Crypto tricks: Proof of work, Hash chaining

CS 161: Computer Security

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A Tangent:
How Can I Prove I Am Rich?

Math Puzzle – Proof of Work

- **Problem.** To prove to Bob I'm not a spammer, Bob wants me to do 10 seconds of computation before I can send him an email. How can I prove to Bob that I wasted 10 seconds of CPU time, in a way that he can verify in milliseconds?

Math Puzzle – Proof of Work

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- Hint: Computing 1 billion SHA256 hashes might take 10 seconds.

Your Solution

- Bob provides a random challenge r
- I compute: find x such that H(r,x) starts with 33 0 bits
  - This will take me 2^33 hash computations, on average
  - Geometric: coin flip, with 1 / 2^33 chance of heads
- Bob verifies by: checking that H(r,x) starts with 33 0 bits
- Problem: replay attacks

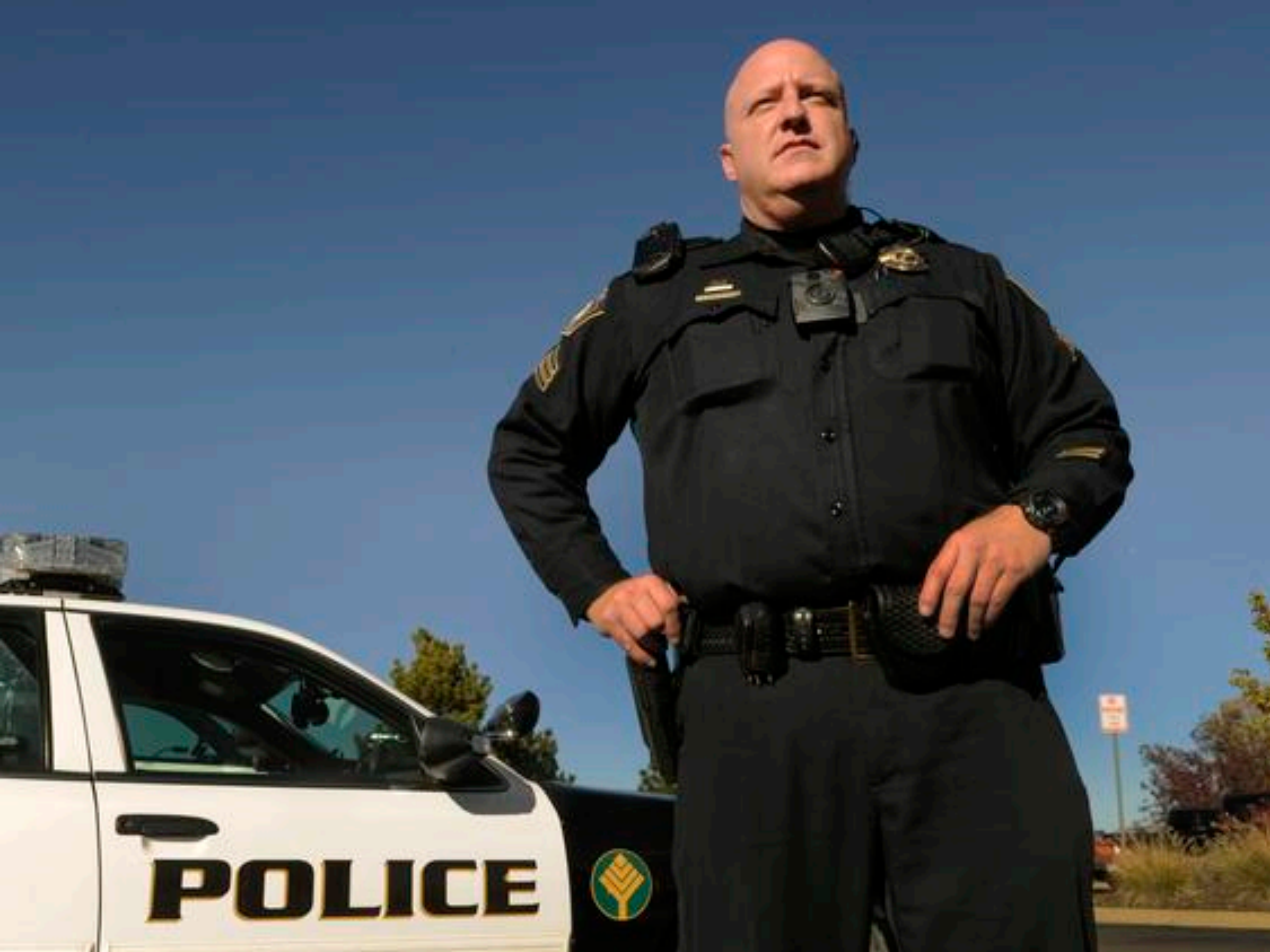
Your Solution

- Bob picks 50-bit primes p,q, sends me n = pq
- I have to factor n, send back p and q
- Bob can verify by multiply p*q

Solution

- To prove that I wasted 10 seconds of CPU time, in a way that he can verify quickly:

- Bob sends me: *r*
- I look for *x* such that first30bits(SHA256(*x* || *r*)) = 0
- I send Bob: *x*
- Bob can verify using a single hash.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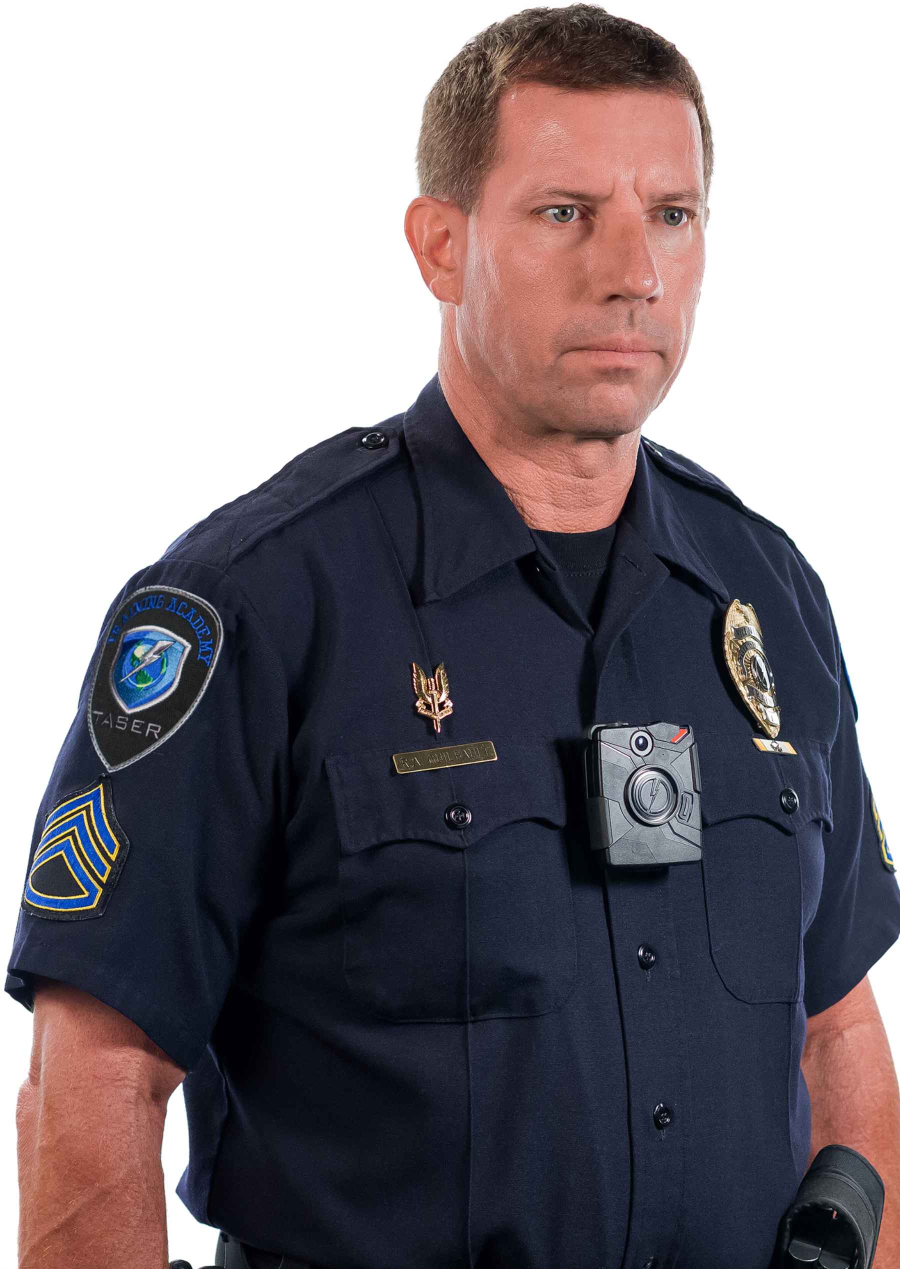
TASER

Tamper-Evident Logging

- We work for the police Electronic Records office.
- To ensure that evidence can’t be questioned in court, we want to make sure that evidence can’t be tampered with, after it is logged with the office.
- In other words: a police officer can log an electronic file at any time; after it is logged, no back-dating or after-the-fact changes to evidence should be possible.
- How should we do it? What crypto or data structures could we use?

Design Problem for You

- Idea: Each day, collect all the files (f_1, f_2, …, f_n) that are logged that day. Then, publish something in the next day's newspaper, to commit to these files.
- Question: What should we publish?
  Needs to be short, and ensure after-the-fact changes or backdating are detectable.
- When a file f_i is exhibited into evidence in a trial, how can judge verify it hasn't been modified post-facto?

Your Solution

- Store in database: f1, .., fn
- Publish: H(f1), H(f2), .., H(fn)
- To verify f_i : reveal fi

Your Solution

- Store in database: f1, .., fn
- Publish: H(H(f1), H(f2), .., H(fn))
- To verify f_i: reveal fi, H(f1), H(f2), .., H(fn)

Your Solution

- Store in database: f1, .., fn
- Publish: Sign(f1), Sign(f2), .., Sign(fn), signed under judge's key
- To verify f_i: reveal fi

Candidate Solution

- Store in database: f_1, Sign(f_1), f_2, Sign(f_2), …, f_n, Sign(f_n)
- Publish: public key
- To verify f_i: reveal f_1, Sign(f_i)

- Critique: Sysadmin can get a copy of the private key, modify database, update the signature, and thus modify old entries or create new backdated entries.

Candidate Solution

- Publish: $H(f_1, f_2, \ldots, f_n)$
- To verify f_i: reveal $f_1, f_2, \ldots, f_n$

Solution

- Each day, collect all the files (f_1, f_2, …, f_n) that are logged that day. Then, publish $H(f_1, f_2, \ldots, f_n)$ in the next day's newspaper, to commit to these files.
- When a file f_i is exhibited into evidence in a trial, reveal f_1, f_2, …, f_n to judge. Judge can hash them, check that their hash was in the right day's newspaper, and check that f_i is in the list.

Better Solution

- Each day, collect all the files (f_1, f_2, …, f_n) that are logged that day. Let f_0 be the previous day's hash. Publish $H(f_0, f_1, f_2, \ldots, f_n)$ in the next day's newspaper, to commit to these files.
- Note that exhibiting file f_i into evidence still requires revealing entire list of other files (f_1, f_2, …, f_n) that were logged that day. Can you think of any way to avoid that?

Tamper-evident Audit Logs

- $X1 = H(X0,$ “opened vault”)
- $X2 = H(X1,$ “disabled alarm”)
- $X3 = H(X2,$ “closed alarm”)
- $X4 = H(X3,$ “front door locked”)
- $X5 = H(X4,$ “closed vault”)

- Publishing any Xi commits to all prior log entries.

Take-away

- Using hash chaining, we can provide tamper-evident audit logs that let us detect after-the-fact modifications and backdated entries.

Bitcoin

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Distributed Logging

- Let's do distributed peer-to-peer logging of public data. We have *n* computers; they all know each others' public keys. Any computer can broadcast to all others (instantaneously, reliably). Any computer should be able to append a signed entry to the log, and to verify integrity of any previous log entry.
- Security goal: Malicious computers should not be able to back-date entries or modify past log entries. Assume ≤ 3 computers are malicious.

- **Problem 1.** Describe a protocol for this. What does Alice do to append an entry? What do other computers need to do?

Your Solution

- To append log entry e:
- Other computers should:

Distributed Logging

- **Problem 2.** Let's generalize. Suppose *m* of the *n* computers are malicious. If we make the obvious change to your protocol, for which *m* can it be made secure?

- (a): for all $m < n$.
- (b): for all $m < n/2$.
- (c): for all $m < n/3$.
- (d): for all $m < \sqrt{n}$.
- (e): for all $m < O(\lg n)$.

Distributed Logging

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Distributed Money

- Donna gets the brilliant idea to use this log to store financial transactions. Each person's initial balance is public.
- To transfer $10 from Alice to Bob, Alice appends a signed log entry saying "I transfer $10 to Bob" and broadcasts it. Everyone can compute the updated balance for Alice and Bob.

- **Problem 3.** What are some ways that a malicious actor might try to attack this scheme? Is this a good scheme?

Your Answers

- Replay
- Denial of service attacks
- Broadcast doesn’t scale
- TOCTTOU vulnerability

Problems with This Scheme

- Initial balance is arbitrary
- Broadcasting is expensive and doesn’t scale
- A conspiracy of *n*/2 malicious computers can fork the audit log and steal all the money
- Sybil attacks: Anyone can set up millions of servers and thus have a 50% majority

Bitcoin

- Public, distributed, peer-to-peer audit log of all transactions.
- To append an entry to the log, the latest value must hash to something whose first 30 bits are zero; then broadcast it to everyone.
- Anyone who appends an entry to the log is given a small reward, in new money (a fraction of a Bitcoin).