Homework #4

Due at 6 pm in 240 Cory on Wednesday, 2/14/07

Total Points: 100

- Put (1) your name and (2) discussion section number on your homework.
- You need to put down all the derivation steps to obtain full credits of the problems. Numerical answers alone will at best receive low percentage partial credits.
- No late submission will be accepted expect those with prior approval from Prof. Chang-Hasnain.

1. Hambley, P3.25 [8 points]

\[ C_{eq} = \frac{1}{1/C_1 + 1/C_2} = \frac{2}{3} \mu F \]

The charges stored on each capacitor

\[ Q = C_{eq} \times 12V = 8 \mu C \]

\[ V_1 = \frac{Q}{C_1} = 8V \]

\[ V_2 = \frac{Q}{C_2} = 4V \]

2. Hambley, P3.43 [8 points – 2 per graph]

\[ i(t) = \frac{1}{L} \int_0^t v_L(t) dt + i_L(0) = \frac{1}{2} \int_0^t v_L(t) dt \]

\[ P(t) = v_L(t) i(t) \]

\[ w(t) = \frac{1}{2} L [i_L(t)]^2 = [i_L(t)]^2 \]

![Graph of current vs time](image1.png)

![Graph of power vs time](image2.png)
3. Hambley, P4.7 [9 points]

Before t=0, v(t)=0
After t=0, apply KCL to the top node,
\[ \frac{v(t)}{R} + C \frac{dv(t)}{dt} = 1mA \]  (1) [3 points]

The solution is of the form [1 point]
\[ v(t) = K_1 + K_2 \exp(-t/RC) = K_1 + K_2 \exp(-100t) \]  (2)

substitute (2) into (1) => K1=10 [2 points]
Another boundary condition v(0)=0 => k1+k2=0 =>k2=-k1=-10 [2 points]
Thus, \[ v(t) = 10 - 10 \exp(-100t) \] [1 point]

4. Hambley, P4.8 [9 points]

The voltage can be written as
\[ v_c(t) = V_i \exp(-t/RC) \] [3 points]

At t=0, \[ v_c(0) = V_i = 50V \] [3 points]
At t=30, \[ v_c(30) = V_i \exp(-30/RC) \] => R= 4.328MΩ [3 points]

5. Hambley, P4.9 [10 points]

During the charging interval, \[ \tau_1 = R_1C = 10s \]
\[ v_c(t) = 1000[1 - \exp(-t/\tau_1)] \] 0 ≤ t ≤ 25 [3 points]

At the end of charging
\[ v_c(25) = 1000[1 - \exp(-25/\tau_1)] = 917.9V \] [2 points]

During the discharging interval, \[ \tau_2 = R_2C = 20s \]
Use another time variable \[ t' = t - 25 \]
\[ v_c(t') = 917.9 \exp(-t'/\tau_2) \] 0 ≤ t' [3 points]

For t=50, which means t'=25,
\[ v_c(25) = 917.9 \exp(-25/\tau_2) = 263V \] [2 points]
6. Hambley, P4.14 [10 points – 2 each]

For steady state, inductors act as a short circuit while capacitors act as an open circuit.

\[ i_2 = \frac{100V}{1k\Omega} = 100mA \]
\[ i_3 = 0 \]
\[ i_4 = \frac{100V}{1k\Omega} = 100mA \]
\[ i_1 = i_2 + i_3 + i_4 = 200mA \]
\[ v_c = 100V \]

7. Hambley, P4.26 [9 points]

Boundary conditions,

\[ i(0) = \frac{100V}{100\Omega} = 1A \] (1)
\[ i(\infty) = \frac{100V}{25\Omega} = 4A \]

For t>0, the current has the form,

\[ i(t) = K_1 + K_2 \exp(-Rt/L) \] (2) [3 points]

Substitute those 2 boundary conditions (1) into (2),

\[ K_1 = 4, K_2 = -3 \] [4 points]

Thus,

\[ i(t) = 1 \quad \text{for } t<0 \]
\[ i(t) = 4 - 3 \exp(-12.5t) \quad \text{for } t>0 \]

[1 point for final answer]

8. Hambley, P3.26. Do not assume constant current as stated in the problem. [10 points]

The current has the form when discharging,

\[ i(t) = 5 \exp(-t / RC_{eq}) \] [2 points]

When t=1ms,

\[ i(1ms) = 5 \exp(-t / RC_{eq}) = 4.9 \quad \Rightarrow \quad C_{eq} = 99\mu F \] [2 points]

Where \( C_{eq} = \frac{C}{2} \) [1 point]

Thus, each capacitor has capacitance of 198 \( \mu \)F

The battery must charge the capacitors from 2.45V to 2.5V in 1 second, which corresponds to average current:

\[ I = \frac{\Delta Q}{\Delta T} = \frac{C_{eq}\Delta V}{\Delta T} = \frac{(2)(198\mu F)(2.5V - 2.45V)}{1sec} = 19.8\mu A \] [3 points]
5k

2mA

30uF

10k

60uF

Assume the switch has been open for a very long time before closing at time \( t=0 \).
We want to derive an expression for \( V_1(t) \) for \( t>0 \).
(a) Re-draw an equivalent series RC circuit (using Thevenin and combining the capacitors in series)
(b) Solve for \( V_1+V_2 \) in your simplified circuit.
(c) Use your result from problem 1 to obtain an expression for \( V_1(t) \).

For \( t >0 \), regard capacitors as a load and \( R_{th} \) would be
\[ 10k/(5k+5k)=5k \Omega \] [2 points]
\[ V_{th} = V_{oc} = 2mA \times \frac{5k}{5k + 5k + 10k} \times 10k = 5V \] [2 points]
\[ C = \frac{1}{1/C_1 + 1/C_2} = 20\mu F \] [2 points]
Initial condition, \( V_1(t)+V_2(t)=0 \)
Voltage has the form,
\[ V_1(t) + V_2(t) = V_0[1 - \exp(-t / RC)] = 5[1 - \exp(-10t)] \] [5 points]
The voltage is inversely proportional to capacitance,
\[ V_1(t) = \frac{C_2}{C_1 + C_2} \left[ V_1(t) + V_2(t) \right] = \frac{10}{3} \left[ 1 - \exp(-10t) \right] \] [1 point]
Assume switch a has been open, and b closed for a very long time. Then at time t=0, switch a closes and b opens. We will solve for the inductor current $i_L$ (going from top to bottom) in two ways:

a. Norton equivalent:
(1) Draw a Norton equivalent for the circuit before time t=0 and find the value of $i_L(0-)$. Before t=0, a is open and b is closed. Regard the inductor as a load, $R_{th} = 6k // 0 = 0$

Isc=2mA

(2) Draw a Norton equivalent for the circuit after time t=0 and use this to solve for $i_L(t)$ for t>0. (Notice that we could not just draw one Norton equivalent for all times, because we can’t include the switches.)

After t=0, a is closed and b is open.

To get $R_{th}$, the current source is treated as an open circuit and the voltage source is treated as a short circuit

$R_{th} = 6k // 3k = 2k$

Apply KCL to the node of $V_x$,

$$\frac{12 - V_x}{3k} + 2mA = \frac{V_x}{6k} \Rightarrow$$

$V_x=12V=Voc=Vth$
\[ I_{sc} = \frac{V_{oc}}{R_{th}} = 6mA \]

and Norton equivalent circuit is shown below.

Apply KVL,
\[ L \frac{di_L}{dt} = i_R R = (6mA - i_L)2k \quad (2) \]

Solve this differential equation and we get,
\[ i_L(t) = 6mA - (6mA - I_0) \exp(-100000t) \quad [3 \text{ points}] \]

Note for this case, the initial current, \( I_0 \), through the inductor could be arbitrary between 0 and 2mA.

b. Superposition:
(1) Zero the current source and solve for the part of \( i_L(t) \) due to the voltage source.
\[ R_{th} = 6k \parallel 3k = 2k \]
\[ V_{th} = V_{oc} = 12V \]
\[ I_{sc} = \frac{V_{oc}}{R_{th}} = \frac{8V}{2k} = 4mA \]

The current through the inductor contributed by the voltage source would be,
\[ i_L(t) = 4mA[1 - \exp(-100000t)] \quad [3 \text{ points}] \]

(2) Zero the voltage source and solve for the part of \( i_L(t) \) due to the current source.
\[ R_{th} = 6k \parallel 3k = 2k \]
\[ I_{sc} = 2mA \]

The current through the conductor contributed by the current source would be
\[ i_L(t) = 2mA - (2mA - I_0) \exp(-100000t) \quad [3 \text{ points}] \]

(3) Add these together to get the total current.
\[ i_L(t) = 6mA - (6mA - I_0) \exp(-100000t) \quad [1 \text{ point}] \]

(4) Do your results agree with part (a)? Why or why not?
Yes. Since RLC circuit is a linear system, superposition will hold. [1 point]