 EE247  
Lecture 5  

• Filters  
  – Effect of integrator non-idealities on filter behavior  
    • Integrator quality factor and its influence on filter frequency characteristics (review for last lecture)  
    • Filter dynamic range limitations due to limited integrator linearity  
      – Measures of linearity: Harmonic distortion, intermodulation distortion, intercept point  
    • Effect of integrator component variations and mismatch on filter response  
  – Various integrator topologies utilized in monolithic filters  
    • Resistor + C based filters  
    • Transconductance (gm) + C based filters  
    • Switched-capacitor filters  
  – Continuous-time filter considerations  
    • Facts about monolithic Rs, gms, & Cs and its effect on integrated filter characteristics

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Summary of Lecture 4  

• Ladder type RLC filters converted to integrator based active filters  
  – All pole ladder type filters  
    • Convert RLC ladder filters to integrator based form  
    • Example: 5th order Butterworth filter  
  – High order ladder type filters incorporating zeros  
    • 7th order elliptic filter in the form of ladder RLC with zeros  
      – Sensitivity to component mismatch  
      – Compare with cascade of biquads  
    ➔ Doubly terminated LC ladder filters ➔ Lowest sensitivity to component variations  
    • Convert to integrator based form utilizing SFG techniques  
    • Example: Differential high order filter implementation  
  – Effect of integrator non-idealities on continuous-time filter behavior  
    • Effect of integrator finite DC gain & non-dominant poles on filter frequency response
Real Integrator Non-Idealities

\[ H(s) = \frac{-a_1}{s} \]  
\[ H(s) = \frac{-a}{1 + s \frac{a}{a_1}} \left(1 + \frac{s}{p_2}\right) \left(1 + \frac{s}{p_3}\right) \ldots \]

Summary

Effect of Integrator Non-Idealities on Q

- Amplifier DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements.
- Amplifier poles located above integrator unity-gain frequency enhance the Q!
  - If non-dominant poles close to unity-gain freq. \( \rightarrow \) Oscillation
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!
- Overall quality factor of the integrator has to be much higher compared to the filter’s highest pole Q
Effect of Integrator Non-Linearities on Overall Integrator-Based Filter Performance

- Dynamic range of a filter is determined by the ratio of maximum signal output with acceptable performance over total noise.

- Maximum signal handling capability of a filter is determined by the non-linearities associated with its building blocks.

- Integrator linearity function of opamp/R/C (or any other component used to build the integrator) linearity.

- Linearity specifications for active filters typically given in terms of:
  - Maximum allowable harmonic distortion @ the output
  - Maximum tolerable intermodulation distortion
  - Intercept points & compression point referred to output or input.

Component Linearity versus Overall Filter Performance

1- Ideal Components

Ideal DC transfer characteristics:
Perfectly linear output versus input transfer function with no clipping.

\[ V_{out} = \alpha V_{in} \quad \text{for} \quad -\infty \leq V_{in} \leq \infty \]

If \( V_{in} = A \sin(\omega t) \) \( \rightarrow \)
\[ V_{out} = \alpha A \sin(\omega t) \]
Component Linearity versus Overall Filter Performance

2- Semi-Ideal Components

Semi-ideal DC transfer characteristics:
Perfectly linear output versus input transfer function with clipping

\[ Vout = \alpha \, Vin \] for \( -\Delta \leq Vin \leq +\Delta \)
\[ Vout = -\Delta \alpha \] for \( Vin \leq -\Delta \)
\[ Vout = \Delta \alpha \] for \( Vin \geq \Delta \)

If \( Vin = A \sin(\omega t) \rightarrow Vout = \alpha \, A \sin(\omega t) \) for \( -\Delta \leq Vin \leq +\Delta \)

Clipped & distorted otherwise

Effect of Component Non-Linearities on Overall Filter Linearity

Real Components including Non-Linearities

Real DC transfer characteristics: Both soft non-linearities & hard (clipping)

\[ Vout = \alpha_1 \, Vin + \alpha_2 \, Vin^2 + \alpha_3 \, Vin^3 + \ldots \ldots \] for \( -\Delta \leq Vin \leq \Delta \)

Clipped otherwise

If \( Vin = A \sin(\omega t) \)
Effect of Component Non-Linearities on Overall Filter Linearity

Real Components including Non-Linearities

Typical real circuit DC transfer characteristics:
\[ V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^3 + \alpha_3 V_{in}^5 + \ldots \ldots \text{ If } V_{in} = A \sin(\omega t) \land A < \Delta \]

Then:
\[ \rightarrow V_{out} = \alpha_1 A \sin(\omega t) + \alpha_2 A^2 \sin^2(\omega t) + \alpha_3 A^3 \sin^3(\omega t) + \ldots \]

or
\[ V_{out} = \alpha_1 \sin(\omega t) + \frac{\alpha_2 A^2}{2} (1 - \cos(2\omega t)) + \frac{\alpha_3 A^3}{4} (3\sin(\omega t) - \sin(3\omega t)) + \ldots \]

Effect of Component Non-Linearities on Overall Filter Linearity

Harmonic Distortion

Harmonic distortion component HD amplitude fundamental

\[ V_{out} = \alpha_1 \sin(\omega t) + \frac{\alpha_2 A^2}{2} (1 - \cos(2\omega t)) \]

\[ + \frac{\alpha_3 A^3}{4} (3\sin(\omega t) - \sin(3\omega t)) + \ldots \]

HD2 = \text{amplitude 2nd harmonic distortion component} / \text{amplitude fundamental}

HD3 = \text{amplitude 3rd harmonic distortion component} / \text{amplitude fundamental}

\[ \rightarrow HD2 = \frac{1}{2} \times \frac{\alpha_2}{\alpha_1} A^2 \quad \text{HD3} = \frac{1}{4} \times \frac{\alpha_3}{\alpha_1} A^3 \]
Example: Significance of Filter Harmonic Distortion in Voice-Band CODECs

- Voice-band CODEC filter (CODEC stands for coder-decoder, telephone circuitry includes CODECs with extensive amount of integrated active filters)
- Specifications includes limits associated with maximum allowable harmonic distortion at the output (< typically < 1% → -40dB)

**CODEC Filter including Output/Input transfer characteristic non-linearity's**

\[ V_{in} \rightarrow V_{out} \]

1kHz \[ f \] 3kHz

Effect of Component Non-Linearities on Overall Filter Linearity Intermodulation Distortion

DC transfer characteristics including nonlinear terms, input 2 sinusoidal waveforms:

\[ V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \ldots \]

If \[ V_{in} = A_1 \sin(\alpha_1 t) + A_2 \sin(\alpha_2 t) \]

Then \[ V_{out} \] will have the following components:

\[ \alpha_1 V_{in} \rightarrow \alpha_1 A_1 \sin(\alpha_1 t) + \alpha_1 A_2 \sin(\alpha_2 t) \]

\[ \alpha_1 V_{in}^2 \rightarrow \alpha_1 A_1^2 \sin^2(\alpha_1 t) + \alpha_1 A_2^2 \sin^2(\alpha_2 t) + \alpha_1 A_1 A_2 \sin(\alpha_1 t)\sin(\alpha_2 t) \]

\[ \alpha_1 V_{in}^3 \rightarrow \alpha_1 A_1^3 \sin^3(\alpha_1 t) + \alpha_1 A_2^3 \sin^3(\alpha_2 t) + \alpha_1 A_1^2 A_2 \sin(2\alpha_1 t) \sin(\alpha_2 t) \]

\[ \alpha_1 V_{in}^4 \rightarrow \alpha_1 A_1^4 \sin^4(\alpha_1 t) + \alpha_1 A_2^4 \sin^4(\alpha_2 t) + 3\alpha_1 A_1^2 A_2^2 \sin(2\alpha_1 t) \sin(2\alpha_2 t) \]

\[ + \frac{3\alpha_1 A_1^3 A_2 \sin(3\alpha_1 t) \sin(\alpha_2 t)}{4} + \frac{3\alpha_1 A_2^3 A_1 \sin(3\alpha_2 t) \sin(\alpha_1 t)}{4} \]
Effect of Component Non-Linearity on Overall Filter Linearity

Intermodulation Distortion

Real DC transfer characteristics, input 2 sin waves:

\[ V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \ldots \]

If \( V_{in} = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t) \)

For \( f_1 \) & \( f_2 \) close in frequency \( \Rightarrow \) Components associated with \( (2f_1-f_2) \) & \( (2f_2-f_1) \) are the closest to the fundamental signals on the frequency axis and thus most harmful.

Intermodulation distortion is measured in terms of \( \text{IM}_2 \) and \( \text{IM}_3 \):

Typically for input two sinusoids with equal amplitude \( (A_1 = A_2 = A) \)

\[
\text{IM}_2 = \frac{\text{amplitude 2nd IM component}}{\text{amplitude fundamental}}
\]

\[
\text{IM}_3 = \frac{\text{amplitude 3rd IM component}}{\text{amplitude fundamental}}
\]

\[
\text{IM}_2 = \frac{\alpha_2}{\alpha_1} A + \ldots \quad \text{IM}_3 = \frac{3}{4} \frac{\alpha_3}{\alpha_1} A^2 + \frac{25}{8} \frac{\alpha_4}{\alpha_1} A^4 + \ldots
\]
Wireless Communications Measure of Linearity

1dB Compression Point

\[ V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \ldots \]

Third Order Intercept Point

\[ I_{IP3} = \frac{3 \alpha_1}{4 \alpha_1} V_{in}^2 + \frac{25 \alpha_2}{8 \alpha_1} V_{in}^3 + \ldots \]

Typically:

\[ I_{IP3} - P_{1dB} = 9.6 \text{dB} \]

Most common measure of linearity for wireless circuits:

\[ \Rightarrow \text{OIP3 & IIP3, Third order output/input intercept point} \]
Example: Significance of Filter Intermodulation Distortion in Wireless Systems

- Typical wireless receiver architecture

Worst case signal scenario wrt linearity of the building blocks
→ Two adjacent channels large compared to desired channel

Example: Significance of Filter Intermodulation Distortion in Wireless Systems

- Adjacent channels can be as much as 60dB higher compared to the desired RX signal!
- Notice that in this example, 3rd order intermodulation component associated with the two adjacent channel, falls on the desired channel signal!
Filter Linearity

- Maximum signal handling capability is usually determined by the specifications wrt harmonic distortion and/or intermodulation distortion.
  Distortion in a filter is a function of linearity of the components.
- Example: In the above circuit linearity of the filter is mainly a function of linearity of the opamp voltage transfer characteristics.

Various Types of Integrator Based Filter

- Continuous Time
  - Resistive element based
    - Opamp-RC
    - Opamp-MOSFET-C
    - Opamp-MOSFET-RC
  - Transconductance (Gm) based
    - Gm-C
    - Opamp-Gm-C
- Sampled Data
  - Switched-capacitor Integrator
Continuous-Time Resistive Element Type Integrators
Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC

Ideal transfer function: 
\[ \frac{V_o}{V_{in}} = \frac{-a_b}{s} \text{ where } a_b = \frac{1}{R_{eq}C} \]

Continuous-Time Transconductance Type Integrator
Gm-C & Opamp-Gm-C

Ideal transfer function: 
\[ \frac{V_o}{V_{in}} = \frac{-a_b}{s} \text{ where } a_b = \frac{G_m}{C} \]
Integrator Implementation
Switched-Capacitor

\[ V_{in} \rightarrow V_o \]

\[ \phi_1 \quad \phi_2 \]

\[ T = \frac{1}{f_{clk}} \]

\[ V_0 = \frac{f_{clk} \times C_s}{C_f} \int V_{in} \, dt \]

Main advantage: Critical frequency function of ratio of caps & clock freq.
\[ \Rightarrow \] Critical filter frequencies (e.g. LPF -3dB freq.) very accurate

Few Facts About Monolithic \( R_s \) & \( C_s \) & \( G_{ms} \)

- Monolithic continuous-time filter critical frequency set by \( R_x C \) or \( C/G_{ms} \)
- Absolute value of integrated \( R_s \) & \( C_s \) & \( G_{ms} \) are quite variable
  - \( R_s \) vary due to doping and etching non-uniformities
    - Could vary by as much as \( \sim \) -20 to 40% due to process & temperature variations
  - \( C_s \) vary because of oxide thickness variations and etching inaccuracies
    - Could vary \( \sim \) +10 to 15%
  - \( G_{ms} \) typically function of mobility, oxide thickness, current, device geometry ...
    - Could vary \( \sim \) -40% or more with process & temp. & supply voltage
  - Continuous-time filter critical frequency could vary by over \( \sim \) 50%
Few Facts About Monolithic Rs & Cs

• While absolute value of monolithic Rs & Cs and gms are quite variable, with special attention paid to layout, C & R & gms quite well-matched
  – *Ratios very accurate* and stable over processing, temperature, and time

• With special attention to layout (e.g. interleaving, use of dummy devices, common-centroid geometries…):
  – Capacitor mismatch << 0.1%
  – Resistor mismatch < 0.1%
  – Gm mismatch < 0.5%

Impact of Component Variations on Filter Characteristics

![RLC Filters](image)

Facts about RLC filters

• $\omega_{3dB}$ determined by absolute value of $Ls$ & $Cs$

\[
C_1^{RLC} = C_1 \times C_1^{Norm} = \frac{C_1^{Norm}}{R \times \omega_{-3dB}}
\]

\[
L_2^{RLC} = L_2 \times L_2^{Norm} = \frac{L_2^{Norm} \times R^*}{\omega_{-3dB}}
\]

• Shape of filter depends on *ratios* of normalized $L$ & $C$
Effect of Monolithic R & C Variations on Filter Characteristics

- Filter shape (whether Elliptic with 0.1dB Rpass or Butterworth..etc) is a function of ratio of normalized $L_s$ & $C_s$ in RLC filters
- Critical frequency (e.g. $\omega_{3dB}$) function of absolute value of $L_s \times C_s$
- Absolute value of integrated $R_s$ & $C_s$ & $G_m$s are quite variable
- Ratios very accurate and stable over time and temperature

→ What is the effect of on-chip component variations on monolithic filter frequency characteristics?

Impact of Process Variations on Filter Characteristics

$$\tau_1 = c^{RLC}_{R} \frac{C^*}{R_{Norm}} = \frac{C^*}{\omega_{3dB}}$$

$$\tau_2 = \frac{c^{RLC}_{R}}{R_{Norm}} = \frac{C^*}{\omega_{3dB}}$$

$$\frac{\tau_1}{\tau_2} = \frac{C^*_{Norm}}{L^*_{Norm}}$$
Impact of Process Variations on Filter Characteristics

Variation in absolute value of integrated $Rs$ & $Cs$ $\rightarrow$ change in critical freq. ($\omega_{-3dB}$)

Since Ratios of $Rs$ & $Cs$ very accurate

$\Rightarrow$ Continuous-time monolithic filters retain their shape due to good component matching even with variability in absolute component values

Example: LPF Worst Case Corner Frequency Variations

- While absolute value of on-chip RC (gm-C) time-constants could vary by as much as 100% (process & temp.)
- With proper precautions, excellent component matching can be achieved:
  $\Rightarrow$ Well-preserved relative amplitude & phase vs freq. characteristics
  $\Rightarrow$ Need to only adjust (tune) continuous-time filter critical frequencies
Tunable Opamp-RC Filters

Example

- 1st order Opamp-RC filter is designed to have a corner frequency of 1.6MHz
- Assuming process variations of:
  - C varies by +10%
  - R varies by +25%
- Build the filter in such a way that the corner frequency can be adjusted post-manufacturing.

Nominal R & C values for 1.6MHz corner frequency

Filter Corner Frequency Variations

- Assuming expected process variations of:
  - Maximum C variations by +-10%
    \[
    C_{nom}=10\text{pF} \rightarrow C_{min}=9\text{pF}, \ C_{max}=11\text{pF}
    \]
  - Maximum R variations by +-25%
    \[
    R_{nom}=10K \rightarrow R_{min}=7.5K, \ R_{max}=12.5K
    \]
  - Corner frequency ranges from
    \[
    2.357\text{MHz} \text{ to } 1.157\text{MHz}
    \]
  - Corner frequency varies by +48% & -27%
Variable Resistor or Capacitor

- In order to make provisions for filter to be tunable either $R$ or $C$ should be adjustable (this example adjustable $R$)
- Monolithic Rs can only be made adjustable in discrete steps (not continuous)

$$\frac{R_{\text{max}}}{R_{\text{nom}}} = \frac{f_{\text{max}}}{f_{\text{nom}}} = 1.48$$

$$\rightarrow R_{\text{max}}^{\text{nom}} = 14.8k\Omega$$

$$\frac{R_{\text{min}}}{R_{\text{nom}}} = \frac{f_{\text{min}}}{f_{\text{nom}}} = 0.72$$

$$\rightarrow R_{\text{min}}^{\text{nom}} = 7.2k\Omega$$

Tunable Resistor

- Maximum $C$ variations by $+10\% \rightarrow C_{\text{min}}=9pF$, $C_{\text{max}}=11pF$
- Maximum $R$ variations by $-25\% \rightarrow R_{\text{min}}=7.5k$, $R_{\text{max}}=12.5k$
  \( \Rightarrow \) Corner frequency varies by $+48\% \& -27\%$

- Assuming control signal has $n = 3$ bit (0 or 1) for adjustment \( \Rightarrow R_2=2R_3=4R_4 \)

$$R_1 = R_{\text{min}}^{\text{nom}} = 7.2k\Omega$$

$$R_2 = \left( \frac{R_{\text{max}}^{\text{nom}} - R_{\text{min}}^{\text{nom}}}{2^n} \right) \times \frac{2^n - 1}{2^n - 1} = (14.8k - 7.2k)\frac{4}{7} = 4.34k\Omega$$

$$R_3 = \left( \frac{R_{\text{max}}^{\text{nom}} - R_{\text{min}}^{\text{nom}}}{2^n} \right) \times \frac{2^{n-2}}{2^n - 1} = (14.8k - 7.2k)\frac{2}{7} = 2.17k\Omega$$

$$R_4 = \left( \frac{R_{\text{max}}^{\text{nom}} - R_{\text{min}}^{\text{nom}}}{2^n} \right) \times \frac{2^{n-3}}{2^n - 1} = (14.8k - 7.2k)\frac{1}{7} = 1.08k\Omega$$

Tuning resolution $= 1.08k/10k \approx 10\%$
Tunable Opamp-RC Filter

Post manufacturing:
- Set all D_x to 100 (mid point)
- Measure -3dB frequency
  - If frequency too high decrement D to D-1
  - If frequency too low increment D to D+1
  - If frequency within 10% of the desired corner frequency ➔ stop
  - else

For higher order filters, all filter integrators tuned simultaneously

Tunable Opamp-RC Filters

Summary

- Program C_s and/or R_s to freq. tune the filter
- All filter integrators tuned simultaneously
- Tuning in discrete steps & not continuous
- Tuning resolution limited
- Switch parasitic C & series R can affect the freq. response of the filter
Opamp RC Filters

- Advantages
  - Since resistors are quite linear, linearity only a function of opamp linearity
    - good linearity

- Disadvantages
  - Opamps have to drive resistive load, low output impedance is required
    - High power consumption
  - Continuous tuning not possible—tuning only in discrete steps
  - Tuning requires programmable Rs and/or Cs
Integrator Implementation
Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC

\[ V_o = \frac{-a_b}{s} V_{in} \]
where \( a_b = \frac{1}{R_{eq} C} \)

Use of MOSFETs as Variable Resistors

R replaced by MOSFET
Operating in triode mode

\( \rightarrow \) Continuously variable resistor:

MOSFET IV characteristic:
Opamp MOSFET-C Integrator
Single-Ended Integrator

Problem: Single-ended MOSFET-C Integrator
- Effective R non-linear

Note that the non-linearity is mainly 2nd order type

\[
I_D = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} \right) \frac{V_{ds}^2}{2}
\]

\[
I_D = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} \right) \frac{V_i^2}{2}
\]

\[
G = \frac{dI_D}{dV_i} = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} - V_i \right)
\]

By varying \( V_G \) effective admittance is tuned
\( \rightarrow \) Tunable integrator time constant

Problem: Single-ended MOSFET-C Integrator
- Effective R non-linear

Note that the non-linearity is mainly 2nd order type

Use of MOSFETs as Resistors
Differential Integrator

Non-linear term is of even order & cancelled!
- Admittance independent of \( V_i \)

Problem: Threshold voltage dependence
Use of MOSFET as Resistor Issues

- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles:
  - Excess phase @ the unity-gain frequency of the integrator
  - Enhanced integrator Q
  - Enhanced filter Q,
  - Peaking in the filter passband

- Tradeoffs affecting the choice of device channel length:
  - Filter performance mandates well-matched MOSFETs → long channel devices desirable
  - Excess phase increases with $L^2$ → Q enhancement and potential for oscillation!
  - Tradeoff between device matching and integrator Q
  - Tradeoff limited to low frequencies
Example: Opamp MOSFET-C Filter

- Suitable for low frequency applications
- Issues with linearity
- Linearity achieved ~40-50dB
- Needs tuning

5th Order Elliptic MOSFET-C LPF with 4kHz Bandwidth


Improved MOSFET-C Integrator

\[ I_D = \mu C_{ox} \frac{W}{L} (V_{g1} - V_{d}) \]  
\[ I_{D1} = \mu C_{ox} \frac{W}{L} (V_{g2} - V_{d}) \]  
\[ I_{D2} = -\mu C_{ox} \frac{W}{L} (V_{g3} - V_{d}) \]  
\[ I_{X1} = I_{D1} + I_{D3} \]  
\[ I_{X2} = \mu C_{ox} \frac{W}{L} (V_{g4} - V_{d}) \]  
\[ I_{X1} - I_{X2} = \mu C_{ox} \frac{W}{L} (V_{g4} - V_{g3}) \]  
\[ G = \frac{\partial (I_{X1} - I_{X2})}{\partial V_{ds}} = \mu C_{ox} \frac{W}{L} (V_{g4} - V_{g3}) \]  

No threshold voltage dependence

Linearity achieved in the order of 50-70dB

R-MOSFET-C Integrator

• Improvement over MOSFET-C by adding resistor in series with MOSFET
• Voltage drop primarily across fixed resistor → small MOSFET Vds → improved linearity & reduced tuning range
• Generally low frequency applications


R-MOSFET-C Lossy Integrator

Negative feedback around the non-linear MOSFETs improves linearity but compromises frequency response accuracy

Example:
Opamp MOSFET-RC Filter

5th Order Bessel MOSFET-RC LPF 22kHz bandwidth
THD $\approx -90\text{dB}$ for $4\text{Vp-p}$, $2\text{kHz}$ input signal

- Suitable for low frequency, low Q applications
- Significant improvement in linearity compared to MOSFET-C
- Needs tuning


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Operational Amplifiers (Opamps) versus Operational Transconductance Amplifiers (OTA)

**Opamp**
- Voltage controlled voltage source
- Output in the form of voltage
- Low output impedance
- Can drive R-loads
- Good for RC filters, OK for SC filters
- Extra buffer adds complexity, power dissipation

**OTA**
- Voltage controlled current source
- Output in the form of current
- High output impedance
- In the context of filter design called gm-cells
- Cannot drive R-loads
- Good for SC & gm-C filters
- Typically, less complex compared to opamp→ higher freq. potential
- Typically lower power
Integrator Implementation

Transconductance-C & Opamp-Transconductance-C

\[
V_o = \frac{-a_b}{s} \quad \text{where} \quad a_b = \frac{G_m}{C}
\]

Gm-C Filters

Simplest Form of CMOS Gm-C Integrator

- Transconductance element formed by the source-coupled pair \( M1 \) & \( M2 \)
- All MOSFETs operating in saturation region
- Current in \( M1 \) & \( M2 \) can be varied by changing \( V_{\text{control}} \)
  \[ \rightarrow \text{Transconductance of } M1 \text{ & } M2 \text{ varied through } V_{\text{control}} \]

Simplest Form of CMOS Gm-C Integrator

**AC Half Circuit**

\[
\begin{align*}
\text{Int } g_C & \quad \text{control } V \\
\text{in } V & \quad - \\
+ & \quad + \\
\text{M1 M2} & \\
\end{align*}
\]

\[
\begin{align*}
\text{Int } g_2C & \quad \text{M1 M2} \\
\text{AC half circuit} & \\
\end{align*}
\]

---

Gm-C Filters

**Simplest Form of CMOS Gm-C Integrator**

- Use ac half circuit & small signal model to derive transfer function:

\[
\begin{align*}
V_o &= -g_m M_{1,2} \times V_{in} \times 2C_{int} g s \\
\frac{V_o}{V_{in}} &= -\frac{g_m M_{1,2}}{2C_{int} g s} \\
\frac{V_o}{V_{in}} &= -\frac{a_b}{s} \\
\rightarrow a_b &= \frac{g_m M_{1,2}}{2 \times C_{int} g}
\end{align*}
\]
Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

- MOSFET in saturation region:
  \[ I_d = \frac{\mu C_{ox} W}{2} \frac{1}{L} (V_{gs} - V_{th})^2 \]
- Gm is given by:
  \[ g_m^{M1 \& M2} = \frac{\partial I_d}{\partial V_{gs}} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th}) \]
  \[ = 2 \frac{I_d}{(V_{gs} - V_{th})} \]
  \[ = 2 \left( \frac{\mu C_{ox} W}{L} I_d \right)^{1/2} \]

\( I_d \) varied via \( V_{control} \)
\( \rightarrow \) \( g_m \) tunable via \( V_{control} \)

Gm-C Filters
2nd Order Gm-C Filter

- Use the Gm-cell to build a 2nd order bandpass filter
2nd Order Bandpass Filter

\[ \tau_1 = R^* \times C \quad \tau_2 = L / R^* \]

2nd Order Integrator-Based Bandpass Filter

\[ V_{BP} = \frac{\tau_2 s}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + 1} \]

\[ \tau_1 = R^* \times C \quad \tau_2 = L / R^* \]

\[ \beta = \frac{R^*}{R} \]

\[ \omega_0 = \frac{1}{\sqrt{\tau_1 \tau_2}} = \frac{1}{\sqrt{L C}} \]

\[ Q = \frac{1}{\beta \sqrt{\tau_1 / \tau_2}} \]

From matching point of view, desirable:

\[ \tau_1 = \tau_2 = \tau = \frac{1}{\omega_0} \quad \rightarrow \quad Q = \frac{R}{R^*} \]
2nd Order Integrator-Based Bandpass Filter

First implement this part
With transfer function:

\[ \frac{V_0}{V_{in}} = \frac{-1}{s\frac{1}{Q} + \frac{1}{Q}} \]

Terminated Gm-C Integrator

AC half circuit
Terminated Gm-C Integrator

\[ V_o = \frac{-1}{s \frac{2C_{int} g}{g_m M1} + \frac{g_{M3}}{g_m M1}} \]

Compare to:
\[ \frac{V_0}{V_{in}} = \frac{-1}{s \frac{g_{M1}}{\omega_b} + \frac{1}{Q}} \]

Question: How to define Q accurately?
Terminated Gm-C Integrator

\[
g_{m}^{M1} = 2 \left( \frac{1}{2} \mu C_{ox} \frac{W_{M1}}{L_{M1}} I_{d}^{M1} \right)^{1/2}
\]

\[
g_{m}^{M3} = 2 \left( \frac{1}{2} \mu C_{ox} \frac{W_{M3}}{L_{M3}} I_{d}^{M3} \right)^{1/2}
\]

Let us assume equal channel lengths for \( M1, M3 \) then:

\[
\frac{g_{m}^{M1}}{g_{m}^{M3}} = \left( \frac{I_{d}^{M1}}{I_{d}^{M3}} \cdot \frac{W_{M1}}{W_{M3}} \right)^{1/2}
\]

Note that:

\[
\frac{I_{d}^{M1}}{I_{d}^{M3}} = \frac{W_{M10}}{W_{M11}}
\]

Assuming equal channel lengths for \( M10, M11 \):

\[
\frac{g_{m}^{M1}}{g_{m}^{M3}} = \left( \frac{W_{M10}}{W_{M11}} \cdot \frac{W_{M1}}{W_{M3}} \right)^{1/2}
\]
2nd Order Gm-C Filter

- Simple design
- Tunable
- Q function of device ratios:

\[ Q = \frac{g_{m1,2}}{g_{m4}} \]