Homework #4

Due at 5 pm in 240 Cory on Thursday, 10/04/07
Total Points: 100

- Put (1) your name and (2) discussion section number on your homework.
- You need to put down all the derivation steps to obtain full credits of the problems. Numerical answers alone will at best receive low percentage partial credits.
- No late submission will be accepted except those with prior approval from Prof. Chang-Hasnain.
- Problems of this HW are from Hambley 4th Edition

P4.57 (Second-Order Circuits) (5 points)
The sketch should look like the = 0.1 curve in Fig 4.29. Severely underdamped ( << 1) second-order circuits display a lot of overshoot and ringing.

P4.61 (Second-Order Circuits) (20 points)
This circuit is considered on pages 185-186.
a) Writing eq. 4.101 using KCL and comparing with 4.106 gives = 1/(2RC) = 2*10^7, 0 = 1/(LC)^.5 = 10^7, and = / 0 = 2. (3 pts)
Therefore our circuit is overdamped. (2 pts)

b) Using equation 4.100 (before differentiating) and evaluating at t=0+, we get
C v'(0+) + 1/R v(0+) + 1/L \int_0^t v(t) dt + i_L(0+) = i_n(0+)
Plugging in v(0+) = 0 and i_L(0+) = 0 and v(t) =0 for t<0, and i_n(0+)=1A, we get V'(0+) =
1/C = 10^9.

c) Under steady-state conditions, the inductor acts as a short circuit. So $i_L(\infty) = 0$, so the particular solution $v_p(t) = 0$. (5 pts)

d) The complementary solution (that which solves the homogenous equation) is of the form $K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$. The particular solution was 0 by part (c). So the complete solution is $v(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t)$. Plugging this into the differential equation in 4.106 gives us $s_1 = -\frac{\sqrt{L^2 + 0^2}}{L}$ and $s_2 = -\frac{-\sqrt{L^2 + 0^2}}{L}$. Evaluating these gives $-2.679 \cdot 10^6$ and $-37.32 \cdot 10^6$. Next, use the initial conditions $v(0^+) = 0$ an $v'(0^+) = 10^9$:

$v(0^+) = 0 = K_1 + K_2$
$v'(0^+) = 10^9 = s_1 K_1 + s_2 K_2$.

Solving, one finds $K_1 = 28.87$ and $K_2 = -28.87$. Therefore the solution is $v(t) = 28.87 \exp(s_1 t) - 28.87 \exp(s_2 t)$.

P5.10 (Sinusoidal Currents and Voltages) You may also sketch the Lissajous figures by hand. (10 points, 2.5 for each plot)

You can do this in MATLAB as follows:

Theta_vect = [90 45 0 0]*pi/180;
for i = 1:4
    Wx = 2*pi;
    Wy = Wy_vect(i);
    Theta = Theta_vect(i);
    t = 0:0.01:20;
    x = cos(Wx*t);
    y = cos(Wy*t + Theta);
Subplot(2,2,i)
plot(x,y)
end

P5.16 (RMS-value) (5 points)
\[ V_{\text{rms}} = \sqrt{ \frac{1}{T} \int_{0}^{T} v^2(t) \, dt } = \sqrt{ \int_{0}^{1} [3 \exp(-t)]^2 \, dt } = \sqrt{ \int_{0}^{1} [9 \exp(-2t)] \, dt } = \sqrt{ \frac{9}{2} \left[ \exp(-2t) \right]_{t=0}^{t=1} } = \sqrt{ 4.5(1-\exp(-2)) } = 1.973 \, \text{V} \]

Elementary operations (Complex Arithmetic) (12 points, 2 each)
Perform the following operations:

a) \((5 + j3) + (3 - j7) = (5+3) + j(3-7) = 8 - j4\)
b) \((2 + j5) - (9 + j4) = (2-9) + j(5-4) = -7 + j\)
c) \((7 + j8)(4 - j2) = 4*7 + j(8*4 - 2*7) - j^2*8 = 28 + 16 + j(18) = 44 + j18\)
\[ \frac{(1 + j3)}{(4 + j9)} = \frac{(4-j9)(1+j3)}{(16+81)} = \frac{(4 + 27 + 3j)}{(97)} = \frac{(31 + 3j)}{97} = 0.3196 + j 0.0309 \]

e) Convert \((1 + j3)\) into polar form. \(= (1+9)^{0.5} \cdot e^{j \arctan(3/1)} = 10^{0.5} \cdot e^{j \arctan(3)} = 3.16 \exp(j 72^\circ)\)

f) Convert \(2e^{j \cdot 35^\circ}\) into rectangular form \(= 2 \cos(35^\circ) + j \sin(35^\circ) = 1.64 + j 1.15\)

P5.25 (Phasors) (10 points)

\(V_1(t) = 100 \cos(wt + 45^\circ)\)
\(V_2(t) = 150 \sin(wt + 60^\circ)\)

Rewriting as phasors, we get:

\(V_1 = 100 \quad 45 = 70.71 + j 70.71\)
\(V_2 = 150 \quad -30 = 129.9 - j 75\)

\(Vs = V_1 + V_2 = 200.6 - j 4.29 = 200.6 - 1.23^\circ\)

\(Vs(t) = 200.6 \cos(wt - 1.23^\circ)\)

V2 lags V1 by 75 °. Vs lags V1 by 46.23 °. Vs leads V2 by 28.77 °.
P5.24 (Phasors) (8 points)
The magnitude of the phasors are $8\sqrt{2}$ and $3\sqrt{2}$. If the phase angle is the same then you get the maximal value by just adding the phasor magnitudes to get $11\sqrt{2}$. The minimal value is achieved when they are $180^\circ$ out of phase, so that you just subtract them to get $5\sqrt{2}$.

Wheatstone Bridge (Complex Impedances) (? points)
The circuit on the right hand side shows a generalized version of the Wheatstone Bridge. $Z_1, Z_2, Z_3, Z_4$ are complex impedances.
Derive the condition for the current through the amperemeter to be zero. (3 points)

\[
\frac{Z_1}{Z_1 + Z_2} = \frac{Z_3}{Z_3 + Z_4} \quad \frac{Z_4}{Z_3} = \frac{Z_2}{Z_1}
\]

Let $f = 60 \text{Hz}$, $Z_1$ consists of a 60Ω resistor and a 0.2H inductance, $Z_2$ is a 100Ω resistor. $Z_3$ consists of a 200Ω resistor and a 10 F capacitor. Calculate the complex impedances of $Z_1, Z_2, Z_3$. (9 points)

\[
Z_1 = R + jL60 = 60 + (2\pi*60) * 0.2 = 60 + j75.4 = (60^2 + 75^2)^{1/2} = 96.4 \quad 51.5^\circ
\]

\[
Z_2 = 100
\]

\[
Z_3 = 200 - j/(2\pi*60*50*10^{-6}) = 200 - j53 \approx 200 = 206.9 - 14.8^\circ
\]

(3 points each)
Calculate $Z_4$ such that there is no current flowing through the amperemeter.

With which circuit elements can you construct $Z_4$? (6 points)

$Z_4 = Z_3*Z_2/Z_1 = (206.9 \cdot -14.8^\circ \cdot 100 / (96.4 \cdot 51.5^\circ) = 214.6 \cdot -66.3^\circ = 86.3 \cdot j196.5$

We can construct $Z_4$ out of a 86.3 resistor and a capacitor satisfying $1/(\omega C) = j196.5$, so $C = 1/(\omega 196.5) = 1/(2\pi 60 \cdot 196.5) = 13.5 \text{ F.}$

(2 point writing right equations, 2 point identifying correct element based on sign, 2 point computations)

P5.38 (Complex Impedances) (10 points)

a) From the plot, we see that $T = 4\text{ms}$, so $f = 1/T = 250\text{ Hz}$ and $\omega = 2\pi f = 500$ (1 point). Current lags (1 point) voltage by $1\text{ms} = T/4 = 90^\circ$, so we have an inductance (1 point for correct, based on previous part). $L = V_m/I_m = 5$, so $L = 3.18\text{ mH}$ (2 points).

b) We see that $T = 8\text{ms}$, so $f = 250$ (1 point). Current leads (1 point) voltage by $2\text{ms}$, or $90^\circ$, so we have a capacitance (1 point). $C = V_m/I_m = (10\text{V})/(4\text{mA}) = 2.5 \cdot 10^3 = 2500$, so $C = .5093 \text{ F}$ (2 points).