EE247
Lecture 4

• Active ladder type filters
  – For simplicity, will start with all pole ladder type filters
    • Convert to integrator based form- example shown
  – Then will attend to high order ladder type filters incorporating zeros
    • Implement the same 7th order elliptic filter in the form of ladder RLC with zeros
      – Find level of sensitivity to component mismatch
      – Compare with cascade of biquads
    • Convert to integrator based form utilizing SFG techniques
      – Effect of integrator non-Idealities on filter frequency characteristics

Summary Lecture 3

• Active Filters
  – Active biquads
    • Integrator-based filters
      – Signal flowgraph concept
      – First order integrator-based filter
      – Second order integrator-based filter & biquads
  – High order & high Q filters
    • Cascaded biquads & first order filters
      – Cascaded biquad sensitivity to component mismatch
    • Ladder type filters
RLC Ladder Filters
Example: 5th Order Lowpass Filter

• Made of resistors, inductors, and capacitors
• Doubly terminated or singly terminated (with or w/o $R_L$)

Doubly terminated LC ladder filters $\rightarrow$ Lowest sensitivity to component mismatch

LC Ladder Filters

• First step in the design process is to find values for $L_s$ and $C_s$ based on specifications:
  – Filter graphs & tables found in:
  – CAD tools
    • Matlab
    • Agilent ADS (includes Filter package $\rightarrow$ does the job of the tables)
    • Spice
LC Ladder Filter Design Example

Design a LPF with maximally flat passband:

\[ f_{-3dB} = 10\text{MHz}, \quad f_{\text{stop}} = 20\text{MHz} \]

\[ R_s > 27\text{dB} @ f_{\text{stop}} \]

- Maximally flat passband \( \rightarrow \) Butterworth

- Find minimum filter order :

- Here standard graphs from filter books are used

\[ f_{\text{stop}} / f_{-3dB} = 2 \]

\[ R_s > 27\text{dB} \]

**Minimum Filter Order**

\( \approx 5\text{th order} \) Butterworth

From: Williams and Taylor, p. 2-37

**Stopband Attenuation**

\[ -30\text{dB} \]

\[ -40\text{dB} \]

**Passband Attenuation**

**Denormalization:**

Multiply all \( L_{\text{Norm}}, C_{\text{Norm}} \) by:

\[ L_r = R / \omega_{-3dB} \]

\[ C_r = 1 / (R \times \omega_{-3dB}) \]

\( R \) is the value of the source and termination resistor

\( \omega_{-3dB} \) = 1

From: Williams and Taylor, p. 11.3

**Find values for L & C from Table:**

Note \( L \) & \( C \) values normalized to

| \( \omega_{-3dB} \) = 1

**TABLE 11.2**

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From: Williams and Taylor, p. 11.3
LC Ladder Filter Design Example

Find values for L & C from Table:

Normalized values:

C1_{Norm} = C5_{Norm} = 0.618
C3_{Norm} = 2.0
L2_{Norm} = L4_{Norm} = 1.618

Denormalization:

Since \( \omega_{3dB} = 2 \times 10 \text{MHz} \)

\[ L_r = \frac{R}{\omega_{3dB}} = 15.9 \text{ nH} \]

\[ C_r = \frac{1}{R \times \omega_{3dB}} = 15.9 \text{ nF} \]

\[ R = 1 \]

\[ C1 = C5 = 9.836 \text{ nF}, \quad C3 = 31.83 \text{ nF} \]

\[ L2 = L4 = 25.75 \text{ nH} \]

From: Williams and Taylor, p. 11.3

Last Lecture:

Example: 5th Order Butterworth Filter

Specifications:

\( f_{-3dB} = 10 \text{MHz}, \)

\( f_{\text{stop}} = 20 \text{MHz} \)

\( Rs > 27 \text{dB} \)

Used filter tables to obtain \( Ls \) & \( Cs \)
To convert RLC ladder prototype to integrator based filter:

- Use Signal Flowgraph technique
  - Name currents and voltages for all components
  - Use KCL & KVL to derive equations
  - Make sure reactive elements expressed as 1/s term
    \[ V(C) = f(I) \text{ and } I(L) = f(V) \]
  - Use state-space description to derive the SFG
  - Modify & simply the SFG for implementation with integrators e.g. convert all current nodes to voltage

Use KCL & KVL to derive equations:

\[
\begin{align*}
I_1 &= \frac{V_{1}}{R_s} \\
I_2 &= \frac{V_{2}}{C_1} \\
V_{1} &= V_{in} - V_2 \\
V_2 &= \frac{I_2}{sC_1} \\
V_3 &= V_2 - V_4 \\
V_4 &= \frac{I_4}{sC_3} \\
V_5 &= V_4 - V_6 \\
V_6 &= \frac{I_6}{sC_5} \\
I_6 &= \frac{V_6}{RL} \\
I_7 &= \frac{V_6}{RL} \\
I_7 &= V_6 \\
I_4 &= I_3 - I_5 \\
I_5 &= \frac{V_5}{sL_4} \\
I_3 &= I_3 - I_2 \\
I_2 &= I_3 - I_1 \\
I_1 &= \frac{V_{1}}{R_s} \\
\end{align*}
\]
Low-Pass RLC Ladder Filter
Signal Flowgraph

\[ V_1 = V_{in} - V_2 \]
\[ V_2 = \frac{i_2}{sC_1} \]
\[ i_1 = \frac{V_1}{R_s} \]
\[ i_2 = i_1 - i_3 \]
\[ i_4 = i_3 - i_5 \]
\[ i_5 = \frac{V_5}{sL_2} \]
\[ V_3 = V_4 - V_6 \]
\[ V_5 = V_6 - V_4 \]
\[ V_6 = \frac{i_6}{sC_5} \]
\[ V_7 = V_8 - V_6 \]

I.1
I.2
I.3
I.4
I.5
I.6
I.7
V_{in}
V_1
V_2
V_3
V_4
V_5
V_6
V_7
V_{out}

SFG

Low-Pass RLC Ladder Filter
Normalize

\[ V_{in} \]
\[ V_1 \]
\[ V_2 \]
\[ V_3 \]
\[ V_4 \]
\[ V_5 \]
\[ V_6 \]
\[ V_{out} \]

I.1
I.2
I.3
I.4
I.5
I.6
I.7

\[ \frac{R^*}{R_s} \]
\[ \frac{1}{sC_1R^*} \]
\[ \frac{R^*}{sL_2} \]
\[ \frac{1}{sC_3R^*} \]
\[ \frac{R^*}{sL_4} \]
\[ \frac{1}{sC_5R^*} \]

\[ \frac{R^*}{R_L} \]
Low-Pass RLC Ladder Filter

Synthesize

$V_{in} \rightarrow V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4 \rightarrow V_5 \rightarrow V_6 \rightarrow V_o$

$\frac{R^*}{R_s}$ $\frac{1}{sC_1R^*}$ $\frac{R^*}{sL_2}$ $\frac{1}{sC_3R^*}$ $\frac{R^*}{sL_4}$ $\frac{1}{sC_5R^*}$ $\frac{R^*}{R_L}$

$v_{in} \rightarrow R^* \frac{R}{R_s}$

$v_2 \rightarrow \frac{1}{sC_1}$

$v_4 \rightarrow \frac{1}{sC_3}$

$v_5 \rightarrow \frac{1}{sC_5}$

$v_o \rightarrow R^* \frac{R}{R_L}$

Low-Pass RLC Ladder Filter

Integrator Based Implementation

Main building block: Integrator
Let us start to build the filter with RC& Opamp type integrator
Opamp-RC Integrator

Single-Ended

\[ V_o = V_{in1} \times \frac{1}{sR_1C_1} - V_{in2} \times \frac{1}{sR_2C_1} \]

Differential

\[ V_{o+} = V_{o-} = (V_{in1+} - V_{in1-}) \times \frac{1}{sR_1C_1} \]
\[ + (V_{in2+} - V_{in2-}) \times \frac{1}{sR_2C_1} \]

Note: Implementation with single-ended integrator requires extra circuitry for sign inversion whereas in differential case both signal polarities are available.

Differential topologies → additional advantage of immunity to parasitic signal injection & superior power-supply rejection.

Differential Integrator Based LP Ladder Filter

Synthesize

- First iteration:
  - All resistors are chosen = 1Ω
  - Values for \( \tau \equiv R_iC_i \) found from RLC analysis
  - Integrating capacitor values: \( C_1 = C_5 = 9.836nF, C_{12} = C_{14} = 25.45nF, C_{13} = 31.83nF \)
Scale Node Voltages

To maximize dynamic range
→ scale node voltages

Scale $V_o$ by factor "s"
Differential Integrator Based LP Ladder Filter
Node Scaling

- Second iteration:
  - Nodes scaled, note output node $X_2$
  - Resistor values scaled according to scaling of nodes
  - Capacitors the same: $C_1 = C_5 = 9.836nF$, $C_2 = C_4 = 25.45nF$, $C_3 = 31.83nF$

Second Iteration
Maximizing Signal Handling by Node Voltage Scaling
Filter Noise

Total noise @ the output:
\(1.4 \mu V \text{ rms}\)
(noiseless opamps)

That’s excellent, but:
- Capacitors too large for integration
  \(\Rightarrow\) Unrealistically large Si area
- Resistors too small
  \(\Rightarrow\) high power dissipation

Typical applications allow higher noise, assuming tolerable noise in the order of \(140 \mu V \text{ rms}\) ...

Scale to Meet Noise Target

Scale capacitors and resistors to meet noise objective

\[ s = 10^{-4} \Rightarrow (V_{n1}/V_{n2})^2 \]

Noise after scaling: \(141 \mu V \text{ rms}\) (assuming noiseless opamps)
Differential Integrator Based LP Ladder Filter Final Design

- Final iteration:
  - Based on scaled nodes and noise considerations
  - Capacitors: $C1=C5=0.9836pF$, $C2=C4=2.545pF$, $C3=3.183pF$
  - Resistors: $R1=11.77k$, $R2=9.677k$, $R3=10k$, $R4=12.82k$, $R5=8.493k$, $R6=11.93k$, $R7=7.8k$, $R8=10.75k$, $R9=8.381k$, $R10=10k$, $R11=9.306k$

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RLC Ladder Filters Including Transmission Zeros

- All poles
- Poles & Zeros
RLC Ladder Filter Design Example

- Design a baseband filter for CDMA IS95 cellular phone receive path with the following specs.
  - Filter frequency mask shown on the next page
  - Allow enough margin for manufacturing variations
    - Assume overall tolerable pass-band magnitude variation of 1.8dB
    - Assume the -3dB frequency can vary by +\%-8% due to manufacturing tolerances & circuit inaccuracies
  - Assume any phase impairment can be compensated in the digital domain

* Note this is the same example as for cascade of biquad while the specifications are given closer to a real product case
RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Since phase impairment can be corrected for, use filter type with max. roll-off slope/pole
  → Filter type → Elliptic
- Design filter freq. response to fall well within the freq. mask
  – Allow margin for component variations & mismatches
- For the passband ripple, allow enough margin for ripple change due to component & temperature variations
  → Design nominal passband ripple of 0.2dB
- For stopband rejection add a few dB margin $44 + 5 = 49$dB
- Final design specifications:
  – $f_{pass} = 650$ kHz $R_{pass} = 0.2$ dB
  – $f_{stop} = 750$ kHz $R_{stop} = 49$ dB
- Use Matlab or ADS or filter tables to decide the min. order for the filter (same as cascaded biquad example)
  – 7th Order Elliptic

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RLC Low-Pass Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Use filter tables & charts to determine LC values
- Can use the CAD tool: Agilent ADS
RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Specifications
  - fpass = 650 kHz  \( R_{\text{pass}} = 0.2 \text{ dB} \)
  - fstop = 750 kHz  \( R_{\text{stop}} = 49 \text{ dB} \)
- Use filter tables to determine LC values
  - Elliptic filters tabulated wrt “reflection coefficient \( \rho \)"

\[
R_{\text{pass}} = -10 \times \log \left( 1 - \rho^2 \right)
\]

- Since \( R_{\text{pass}} = 0.2 \text{ dB} \) \( \Rightarrow \rho = 20\% \)
- Use table accordingly

---

RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Table from Zverev book page #281 & 282:
- Since our spec. is \( A_{\min} = 44 \text{ dB} \) add 5dB margin & design for \( A_{\min} = 49 \text{ dB} \)
- Table from Zverev page #281 & 282:

- Normalized component values:
  - \( C_1 = 1.17677 \)
  - \( C_2 = 0.19393 \)
  - \( L_2 = 1.19467 \)
  - \( C_3 = 1.51134 \)
  - \( C_4 = 1.01098 \)
  - \( L_4 = 0.72398 \)
  - \( C_5 = 1.27776 \)
  - \( C_6 = 0.71211 \)
  - \( L_6 = 0.80165 \)
  - \( C_7 = 0.83597 \)

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**RLC Filter Frequency Response**

- Component values denormalized
- Frequency response simulated
- Frequency mask superimposed
- Frequency response well within spec.
Frequency Response Passband Detail

- Passband well within spec.
- Make sure enough margin is allowed for variations due to process & temperature

![Frequency Response Graph](image)

RC Ladder Filter Sensitivity

- The design has the same specifications as the previous example implemented with cascaded biquads

- To compare the sensitivity of RLC ladder versus cascaded-biquads:
  - Changed all Ls & Cs one by one by 2% in order to change the pole/zeros by 1% (similar test as for cascaded biquad)
  - Found frequency response → most sensitive to L4 variations
  - Note that by varying L4 both poles & zeros are varied
RCL Ladder Filter Sensitivity

Component mismatch in RLC filter:

- Increase $L_4$ from its nominal value by 2%
- Decrease $L_4$ by 2%

![Graph showing the sensitivity of $L_4$ to frequency changes.]

**Summary:**

- 1.7 dB change at 500 kHz
- 0.2 dB change at 600 kHz
- 1.1 dB change at 1100 kHz
Sensitivity of Cascade of Biquads

Component mismatch in Biquad 4 (highest Q pole):
- Increase $\omega_{p4}$ by 1%
- Decrease $\omega_{z4}$ by 1%

High Q poles $\rightarrow$ High sensitivity in Biquad realizations

Sensitivity Comparison for Cascaded-Biquads versus RLC Ladder

- 7th Order elliptic filter
  - 1% change in pole & zero pair

<table>
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<tr>
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<th>Cascaded Biquad</th>
<th>RLC Ladder</th>
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<tbody>
<tr>
<td>Passband deviation</td>
<td>2.2dB (29%)</td>
<td>0.2dB (2%)</td>
</tr>
<tr>
<td>Stopband deviation</td>
<td>3dB (40%)</td>
<td>1.7dB (21%)</td>
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Doubly terminated LC ladder filters $\rightarrow$ Significantly lower sensitivity compared to cascaded-biquads particularly within the passband
• Previously learned to design integrator based ladder filters without transmission zeros
  → Question:
    o How do we implement the transmission zeros in the integrator-based version?
    o Preferred method → no extra power dissipation → no extra active elements

Integrator Based Ladder Filters
How Do to Implement Transmission zeros?

• Use KCL & KVL to derive:

\[ I_2 = I_1 - I_3 - I_{C_a}, \quad I_{C_a} = (V_2 - V_4)C_a, \quad V_2 = \frac{I_2}{sC_1} \]

Substituting for \( I_2 \):

\[ V_2 = \frac{I_1 - I_3 - I_{C_a}}{sC_1} \]

Substituting for \( I_{C_a} \) and rearranging:

\[ V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \times \frac{C_a}{C_1 + C_a} \]
Integrator Based Ladder Filters

How Do to Implement Transmission zeros?

- Use KCL & KVL to derive:

\[
V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \frac{C_a}{C_1 + C_a}.
\]

\[
V_d = \frac{I_3 - I_5}{s(C_3 + C_a)} + V_2 \frac{C_a}{C_3 + C_a}.
\]

- Frequency independent constants
  Can be substituted by:
    Voltage-Controlled Voltage Source

- Replace shunt capacitors with voltage controlled voltage sources:

\[
\begin{align*}
V_2 &= \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \frac{C_a}{C_1 + C_a} \\
V_d &= \frac{I_3 - I_5}{s(C_3 + C_a)} + V_2 \frac{C_a}{C_3 + C_a}
\end{align*}
\]
3rd Order Lowpass Filter
All Poles & No Zeros

Implementation of Zeros in Active Ladder Filters
Without Use of Active Elements
Integrator Based Ladder Filters
Higher Order Transmission zeros

Convert zero generating Cs in C loops to voltage-controlled voltage sources

Higher Order Transmission zeros
Example: 5th Order Chebyshev II Filter

- 5th order Chebyshev II
- Table from: Williams & Taylor book, p. 11.112
- 50dB stopband attenuation
- $f_{3dB} = 10MHz$

Transmission Zero Generation
Opamp-RC Integrator

$$V_o = -\frac{I}{s(C+C_x)} \left[ \frac{V_{in1} + V_{in2} + V_o}{R_f} \frac{V_{in1}}{R_2} \right]$$

$$V_{in3} \times \frac{C_x}{C+C_x}$$
Differential Integrator Based LP Ladder Filter
Final Design 5th Order All-Pole

Differential 5th Order Chebychev Lowpass Filter

- All resistors 1Ω
- Capacitors: $C_1 = 36.11nF$, $C_2 = 14.05nF$, $C_3 = 12.15nF$, $C_4 = 5.344nF$, $C_5 = 2.439nF$
- Coupling capacitors: $C_a = 1.36nF$, $C_b = 1.36nF$, $C_c = 1.31nF$, $C_d = 1.31nF$
5th Order Chebyshev II Filter
Simulated Frequency Response

7th Order Differential Lowpass Filter
Including Transmission Zeros

Transmission zeros implemented with pair of coupling capacitors
Effect of Integrator Non-Idealities on Filter Frequency Characteristics

• In the passive filter design (RLC filters) section:
  – Reactive element (L & C) non-idealities \( \Rightarrow \) expressed in the form of Quality Factor (Q)
  – Filter impairments due to component non-idealities explained in terms of component Q

• In the context of active filter design (integrator-based filters)
  – Integrator non-idealities \( \Rightarrow \) Translates to the form of Quality Factor (Q)
  – Filter impairments due to integrator non-idealities explained in terms of integrator Q

Effect of Integrator Non-Idealities on Filter Performance

• Ideal integrator characteristics

• Real integrator characteristics:
  – Effect of opamp finite DC gain
  – Effect of integrator non-dominant poles
Effect of Integrator Non-Idealities on Filter Performance

**Ideal Integrator**

- **DC gain** = $\infty$
- **Single pole @ DC**
- $\rightarrow$ no non-dominant poles

\[
H(s) = \frac{-\omega_0}{s}
\]

\[
\omega_0 = \frac{1}{RC}
\]

- **0dB Phase**
- **$-90^\circ$**

\[
20 \log |H(\omega)|
\]

**Ideal Integrator Quality Factor**

**Ideal intg. transfer function:**

\[
H(\omega) = \frac{1}{R(\omega) + jX(\omega)}
\]

Since component Q is defined as:

\[
Q = \frac{X(\omega)}{R(\omega)}
\]

Then Q factor at the unity-gain frequency ($\omega_0$):

\[
Q_{\text{ideal}} = \infty
\]
Real Integrator Opamp Related Non-Idealities

\[ H(s) = \frac{-\omega_b}{s} \]

\[ H(s) \approx \frac{-a}{1 + s \frac{a}{\omega_b}} (1 + \frac{s}{p_2}) (1 + \frac{s}{p_3}) \ldots \]

Effect of Integrator Finite DC Gain on Q

Example: \( a = 100 \rightarrow P1/\omega_0 = 1/100 \)
\( \rightarrow \) phase error \( \geq +0.5 \) degree
Effect of Integrator Finite DC Gain on Q
Example: Lowpass Filter

- Finite opamp DC gain
  → Phase lead @ \( \Omega_0 \)
  → Droop in the passband

- Effect of opamp finite DC gain on filter singularities
- Pushes the ideal poles away from the \( j\omega \) axis
- Results in Q reduction of the poles and thus droop in the passband
Effect of Integrator Opamp Related Non-Dominant Poles

\[ \omega \rightarrow \infty \Rightarrow \arctan \left( \sum_{i=2}^{\infty} \frac{\omega_0}{p_i} \right) \rightarrow \text{Phase lag @ } \omega_0 \]

Example: \( \omega_0 P_2 = 1/100 \rightarrow \text{phase error } \approx -0.5 \text{ degree} \)

Effect of Integrator Non-Dominant Poles
Example: Lowpass Filter

- Additional poles due to opamp poles:
  - Phase lag @ \( \omega_0 \)
  - Peaking in the passband
  - In extreme cases could result in oscillation!
Effect of Integrator Non-Dominant Poles
Example: Lowpass Filter

- Effect of opamp finite bandwidth on filter singularities
- Pushes the ideal poles towards $j\omega$ axis
- Results in $Q$ enhancement of the poles and thus peaking in the passband

Opamp with finite bandwidth
Ideal opamps & ideal pole locations

Effect of Integrator Non-Dominant Poles & Finite DC Gain on $Q$

$\angle -\frac{\pi}{2} + \arctan \frac{P1}{\omega_0}$
$\angle -\arctan \sum_{i=2}^{\infty} \frac{\omega_0}{P_i}$

$P1/\omega_0 = \sum_{i=2}^{\infty} \frac{\omega_0}{P_i}$

Note that the two terms have different signs ➔ Can cancel each other’s effect!
Integrator Quality Factor

Real intg. transfer function: \[ H(s) \approx \frac{-a}{\left(1 + \frac{s}{a_0}\right) \left(1 + \frac{s}{p_2}\right) \left(1 + \frac{s}{p_3}\right)} \ldots \]

Based on the definition of Q and assuming that:
\[ \frac{a_0}{p_{2,3},\ldots} \ll 1 \quad \text{&} \quad a > 1 \]

It can be shown that in the vicinity of unity-gain-frequency:

\[ Q_{\text{real}}^{\text{intg.}} \approx \frac{1}{\frac{1}{a} - \frac{a_0}{a} \sum_{i=2}^{\infty} \frac{1}{p_i}} \]

Example:
Effect of Integrator Finite Q on Bandpass Filter Behavior

Integrator DC gain=100

Integrator P2 @ 100, \( \omega_0 \)
Example:  
Effect of Integrator Q on Filter Behavior

Integrator DC gain = 100 & P2 @ 100. \( \omega_o \)

Effect of Integrating Capacitor Series Resistance on Integrator Q

Finite \( R_{sc} \) adds LHP zero @ \( \frac{1}{R_{sc}C} \)

\[
H(s) = \frac{-\omega_0(1+R_{sc}C)}{s}
\]

\( \rightarrow Q_{intg} = \frac{R_{sc}}{R_{sc}C} \)

Typically, opamp non-idealites dominate \( Q_{intg} \)
Summary

Effect of Integrator Non-Idealities on Q

- Amplifier finite DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements.
- Amplifier poles located above integrator unity-gain frequency enhance the Q!
  - If non-dominant poles close to unity-gain freq. → Oscillation.
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!
- Overall quality factor of the integrator has to be much higher compared to the filter’s highest pole Q.

\[ Q_{\text{ideal}} = \infty \]

\[ Q_{\text{real}} = \frac{1}{\frac{1}{a_0} - \sum_{i=2}^{\infty} \frac{1}{p_i}} \]