip DC source
  - AC signal generator outputs a sinusoid with peak voltage equal to the DC signal source
  - AC Signal Power = 1/2 DC signal power @ the input of the filter

AC signal @ a frequency on the roll-off of the desired filter frequency response (e.g. -3dB frequency)  
\[ V_{AC} = V_{DC} \times \sin \left( 2\pi f_{\text{desired}} \right) \]

Provision can be made during the tuning cycle, the input of the filter is disconnected from the previous stage (e.g. mixer) and connected to:
1. DC source
2. AC source
   under the control of the DSP
Digitally Assisted Filter Tuning Concept

Tuning Cycle:
- Connect the filter input to DC source
- DSP measures the DC power level
- Connect the filter input to AC source (freq. -> desired -3dB freq.)
- DSP measures the AC signal power level

If DC = 4*AC
- Then filter is tuned
- Else if DC > 4*AC
  - Then widen the filter bandwidth & repeat
  - Else narrow the filter bandwidth & repeat

Practical Implementation of Frequency Tuning
AC Signal Generation From DC Source

Clock=high

ClockB=high
Practical Implementation of Frequency Tuning

Effect of Using a Square Waveform

- Input signal chosen to be a square wave due to ease of generation
- Filter input signal comprises a sinusoidal waveform @ the fundamental frequency + its odd harmonics:

\[ V_{in}(t) = \frac{4}{\pi} \sum_{n=1,3,5,...}^{\infty} \frac{\Delta}{n} \sin(\omega_n t) \]

\[ V_{out}(t) = \frac{\Delta}{\pi} \sin(\omega t) \times \frac{1}{\sqrt{2}} \]

Key Point: The filter itself attenuates unwanted odd harmonics
\[ \Rightarrow \text{Inaccuracy incurred by the harmonics negligible} \]
Simplified Frequency Tuning Flowchart

Digitally Assisted Offset Compensation

In cases where the filter DC offset cause significant error in tuning (i.e. high passband gain)
- Offset compensation needed:
  - DC measurement performed in two steps:

\[ V_{out1} = A (\Delta + V_{os}) \]
\[ V_{out2} = A (\Delta - V_{os}) \]

Passband Gain

- DC extracts: Offset component \[ \frac{1}{2} (V_{out1} + V_{out2}) - A \cdot V_{os} \]
- DC component \[ \frac{1}{2} (V_{out1} - V_{out2}) = A \cdot \Delta \]
- DSP substracks \( V_{os} \) from all subsequent AC measurement
Filter Tuning Prototype Diagram

Measured Frequency Response

21 dB

618 kHz

Automatically tuned response

3 dB/div.

Tuning bits varied from all 0's to all 1's

-9 dB

100 kHz

1.3 MHz
### Measured Tuning Characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunable frequency range (nom. process)</td>
<td>370kHz to 1.1MHz</td>
</tr>
<tr>
<td>Variations due to process</td>
<td>±50%</td>
</tr>
<tr>
<td>I/Q bandwidth imbalance</td>
<td>0.1%</td>
</tr>
<tr>
<td>Tuning resolution</td>
<td></td>
</tr>
<tr>
<td>(620kHz frequency range)</td>
<td></td>
</tr>
<tr>
<td>Tuning resolution Measured</td>
<td>3.8%</td>
</tr>
<tr>
<td>Tuning resolution Expected</td>
<td>2-5%</td>
</tr>
<tr>
<td>Tuning time Coarse+Fine</td>
<td>max. 800μsec</td>
</tr>
<tr>
<td>Tuning time Fine only</td>
<td>min. 50μsec</td>
</tr>
<tr>
<td>Memory space required for tuning routine</td>
<td>250 byte</td>
</tr>
</tbody>
</table>
Off-line Digitally Assisted Tuning

• Advantages:
  – No reference signal feedthrough since tuning does not take place during data transmission (off-line)
  – Minimal additional hardware
  – Small amount of programming

• Disadvantages:
  – If acute temperature change during data transmission, filter may slip out of tune!
    • Can add fine tuning cycles during periods of data is not transmitted or received


Summary: Continuous-Time Filter Frequency Tuning

• Trimming
  • Expensive & does not account for temperature and supply etc… variations

• Automatic frequency tuning
  – Continuous tuning
    • Master VCF used in tuning loop
      – Tuning quite accurate
      – Issue \( \rightarrow \) reference signal feedthrough to the filter output
    • Master VCO used in tuning loop
      – Design of reliable & stable VCO challenging
      – Issue \( \rightarrow \) reference signal feedthrough
    • Single integrator in negative feedback loop forces time-constant to be a function of accurate clock frequency
      – More flexibility in choice of reference frequency \( \rightarrow \) less feedthrough issues
    • DC locking of a replica of the integrator to an external resistor
      – DC offset issues & does not account for integrating capacitor variations
  – Periodic digitally assisted tuning
    • Requires digital capability + minimal additional hardware
    • Advantage of no reference signal feedthrough since tuning performed off-line
Integrator Based High-Pass Filters
1st Order

- Conversion of simple high-pass RC filter to integrator-based type by using signal flowgraph technique

\[
\frac{V_o}{V_{in}} = \frac{sRC}{1 + sRC}
\]

1st Order Integrator Based High-Pass Filter

\[
\begin{align*}
V_R &= V_{in} - V_C \\
V_C &= I_C \times \frac{1}{sC} \\
V_o &= V_R \\
I_R &= V_R \times \frac{1}{R} \\
I_C &= I_R
\end{align*}
\]
1st Order Integrator Based High-Pass Filter

SGF

\[ V_o \]
\[ sC \]
\[ R \]
\[ -oVinV \]
\[ 1 \]
\[ -oV\]
\[ inV \]
\[ \int \]
\[ -SGF \]

Note: Addition of an integrator in the feedback path \( \rightarrow \) High pass frequency shaping

Addition of Integrator in Feedback Path

Let us assume flat gain in forward path \( a \)
Effect of addition of an integrator in the feedback path:

\[
\begin{align*}
\frac{V_o}{V_{in}} &= \frac{a}{1+af} \\
\frac{V_o}{V_{in}} &= \frac{a}{1+a/s \tau} = \frac{s \tau}{1+s \tau/a}
\end{align*}
\]

\( \rightarrow \) zero @ DC \(&\) pole @ \( \omega_{pole} = -\frac{a}{\tau} \)

Note: For large forward path gain, \( a \), can implement high pass function with low corner frequency without the need for large value of \( C \) or \( R \)
Addition of an integrator in the feedback path \( \rightarrow \) zero @ DC \& pole @ \( ax \int \)
This technique used for offset cancellation in systems where the low frequency content is not important and thus disposable
**Bandpass Filters**

- Bandpass filters → two cases:
  1. Low $Q$ ($Q < 5$)
     → Combination of lowpass & highpass
  2. High $Q$ or narrow-band ($Q > 5$)
     → Direct implementation

**Narrow-Band Bandpass Filters**

- Narrow-band BP filters → Design based on lowpass prototype
- Same tables used for LPFs are also used for BPFs

\[
\begin{align*}
\Omega_3 & \Rightarrow \Omega_3 - \Omega_1 \\
\Omega_c & \Rightarrow \Omega_c - \Omega_{B1}
\end{align*}
\]
Lowpass to Bandpass Transformation


Lowpass to Bandpass Transformation Table

<table>
<thead>
<tr>
<th>Lowpass filter structures &amp; tables used to derive bandpass filters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q = Q_{\text{filter}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LP</th>
<th>BP</th>
<th>BP Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C'$</td>
<td>$C$</td>
<td>$C = QC' \times \frac{1}{R, \omega_L}$</td>
</tr>
<tr>
<td>$L$</td>
<td>$L$</td>
<td>$L = \frac{1}{QC'} \times \frac{R}{\omega_L}$</td>
</tr>
<tr>
<td>$L'$</td>
<td>$C$</td>
<td>$C = \frac{1}{Q L} \times \frac{1}{R, \omega_L}$</td>
</tr>
</tbody>
</table>

$C'$ & $L'$ are normalized LP values

Lowpass to Bandpass Transformation
Example: 3rd Order LPF → 6th Order BPF

- Each capacitor replaced by parallel L & C
- Each inductor replaced by series L & C

\[ C_1 = Q C'_1 \times \frac{1}{R \omega_0} \]
\[ L_1 = \frac{1}{Q C'_1} \times \frac{R}{\omega_0} \]
\[ C_2 = \frac{1}{Q L_2} \times \frac{1}{R \omega_0} \]
\[ L_2 = Q L_2 \times \frac{R}{\omega_0} \]
\[ C_3 = Q C'_3 \times \frac{1}{R \omega_0} \]
\[ L_3 = \frac{1}{Q C'_3} \times \frac{R}{\omega_0} \]

Where:
- \( C'_1, L'_2, C'_3 \) → Normalized lowpass values
- \( Q \) → Bandpass filter quality factor
- \( \omega_0 \) → Filter center frequency
Lowpass to Bandpass Transformation
Signal Flowgraph

1- Voltages & currents named for all components
2- Use KCL & KVL to derive state space description
3- To have BMFs in the integrator form
   Cap voltage expressed as function of its current
   \( V_C = f(I_C) \)
   Ind current as a function of its voltage \( I_L = f(V_L) \)
4- Use state space description to draw SFG
5- Convert all current nodes to voltage

Signal Flowgraph
6th Order Bandpass Filter

Note: each \( C \) & \( L \) in the original lowpass prototype \( \to \) replaced by a resonator
Substituting the bandpass \( L1, C1, \ldots \) by their normalized lowpass equivalent from page 29
The resulting SFG is:
Signal Flowgraph

6th Order BPF versus 3rd Order LPF

- Note the integrators → different time constants
- Ratio of time constants for two integrator in each resonator → $Q^2$
- Typically, requires high component ratios
- Poor matching
- Desirable to modify SFG so that all integrators have equal time constants for optimum matching.
  - To obtain equal integrator time constant → use node scaling
Signal Flowgraph
6th Order Bandpass Filter

All integrator time-constants \( \rightarrow \) equal
To simplify implementation \( \rightarrow \) choose \( RL=Rs=R^* \)

Let us try to build this bandpass filter using the simple Gm-C structure
Second Order Gm-C Filter
Using Simple Source-Couple Pair Gm-Cell

- Center frequency:
  \[ a_b = \frac{g_m^{M+1,2}}{2 \times C_{int}} \]
- Q function of:
  \[ Q = \frac{g_m^{M+1,2}}{g_m^{M+1,2}} \]

Use this structure for the 1st and the 3rd resonator
Use similar structure w/o M3, M4 for the 2nd resonator
How to couple the resonators?

Coupling of the Resonators
1- Additional Set of Input Devices

Coupling of resonators:
Use additional input source coupled pairs for the highlighted integrators
For example, the middle integrator requires 3 sets of inputs
Example: Coupling of the Resonators
1- Additional Set of Input Devices

- Add one source couple pair for each additional input
- Coupling level \( \rightarrow \) ratio of device widths
- Disadvantage \( \rightarrow \) extra power dissipation

![Coupling Diagram]

Coupling of the Resonators
2- Modify SFG \( \rightarrow \) Bidirectional Coupling Paths

Modified signal flowgraph to have equal coupling between resonators
- In most filter cases \( C_i' = C_i \)
- Example: For a butterworth lowpass filter \( C_i' = C_j' = L \& L_j' = 2 \)
- Assume desired overall bandpass filter \( Q=10 \)
Sixth Order Bandpass Filter Signal Flowgraph

\[ \gamma = \frac{1}{Q\sqrt{2}} \]

Since \( Q=10 \) then:

\[ \gamma = \frac{1}{14} \]

Sixth Order Bandpass Filter Signal Flowgraph

SFG Modification
Sixth Order Bandpass Filter Signal Flowgraph
SFG Modification

For narrow band filters (high Q) where frequencies within the passband are close to \( \omega_0 \), a narrow-band approximation can be used:

Within filter passband:

\[
\left( \frac{\omega}{\omega_0} \right)^2 = 1
\]

\[
\gamma \times \left( \frac{\omega}{s} \right)^2 = \gamma \times \left( \frac{\omega}{j\omega} \right)^2 = -\gamma
\]

The resulting SFG:

Bidirectional coupling paths, can easily be implemented with coupling capacitors \( \rightarrow \) no extra power dissipation
Sixth Order Gm-C Bandpass Filter
Utilizing Simple Source-Coupled Pair Gm-Cell

Parasitic cap. at integrator output, if unaccounted for, will result in inaccuracy in $\gamma$

$$\gamma = \frac{C_k}{2 \times C_{int} g + C_k}$$

$$C_k = \frac{2 \times C_{int} g}{1 - \gamma}$$

$$\gamma = \frac{1}{14}$$

$$\rightarrow C_k = \frac{2}{13} C_{int} g$$
Simplest Form of CMOS Gm-Cell
Nonidealities

- DC gain (integrator Q)
  \[ a = \frac{g_{m1,2}}{sC_{M1,2} + sC_{load}} \]
  \[ a = \frac{2L}{\theta(V_{gs} - V_{th})_{M1,2}} \]

- Where \( a \) denotes DC gain & \( \theta \) is related to channel length modulation by:
  \[ \lambda = \frac{\theta}{L} \]
- Seems no extra poles!

CMOS Gm-Cell High-Frequency Poles

- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles
CMOS Gm-Cell High-Frequency Poles

\[ p_{\text{effective}} = 2.5 \omega_1^{M,2} \]

\[ \omega_1^{M,2} = \frac{g_{M,2}}{2 \cdot \beta \cdot \alpha \cdot W \cdot L} = \frac{\mu (V_{gs} - V_{th})}{2 L^2} \]

- Distributed nature of gate capacitance & channel resistance results in an effective pole at 2.5 times input device cut-off frequency

Simple Gm-Cell Quality Factor

\[ a = \frac{2L}{\theta (V_{gs} - V_{th})_{M,2}} \]

\[ p_{\text{effective}} = \frac{15 \mu (V_{gs} - V_{th})_{M,2}}{4} \]

\[ Q_{\text{real}}^{\int \text{g}} = \frac{1}{\frac{1}{a} - \alpha_{th} \sum_{l=2}^{\infty} \frac{1}{l}} \]

\[ Q_{\text{intg.}}^{\int \text{g}} = \frac{1}{\frac{\theta (V_{gs} - V_{th})_{M,2}}{2L}} - \frac{4 \alpha_{th} L^2}{13 \mu (V_{gs} - V_{th})_{M,2}} \]

- Note that phase lead associated with DC gain is inversely prop. to L
- Phase lag due to high-freq. poles directly prop. to L
  \[ \Rightarrow \] For a given \( \Omega_0 \) there exists an optimum \( L \) which cancel the lead/lag phase error resulting in high integrator \( Q \)
Simple Gm-Cell Channel Length for Optimum Integrator Quality Factor

\[ L_{\text{opt}} \approx \left( \frac{\mu V_{gs th}^2}{\omega_b} \right)^{1/3} \]

- Optimum channel length computed based on process parameters (could vary from process to process)

Source-Coupled Pair CMOS Gm-Cell Transconductance

For a source-coupled pair the differential output current (\( \Delta I_d \)) as a function of the input voltage (\( \Delta V_i \)):

\[ \Delta I_d = I_{si} \left[ \frac{\Delta V_i}{(V_{gs th})_{M1,2}} \right] \left[ 1 - \frac{\Delta V_i}{(V_{gs th})_{M1,2}} \right]^{1/2} \]

Note: For small \( \frac{\Delta V_i}{(V_{gs th})_{M1,2}} \) \( \Delta I_d = g_{M1M2} \Delta V_i \)

Note: As \( \Delta V_i \) increases \( \frac{\Delta I_d}{\Delta V_i} \) or the effective transconductance decreases

\[ \Delta V_i = V_{i1} - V_{i2} \]

\[ \Delta I_d = I_{d1} - I_{d2} \]
Source-Coupled Pair CMOS Gm-Cell Linearity

- Large signal \( G_m \) drops as input voltage increases
  \( \rightarrow \) Gives rise to nonlinearity

Measure of Linearity

\[
V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \ldots
\]

\[
HD_3 = \frac{\text{amplitude} 3\text{rd harmonic dist. comp.}}{\text{amplitude fundamental}}
\]

\[
= \frac{1}{4} \frac{\alpha_1}{\alpha_i} V_{in}^3 + \ldots
\]

\[
IM_3 = \frac{\text{amplitude} 3\text{rd order IM comp.}}{\text{amplitude fundamental}}
\]

\[
= \frac{3}{8} \frac{\alpha_1}{\alpha_i} V_{in}^3 + \frac{25}{8} \frac{\alpha_1}{\alpha_i} V_{in}^4 + \ldots
\]
Source-Coupled Pair Gm-Cell Linearity

\[ \Delta I_d = I_{ss} \left[ \frac{\Delta V_i}{4(V_{gs}-V_{th})_{M1,2}} \right] \left[ 1 - 2^\frac{1}{2} \left( \frac{\Delta V_i}{4(V_{gs}-V_{th})_{M1,2}} \right) \right]^{1/2} \] (1)

\[ \Delta I_d = a_1 \times \Delta V_i + a_2 \times \Delta V_i^2 + a_3 \times \Delta V_i^3 + \ldots \ldots \ldots \]

Series expansion used in (1)

\[ a_1 = \frac{I_{ss}}{(V_{gs}-V_{th})_{M1,2}} \quad \text{&} \quad a_2 = 0 \]

\[ a_3 = -\frac{8I_{ss}}{V_{gs}-V_{th} \ M1,2} \quad \text{&} \quad a_4 = 0 \]

\[ a_5 = -\frac{128I_{ss}}{V_{gs}-V_{th} \ M1,2} \quad \text{&} \quad a_6 = 0 \]

Linearity of the Source-Coupled Pair CMOS Gm-Cell

\[ IM_3 = \frac{3a_1}{4a_1} \Delta V_i^2 + \frac{25a_3}{8a_1} \Delta V_i^4 + \ldots \ldots \ldots \]

Substituting for \( a_1, a_3, \ldots \)

\[ IM_3 = \frac{3}{32} \left( \frac{\Delta V_i}{V_{gs}-V_{th}} \right)^2 + \frac{25}{1024} \left( \frac{\Delta V_i}{V_{gs}-V_{th}} \right)^4 + \ldots \ldots \ldots \]

\[ \hat{v}_{i_{\text{max}}} = 4(V_{gs}-V_{th}) \times \sqrt{\frac{2}{3} \times IM_3} \]

\[ IM_3 = 1\% \quad \text{&} \quad (V_{gs}-V_{th}) = IV \Rightarrow \hat{v}_{i_{\text{rms}}} = 230mV \]

- Key point: Max. signal handling capability function of gate-overdrive voltage
Simplest Form of CMOS Gm Cell

Disadvantages

• Max. signal handling capability function of gate-overdrive
  \[ IM_{ss} \propto (V_{GS} - V_{th})^2 \]

• Critical freq. is also a function of gate-overdrive
  \[
  \omega_0 = \frac{s_M \omega}{2 \times C_{int} g_m}
  \]
  since \[ s_m = \mu C \frac{W}{L} (V_{GS} - V_{th}) \]
  then \[ \omega_0 \propto (V_{GS} - V_{th}) \]

→ Filter tuning affects max. signal handling capability!

Simplest Form of CMOS Gm Cell

Removing Dependence of Maximum Signal Handling Capability on Tuning

• Can overcome problem of max. signal handling capability being a function of tuning by providing tuning through:
  - Coarse tuning via switching in/out binary-weighted cross-coupled pairs
  - Try to keep gate overdrive voltage constant
  - Fine tuning through varying current sources

→ Dynamic range dependence on tuning removed (to 1st order)

Ref: R. Castello, J. Bietti, F. Svelto, “High-Frequency Analog Filters in Deep Submicron Technology”
Dynamic Range for Source-Coupled Pair Based Filter

\[ IM_S = 1\% \text{ and } (V_{GS} - V_{th}) = W \Rightarrow V_{in}^{\text{rms}} = 230mV \]

- Minimum detectable signal determined by total noise voltage
- It can be shown for the 6th order Butterworth bandpass filter fundamental noise contribution is given by:

\[
\sqrt{V_n^2} \approx \sqrt{3 Q \frac{k T}{C_{int}}} \\
\text{Assuming } Q = 10 \quad C_{int} = 5pF \\
\frac{V_{rms_{\text{noise}}}}{V_{rms_{\text{max}}}} = 160\mu V \\
since \frac{V_{rms}}{V_{max}} = 230mV \\
\text{Dynamic Range} = 20\log \frac{230 \times 10^{-3}}{160 \times 10^{-6}} \approx 63dB
\]

---

Improving the Max. Signal Handling Capability of the Source-Coupled Pair Gm-Cell

- 2nd source-coupled pair added to subtract current proportional to nonlinear component associated with the main SCP

\[
\frac{I_{ss1}}{I_{ss3}} = b \quad \text{and} \quad \frac{(V_{gs} - V_{th})_{M1,2}}{(V_{gs} - V_{th})_{M3,4}} = a \\
\frac{W}{L}_{M1,2} = \frac{b}{a^2} \\
\frac{W}{L}_{M3,4}
\]
Improving the Max. Signal Handling Capability of the Source-Coupled Pair Gm


• Improves maximum signal handling capability by about 12dB
  \[ \Delta V_{th} \left( \frac{G_m}{V_{GS}-V_{th}} \right) \]

\[ \rightarrow \text{Dynamic range theoretically improved to } 63+12=75\text{dB} \]
Simplest Form of CMOS Gm-Cell

**Pros**
- Capable of very high frequency performance (highest?)
- Simple design

**Cons**
- Tuning affects power dissipation
- Tuning affects max. signal handling capability (can overcome)
- Limited linearity (possible to improve)


Gm-Cell

Source-Coupled Pair with Degeneration

\[ I_d = \frac{\mu C_{ox} W}{2} \left( 2(V_{gs} - V_{th})V_{ds} - V_{ds}^2 \right) \]

\[ g_{ds} = \frac{\partial I_d}{\partial V_{ds}} = \frac{\mu C_{ox} W}{L} \left( V_{gs} - V_{th} \right) \]

\[ g_{eff} = \frac{1}{g_{ds}} + \frac{2}{g_{m} M^{1.2}} \]

for \( g_m \gg g_{ds} \)

\[ g_{eff} = g_{ds} M^3 \]

M3 operating in triode mode → source degeneration→ determines overall gm
Provides tuning through varing \( V_c \)
Gm-Cell
Source-Coupled Pair with Degeneration

• Pros
  – Moderate linearity
  – Continuous tuning provided by \( V_c \)
  – Tuning does not affect power dissipation

• Cons
  – Extra poles associated with the source of \( M_{1,2} \)
  – Low frequency applications only


---

BiCMOS Gm-Cell
Example

• MOSFET in triode mode:

\[
I_d = \frac{\mu C_{ox} W}{2} \left[ 2(V_{gs} - V_{th})V_{ds} - V_{ds}^2 \right]
\]

• Note that if \( V_{ds} \) is kept constant:

\[
\frac{\partial I_d}{\partial V_{gs}} = \frac{\mu C_{ox} W}{L} V_{ds}
\]

• Linearity performance \( \Rightarrow \) keep \( g_m \) constant \( \Rightarrow \) function of how constant \( V_{ds} \) can be held
  – Need to minimize Gain @ Node X

\[
A_X = \frac{g_{m1}}{s_{m1}} \frac{B1}{B1}
\]

• Since for a given current, \( g_m \) of BJT is larger compared to MOS- preferable to use BJT
• Extra pole at node X

\[
g_m \text{ can be varied by changing } V_b \text{ and thus } V_{ds}
\]
Alternative Fully CMOS Gm-Cell Example

- BJT replaced by a MOS transistor with boosted gm

- Lower frequency of operation compared to the BiCMOS version due to more parasitic capacitance at nodes A & B

BiCMOS Gm-C Integrator

- Differential- needs common-mode feedback ckt

- Freq.tuned by varying Vb

- Design tradeoffs:
  - Extra poles at the input device drain junctions
  - Input devices have to be small to minimize parasitic poles
  - Results in high input-referred offset voltage → could drive ckt into nonlinear region
  - Small devices → high 1/f noise