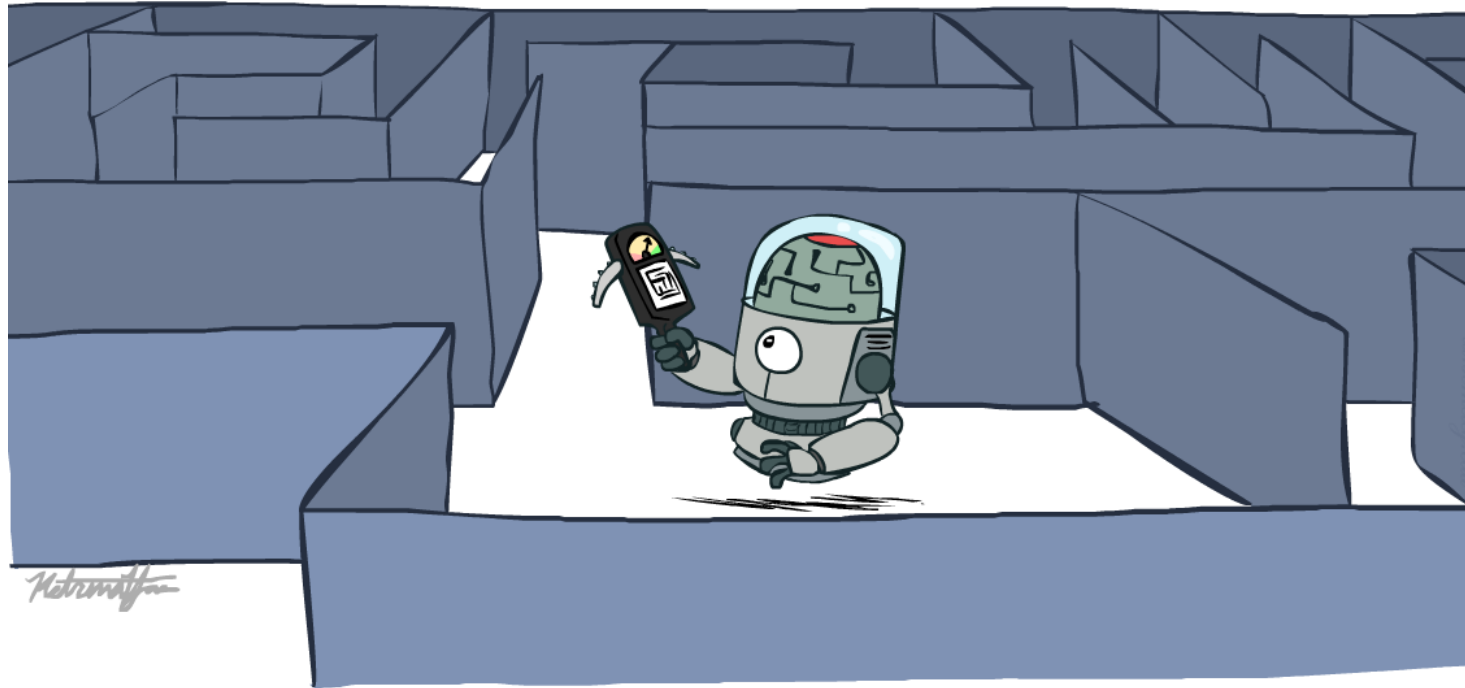


Announcements

- **Homework 1: Search**
 - Has been released! Due **Feb 4th, at 11:59pm.**
 - Electronic component: on Gradescope, instant grading, submit as often as you like.
 - Written component: exam-style template to be completed (we recommend on paper) and to be submitted into Gradescope (graded on effort/completion)
- **Project 1: Search**
 - Has been released! Due **Feb 8th, at 4pm.**
 - Start early and ask questions. It's longer than most!
- **Mini-Contest 1 released (optional)**
 - Due **Feb 6th, at 9 pm.**
- **Sections**
 - Check Piazza poll for section attendance! Try to load balance!
- **My office hours after class – still no room ☹️**

CS 188: Artificial Intelligence

Informed Search



Instructors: Sergey Levine & Stuart Russell

University of California, Berkeley

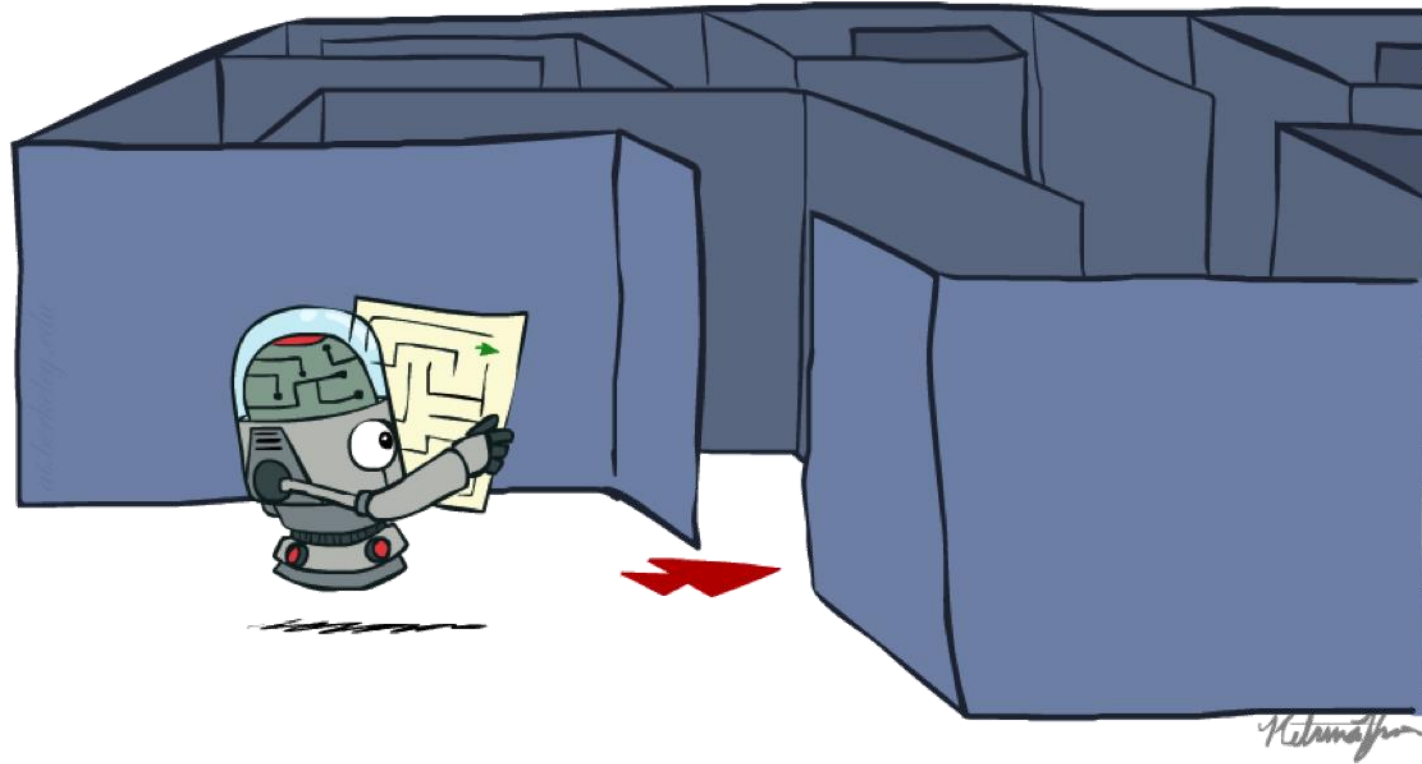
[slides adapted from Dan Klein, Pieter Abbeel]

Today

- Informed Search
 - Heuristics
 - Greedy Search
 - A* Search
- Graph Search

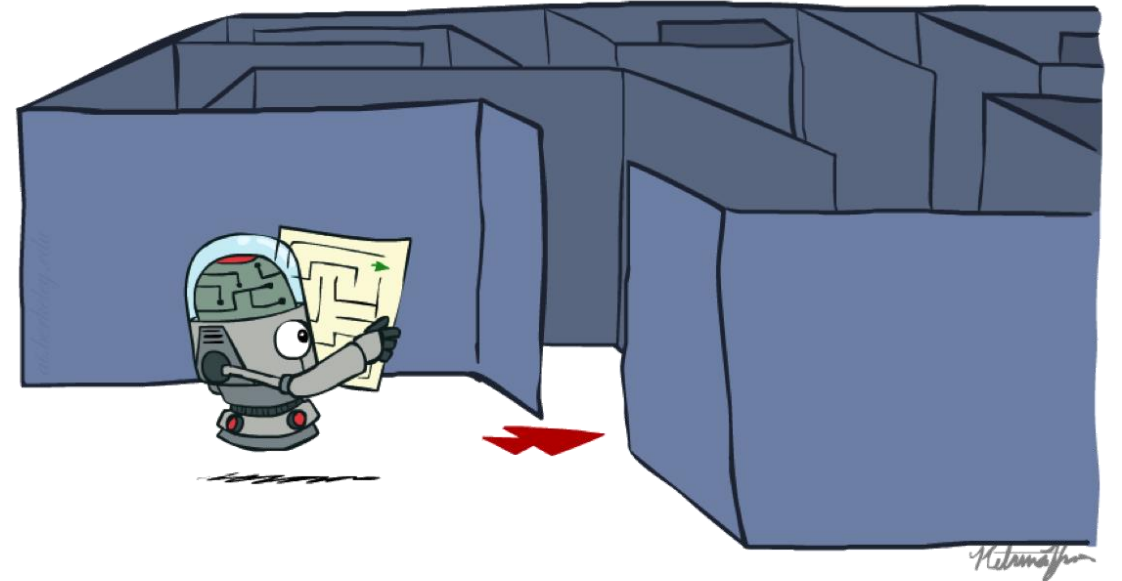


Recap: Search

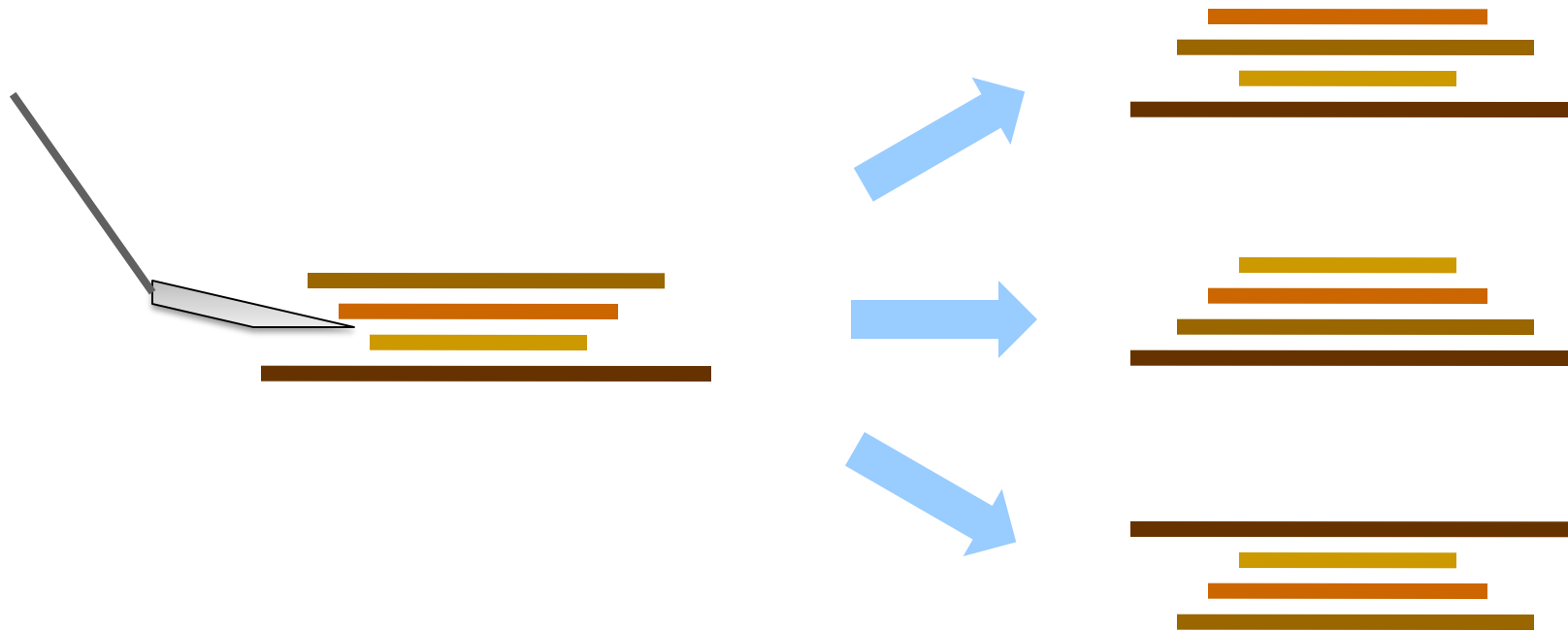


Recap: Search

- **Search problem:**
 - States (configurations of the world)
 - Actions and costs
 - Successor function (world dynamics)
 - Start state and goal test
- **Search tree:**
 - Nodes: represent plans for reaching states
 - Plans have costs (sum of action costs)
- **Search algorithm:**
 - Systematically builds a search tree
 - Chooses an ordering of the fringe (unexplored nodes)
 - Optimal: finds least-cost plans



Example: Pancake Problem



Cost: Number of pancakes flipped

Example: Pancake Problem

BOUNDS FOR SORTING BY PREFIX REVERSAL

William H. GATES

Microsoft, Albuquerque, New Mexico

Christos H. PAPANIMITRIOU*†

Department of Electrical Engineering, University of California, Berkeley, CA 94720, U.S.A.

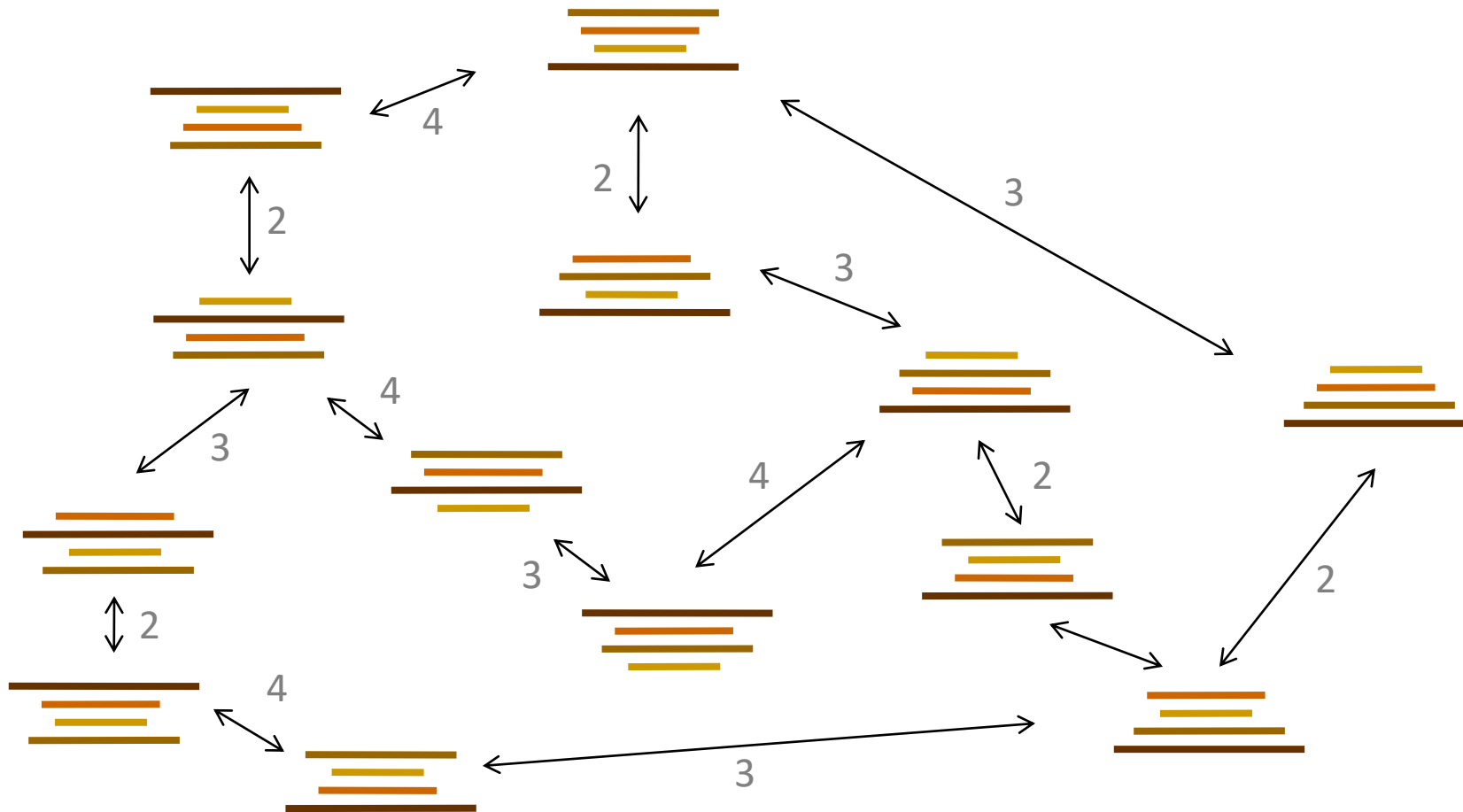
Received 18 January 1978

Revised 28 August 1978

For a permutation σ of the integers from 1 to n , let $f(\sigma)$ be the smallest number of prefix reversals that will transform σ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all σ in (the symmetric group) S_n . We show that $f(n) \leq (5n+5)/3$, and that $f(n) \geq 17n/16$ for n a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leq g(n) \leq 2n + 3$.

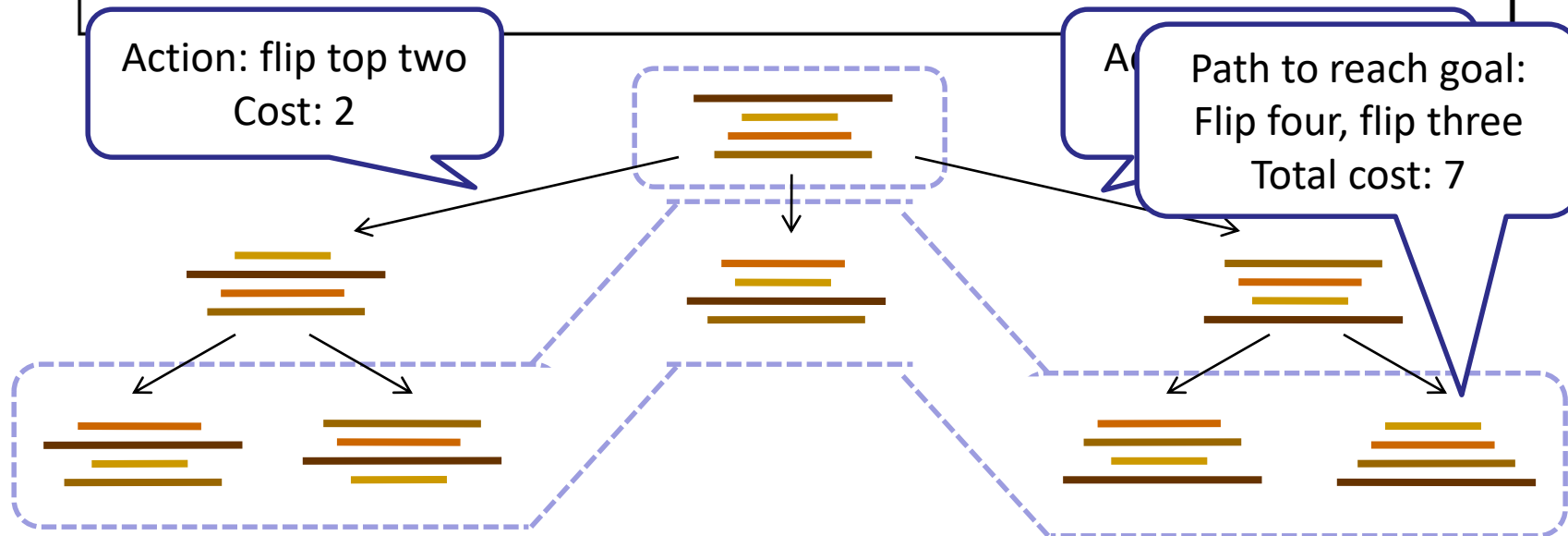
Example: Pancake Problem

State space graph with costs as weights



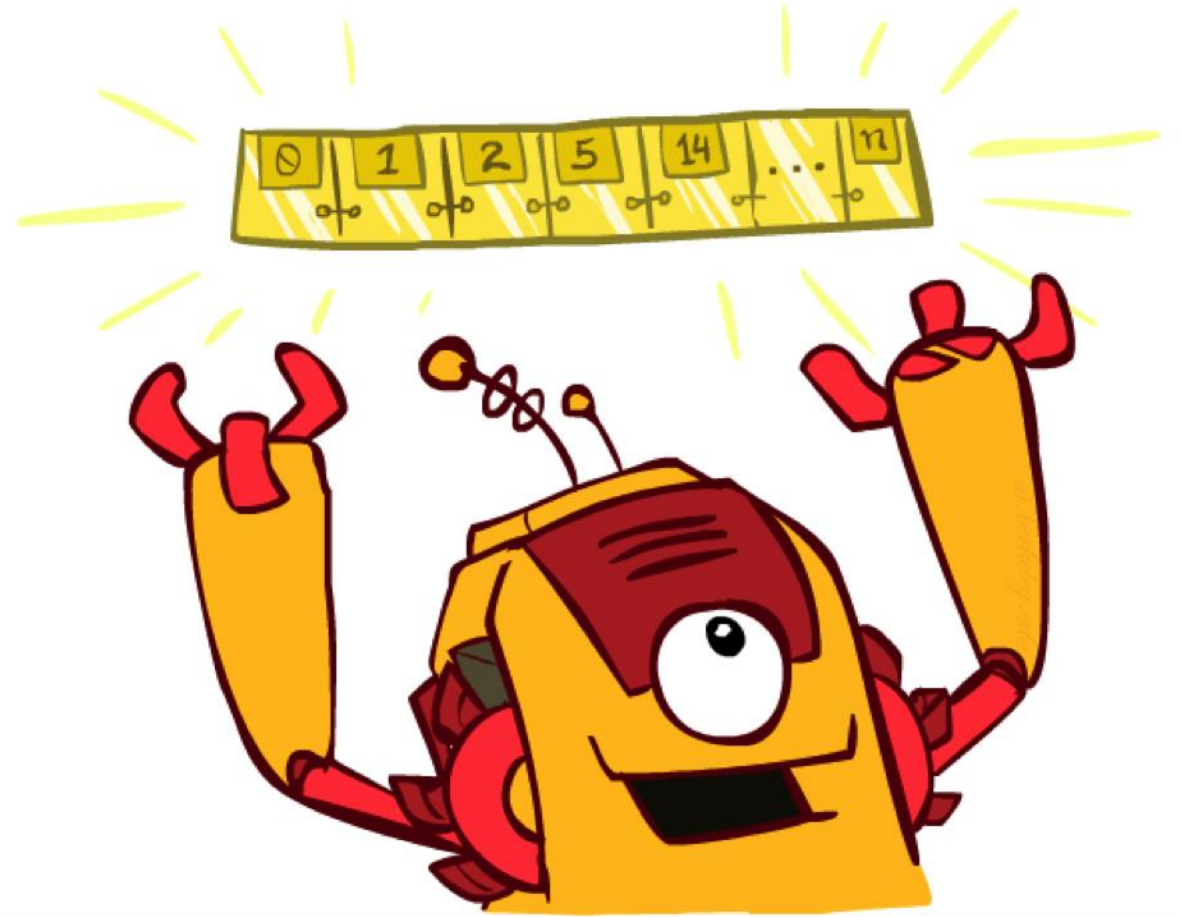
General Tree Search

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

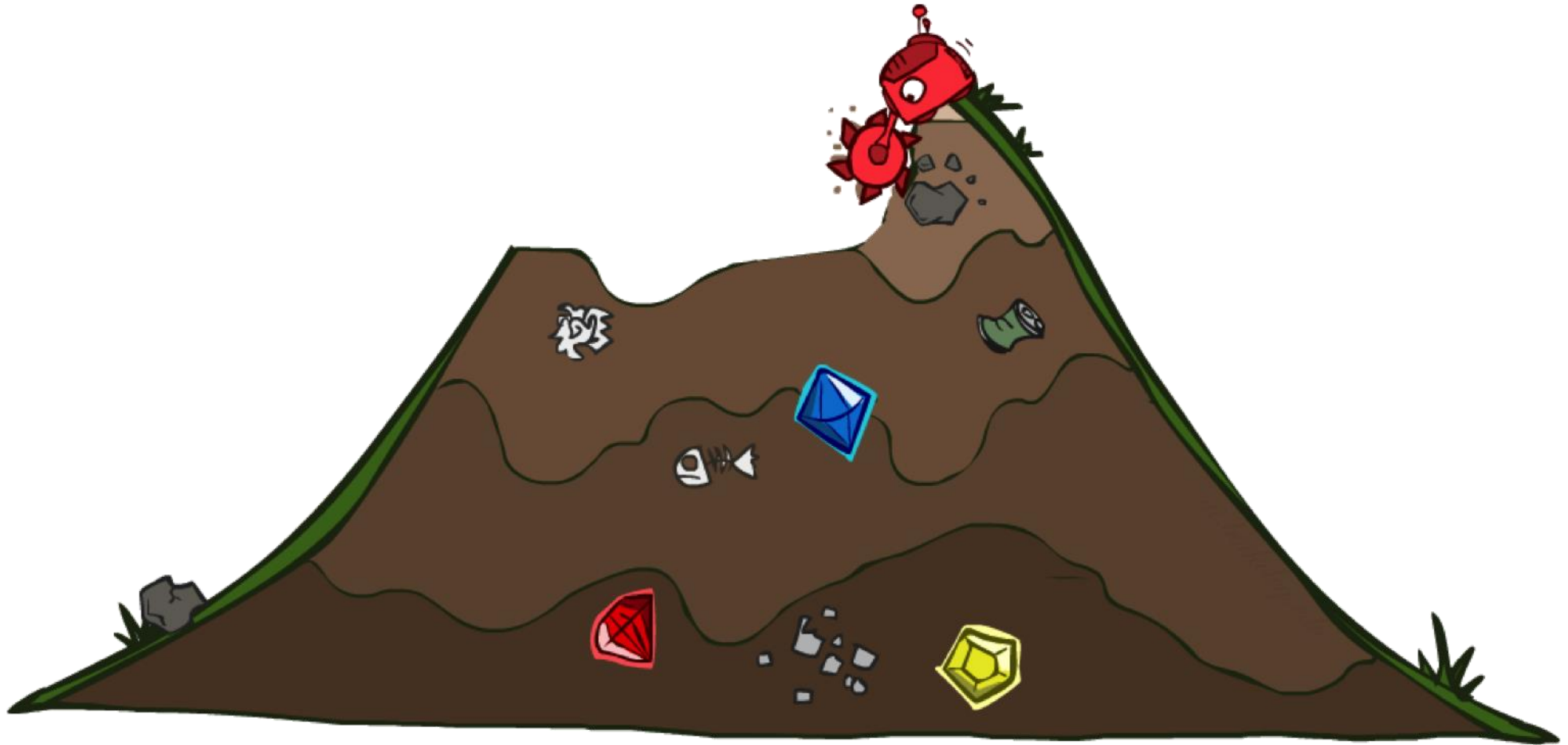


The One Queue

- All these search algorithms are the same except for fringe strategies
 - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
 - Practically, for DFS and BFS, you can avoid the $\log(n)$ overhead from an actual priority queue, by using stacks and queues
 - Can even code one implementation that takes a variable queuing object

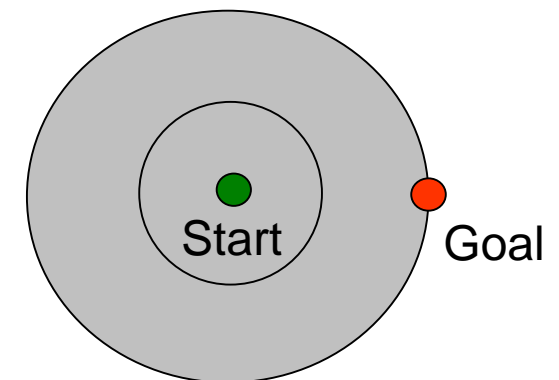
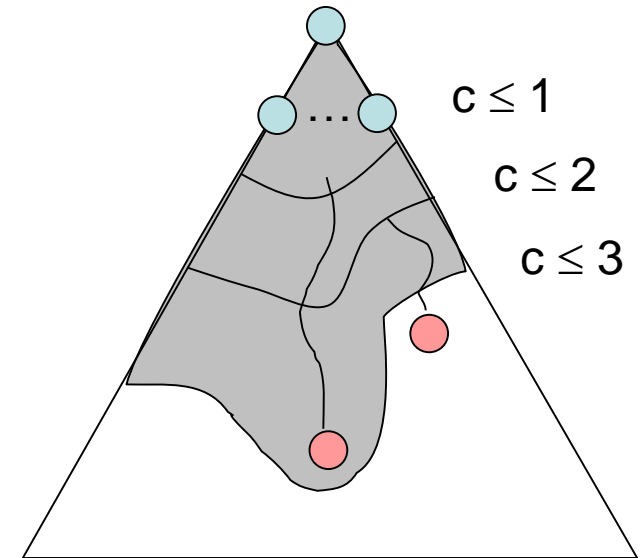


Uninformed Search



Uniform Cost Search

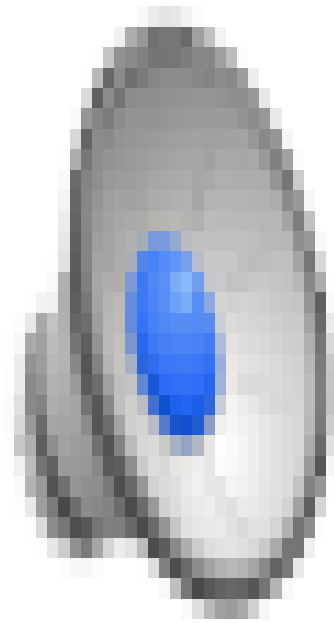
- Strategy: expand lowest path cost
- The good: UCS is complete and optimal!
- The bad:
 - Explores options in every “direction”
 - No information about goal location



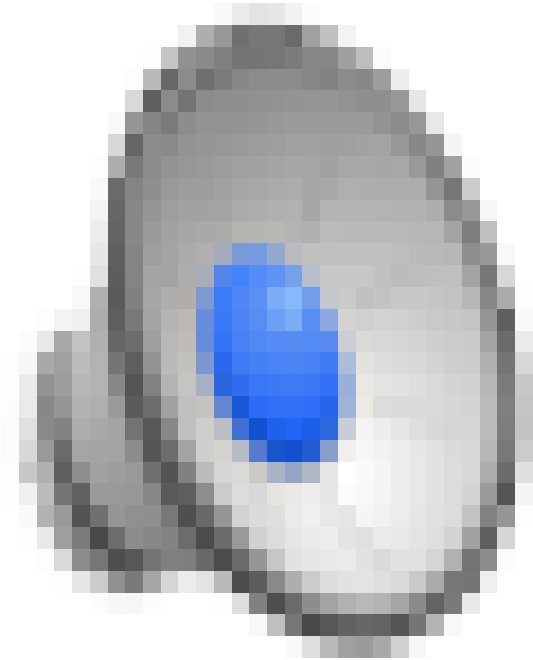
[Demo: contours UCS empty (L3D1)]

[Demo: contours UCS pacman small maze (L3D3)]

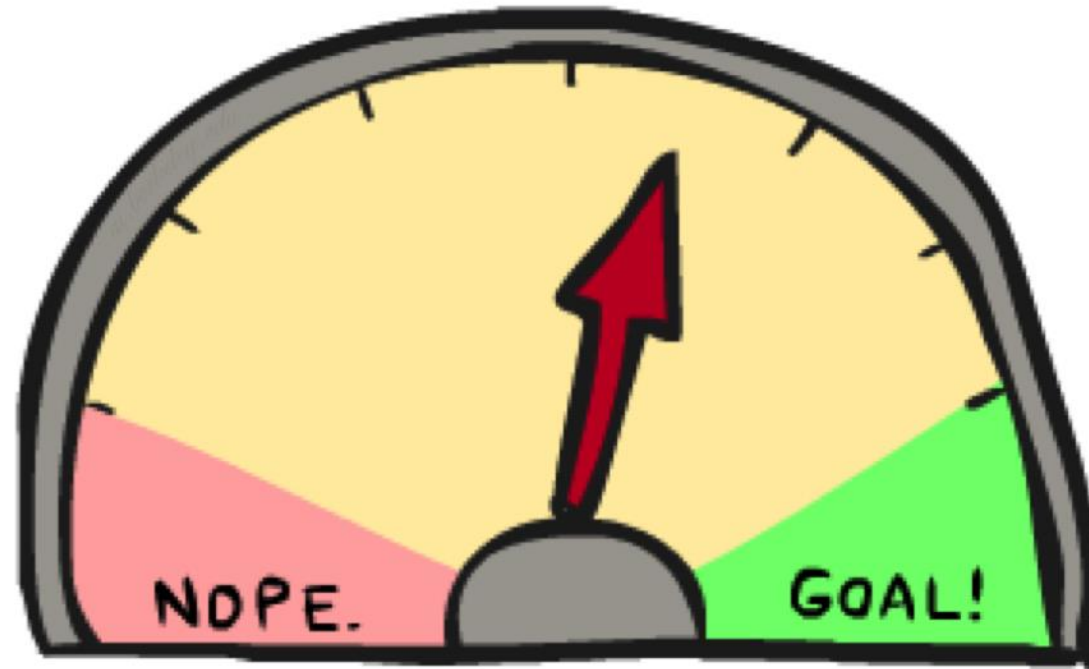
Video of Demo Contours UCS Empty



Video of Demo Contours UCS Pacman Small Maze

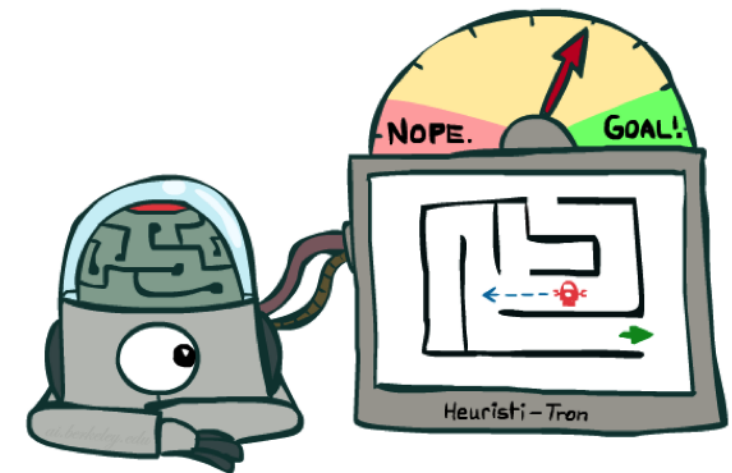
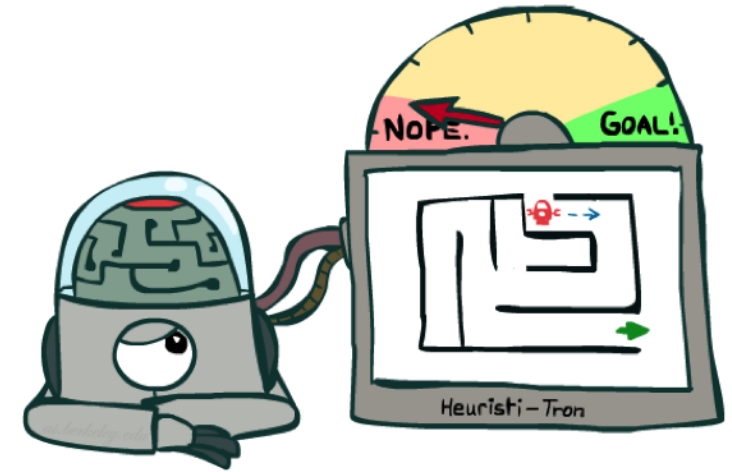
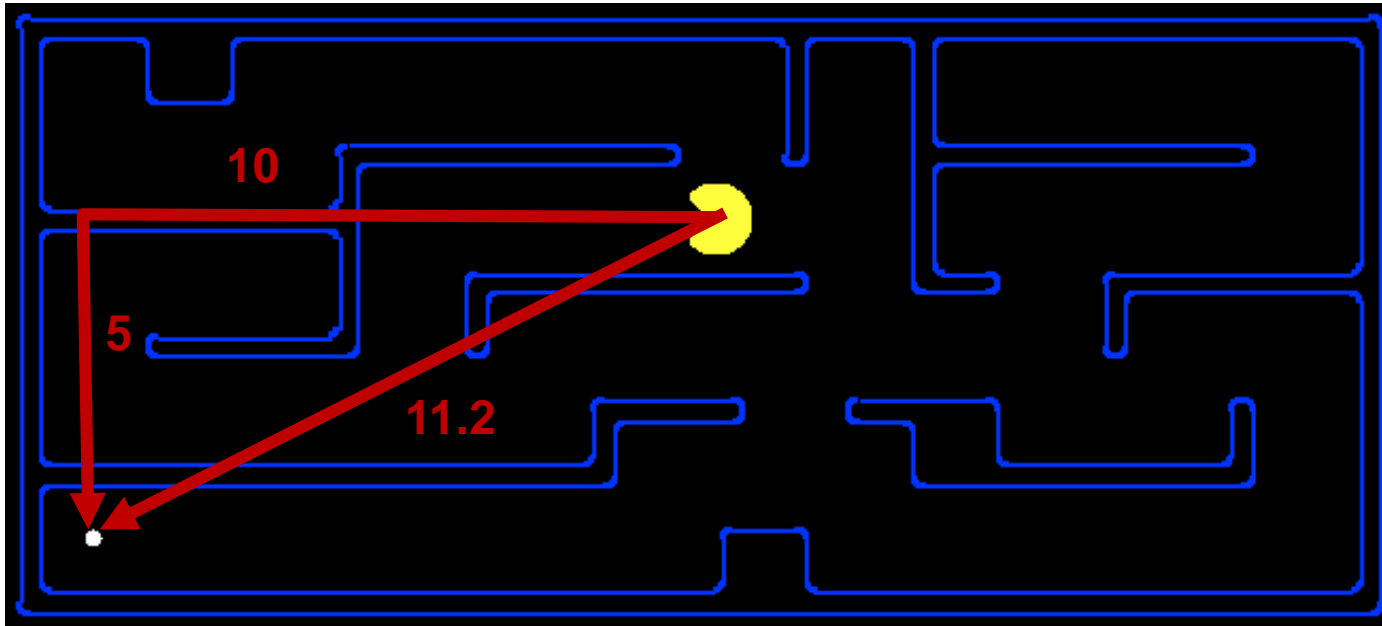


Informed Search

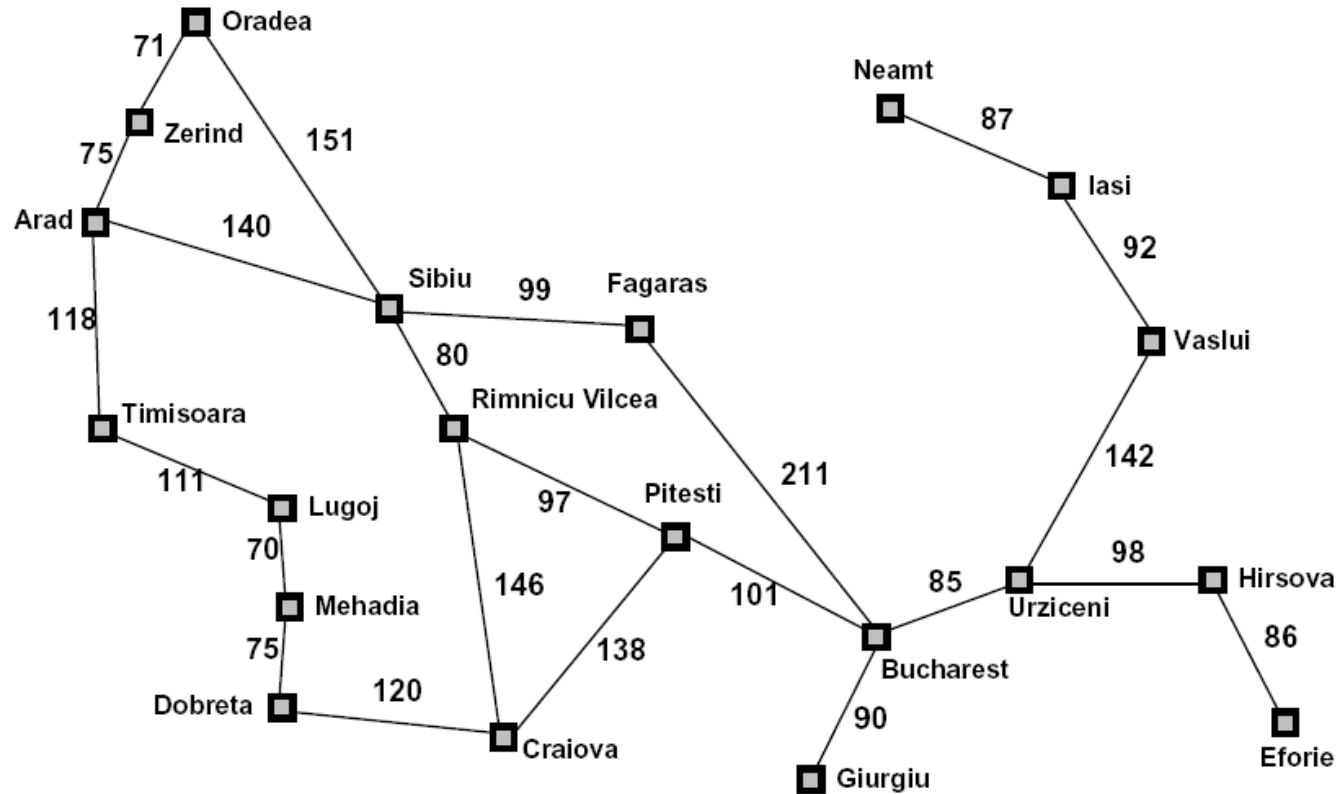


Search Heuristics

- A heuristic is:
 - A function that *estimates* how close a state is to a goal
 - Designed for a particular search problem
 - Examples: Manhattan distance, Euclidean distance for pathing



Example: Heuristic Function

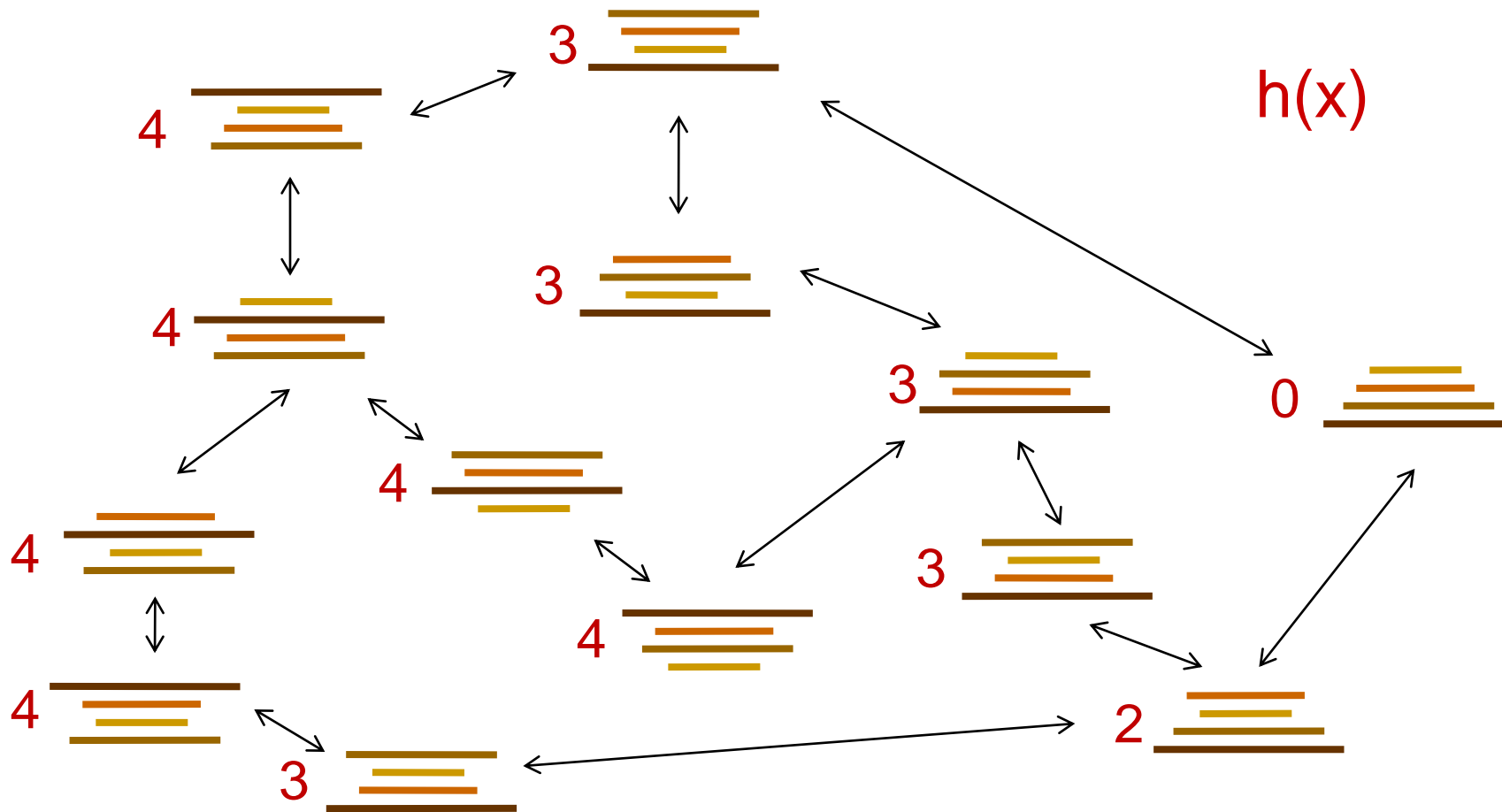


| Straight-line distance to Bucharest | |
|-------------------------------------|-----|
| Arad | 366 |
| Bucharest | 0 |
| Craiova | 160 |
| Dobreta | 242 |
| Eforie | 161 |
| Fagaras | 178 |
| Giurgiu | 77 |
| Hirsova | 151 |
| Iasi | 226 |
| Lugoj | 244 |
| Mehadia | 241 |
| Neamt | 234 |
| Oradea | 380 |
| Pitesti | 98 |
| Rimnicu Vilcea | 193 |
| Sibiu | 253 |
| Timisoara | 329 |
| Urziceni | 80 |
| Vaslui | 199 |
| Zerind | 374 |

$h(x)$

Example: Heuristic Function

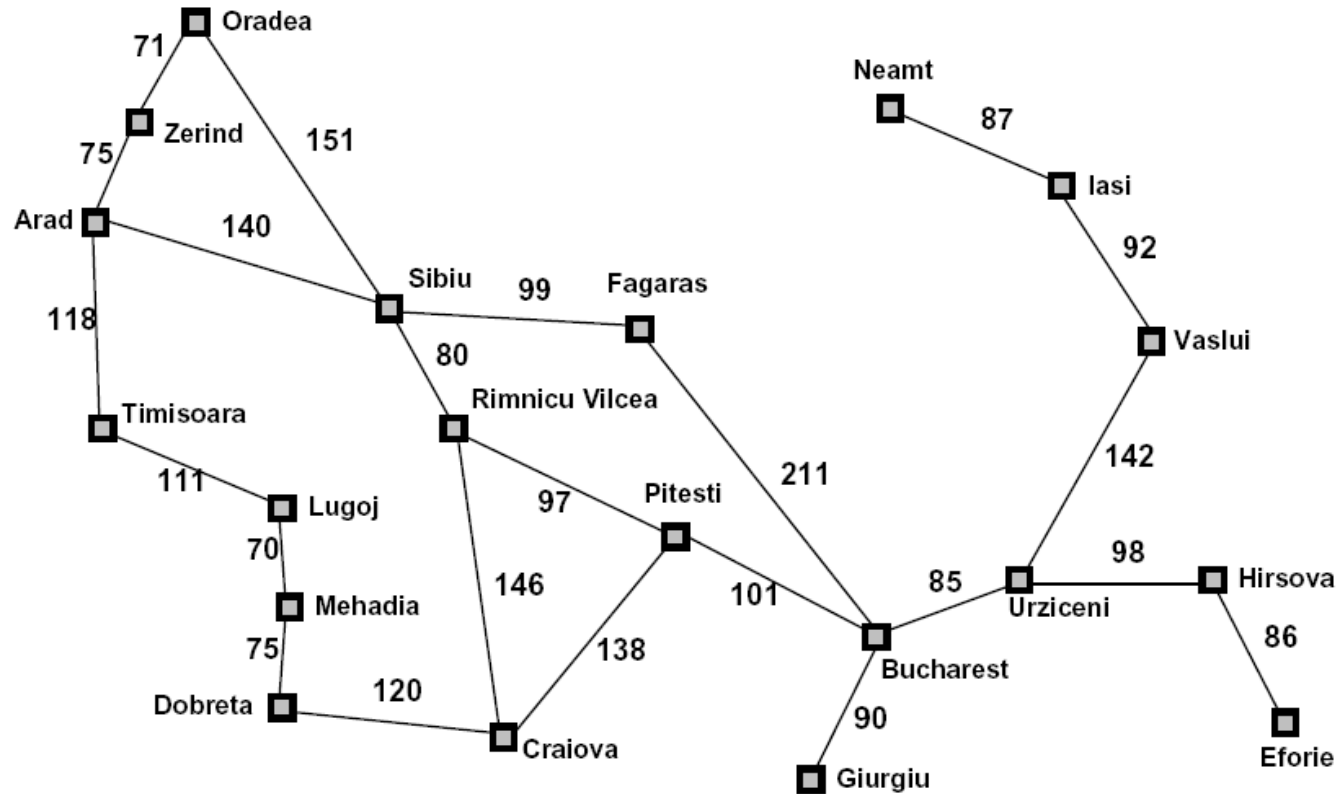
Heuristic: the number of the largest pancake that is still out of place



Greedy Search



Example: Heuristic Function

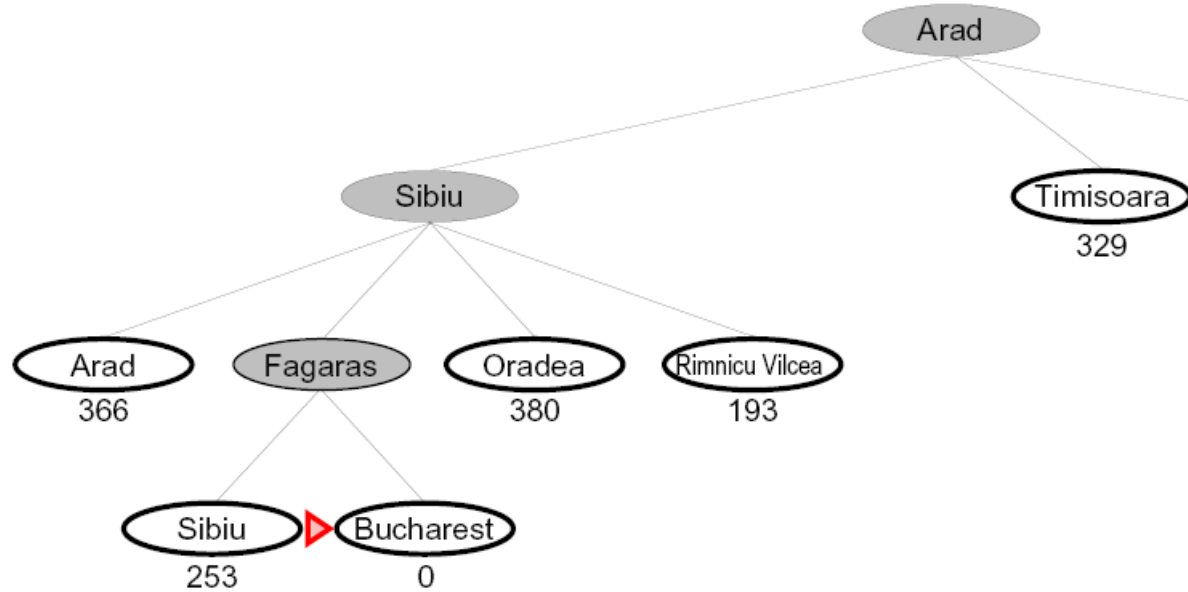


| Straight-line distance to Bucharest | |
|-------------------------------------|-----|
| Arad | 366 |
| Bucharest | 0 |
| Craiova | 160 |
| Dobreta | 242 |
| Eforie | 161 |
| Fagaras | 178 |
| Giurgiu | 77 |
| Hirsova | 151 |
| Iasi | 226 |
| Lugoj | 244 |
| Mehadia | 241 |
| Neamt | 234 |
| Oradea | 380 |
| Pitesti | 98 |
| Rimnicu Vilcea | 193 |
| Sibiu | 253 |
| Timisoara | 329 |
| Urziceni | 80 |
| Vaslui | 199 |
| Zerind | 374 |

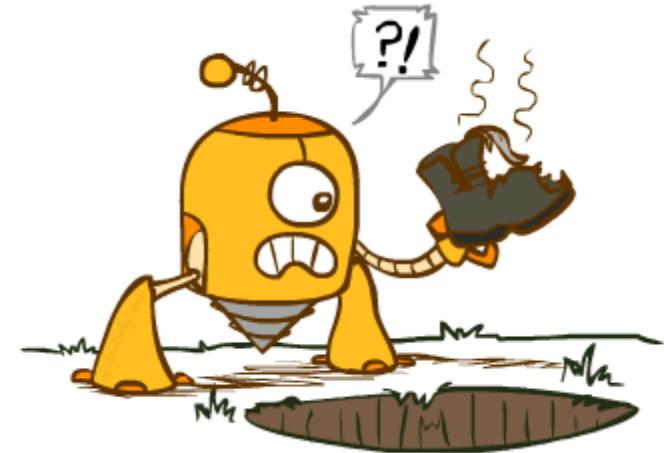
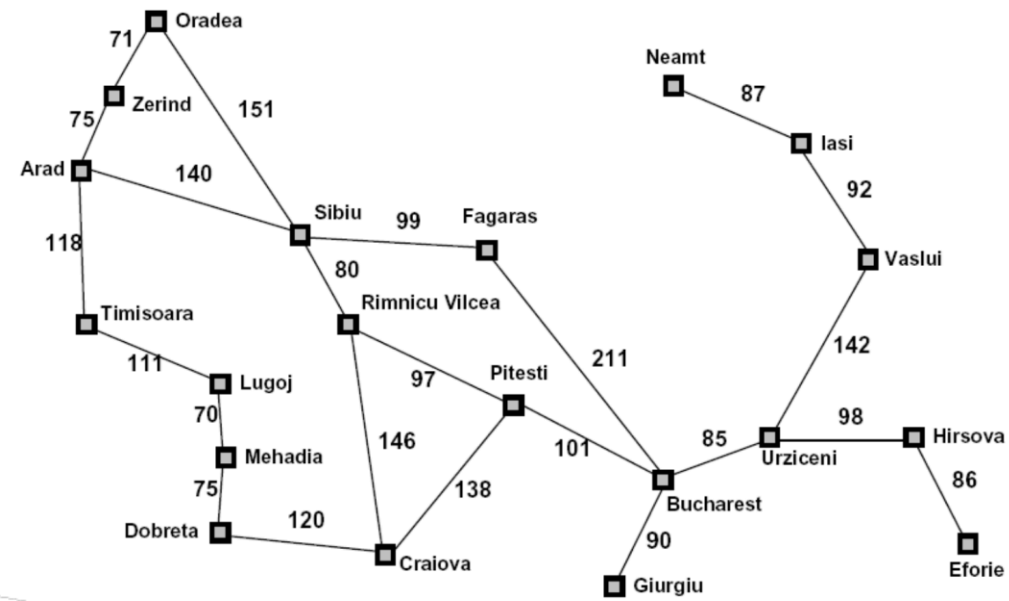
$h(x)$

Greedy Search

- Expand the node that seems closest...

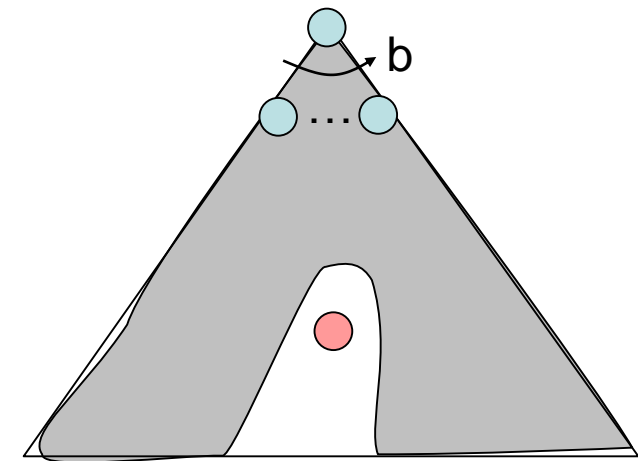
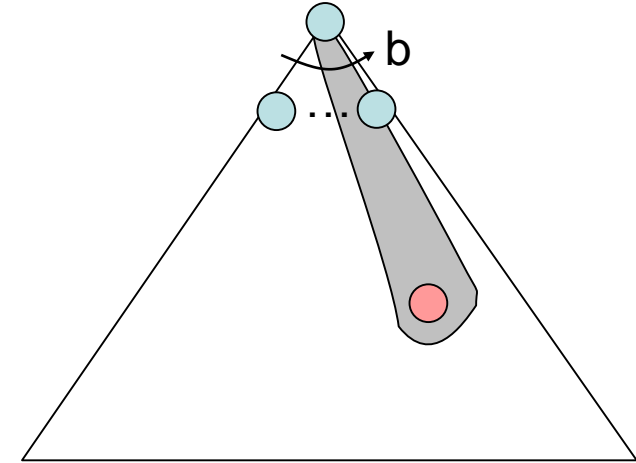


- What can go wrong?



Greedy Search

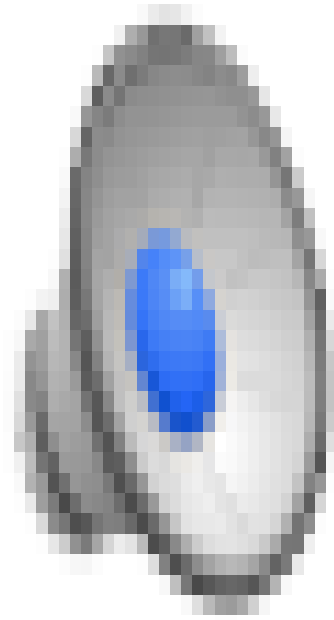
- Strategy: expand a node that you think is closest to a goal state
 - Heuristic: estimate of distance to nearest goal for each state
- A common case:
 - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS



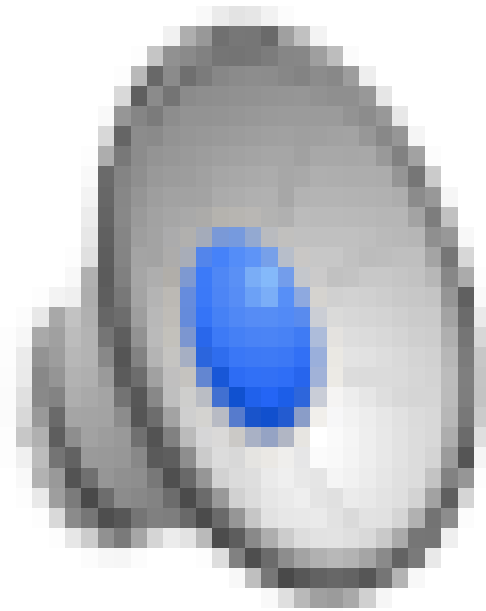
[Demo: contours greedy empty (L3D1)]

[Demo: contours greedy pacman small maze (L3D4)]

Video of Demo Contours Greedy (Empty)



Video of Demo Contours Greedy (Pacman Small Maze)



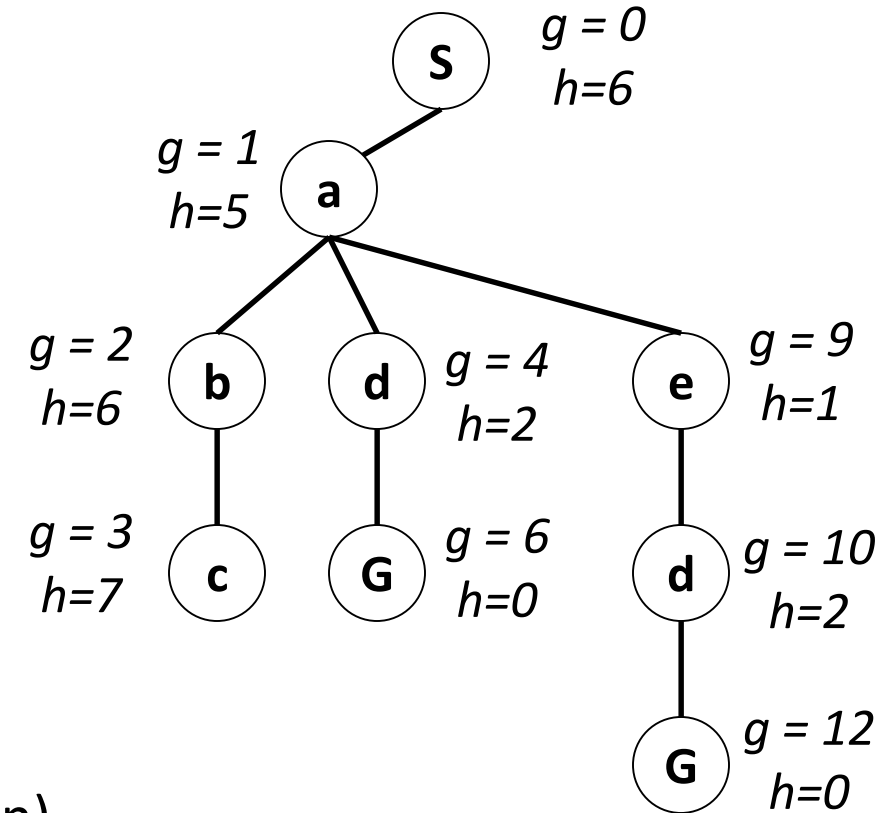
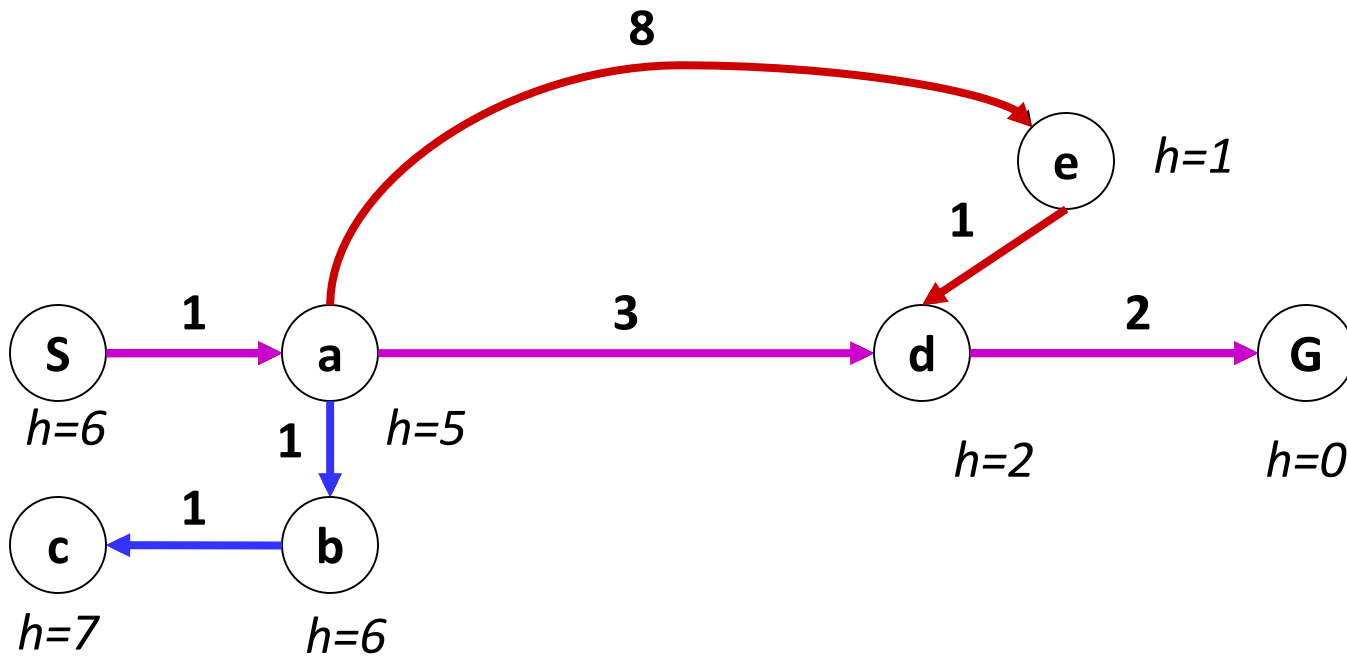
A* Search



A* Search

Combining UCS and Greedy

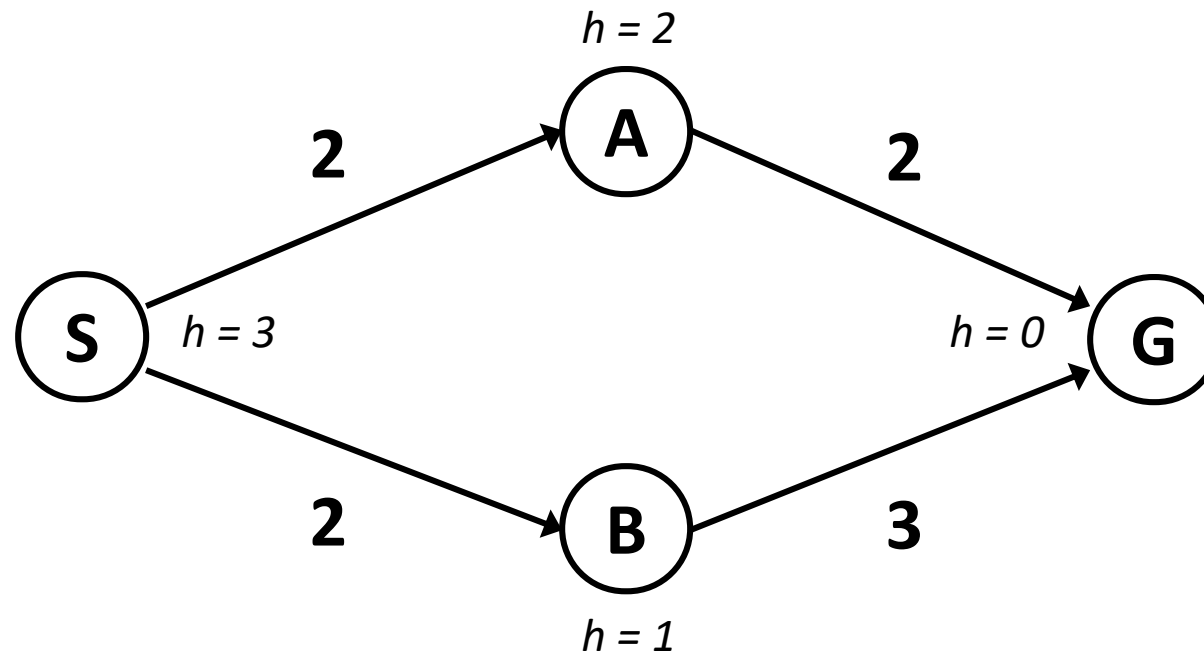
- Uniform-cost orders by path cost, or *backward cost* $g(n)$
- Greedy orders by goal proximity, or *forward cost* $h(n)$



- A* Search orders by the sum: $f(n) = g(n) + h(n)$

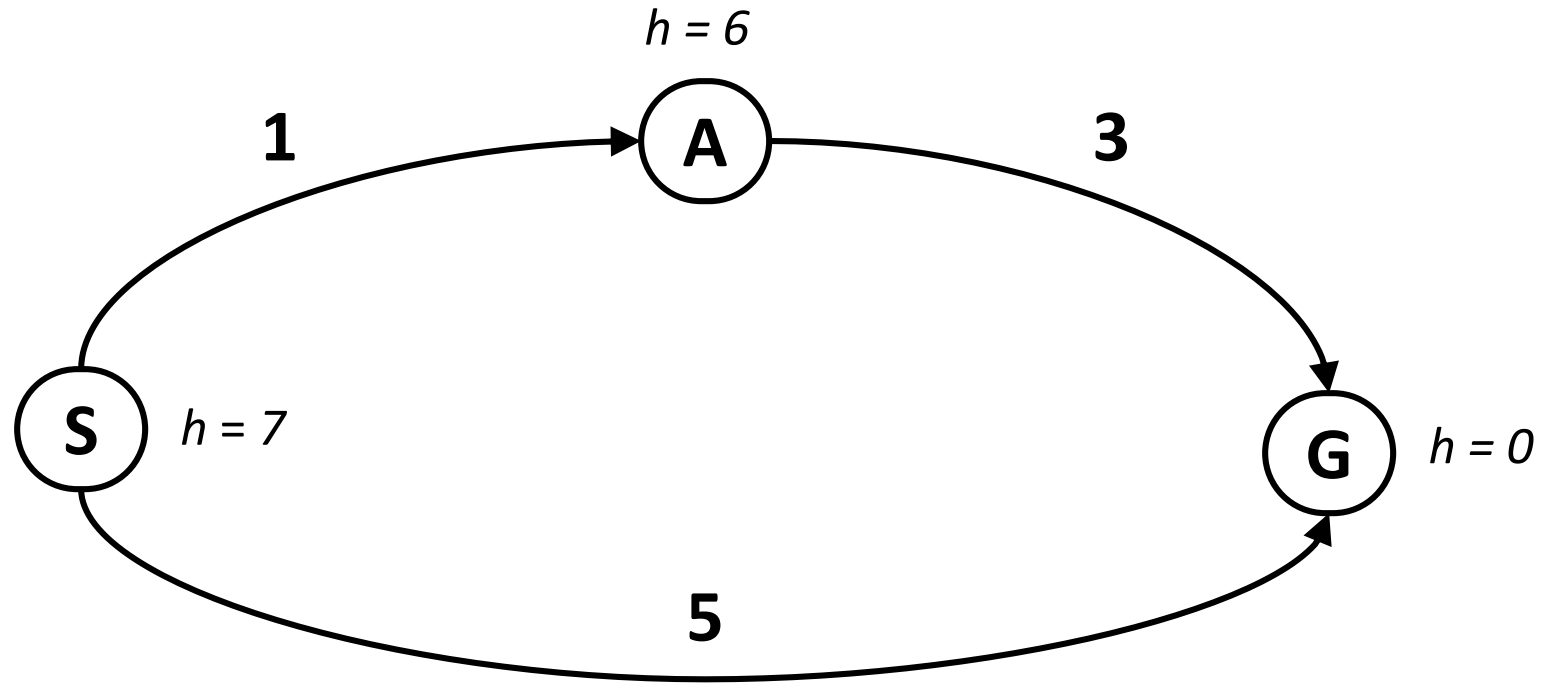
When should A* terminate?

- Should we stop when we enqueue a goal?



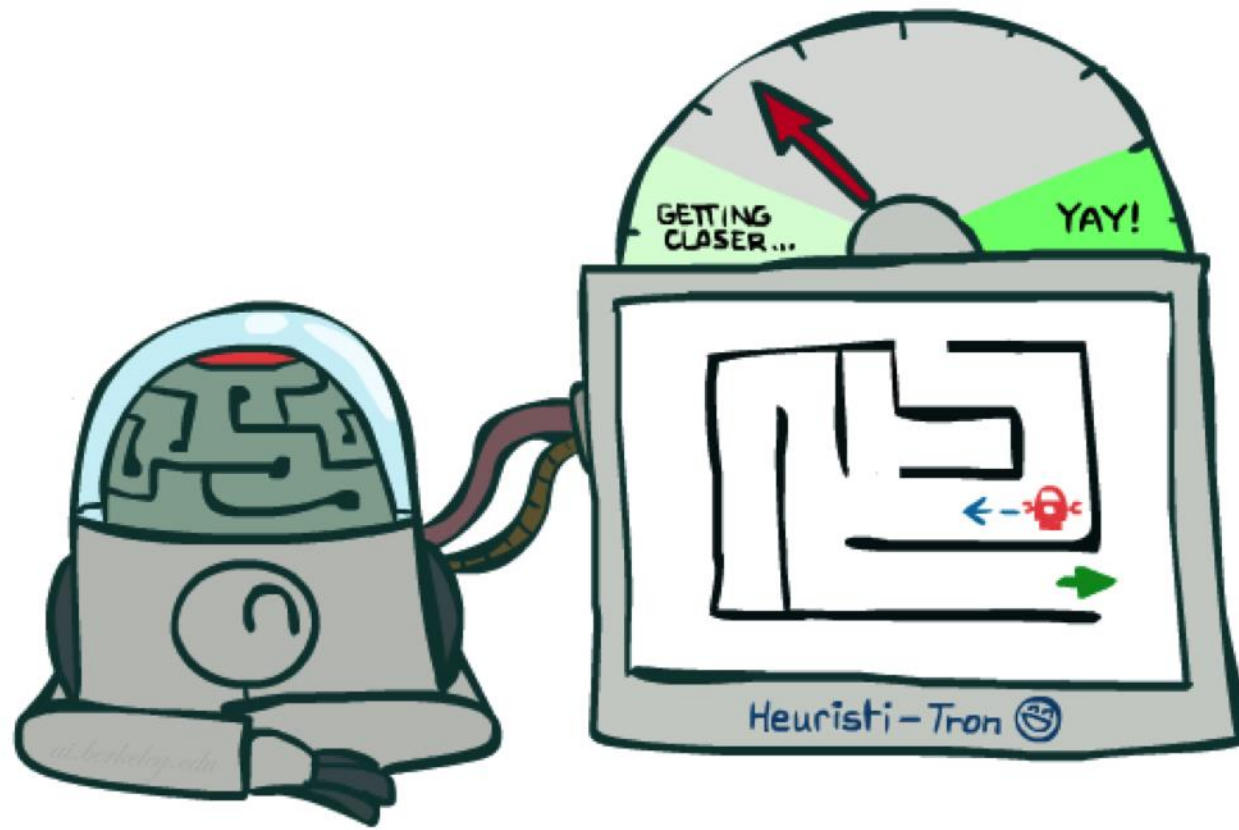
- No: only stop when we dequeue a goal

Is A* Optimal?

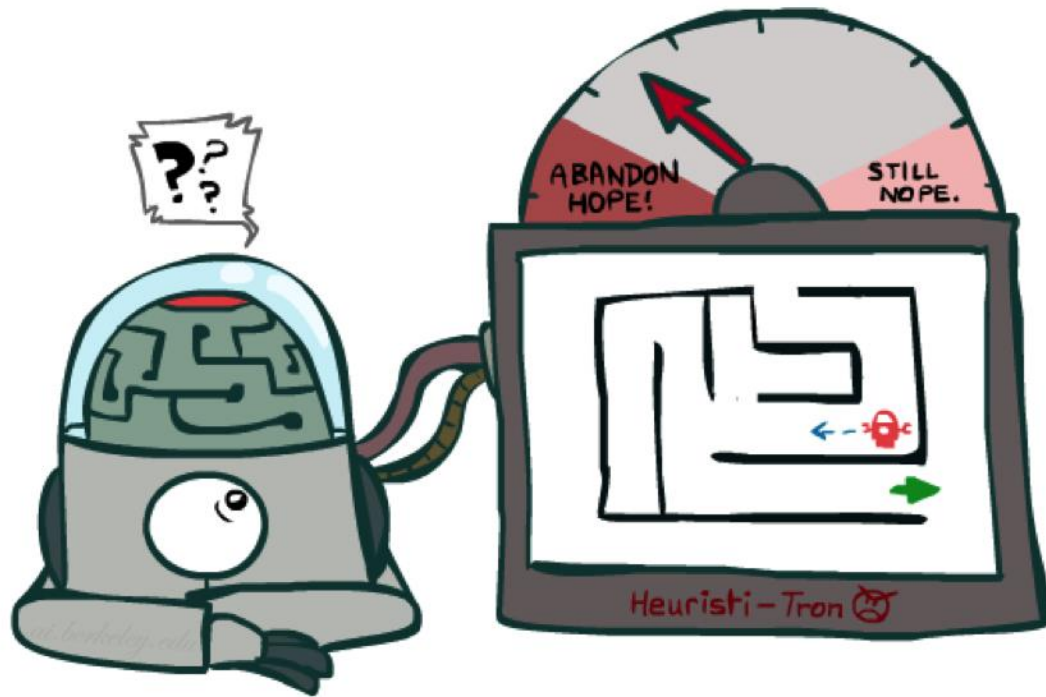


- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

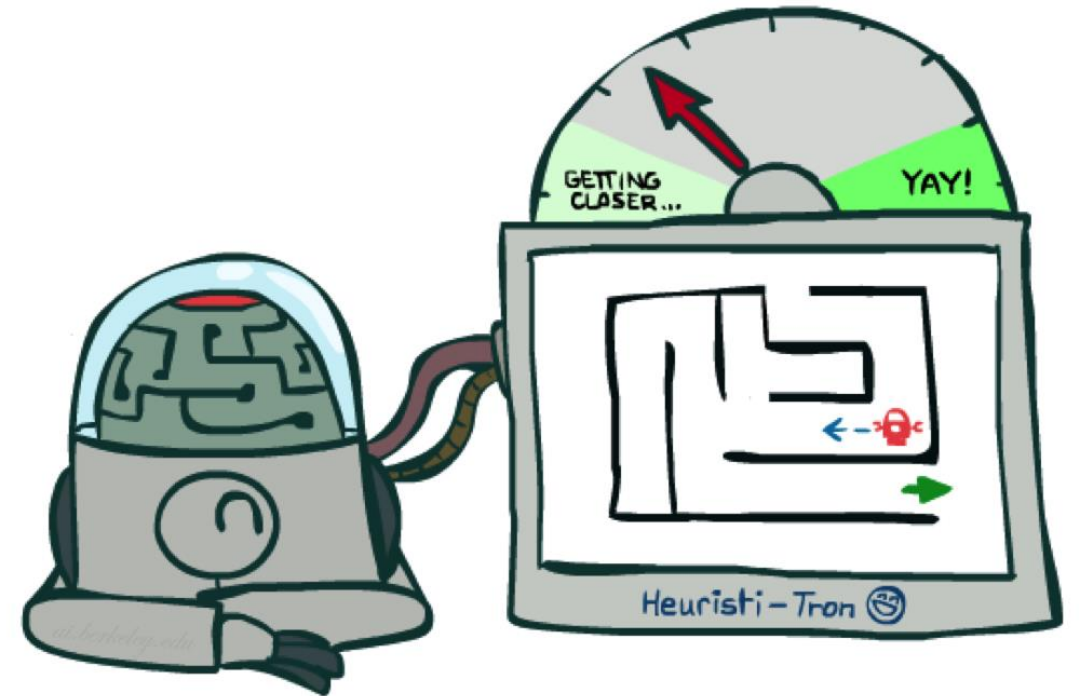
Admissible Heuristics



Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

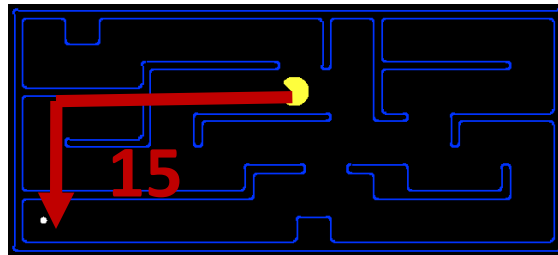
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:

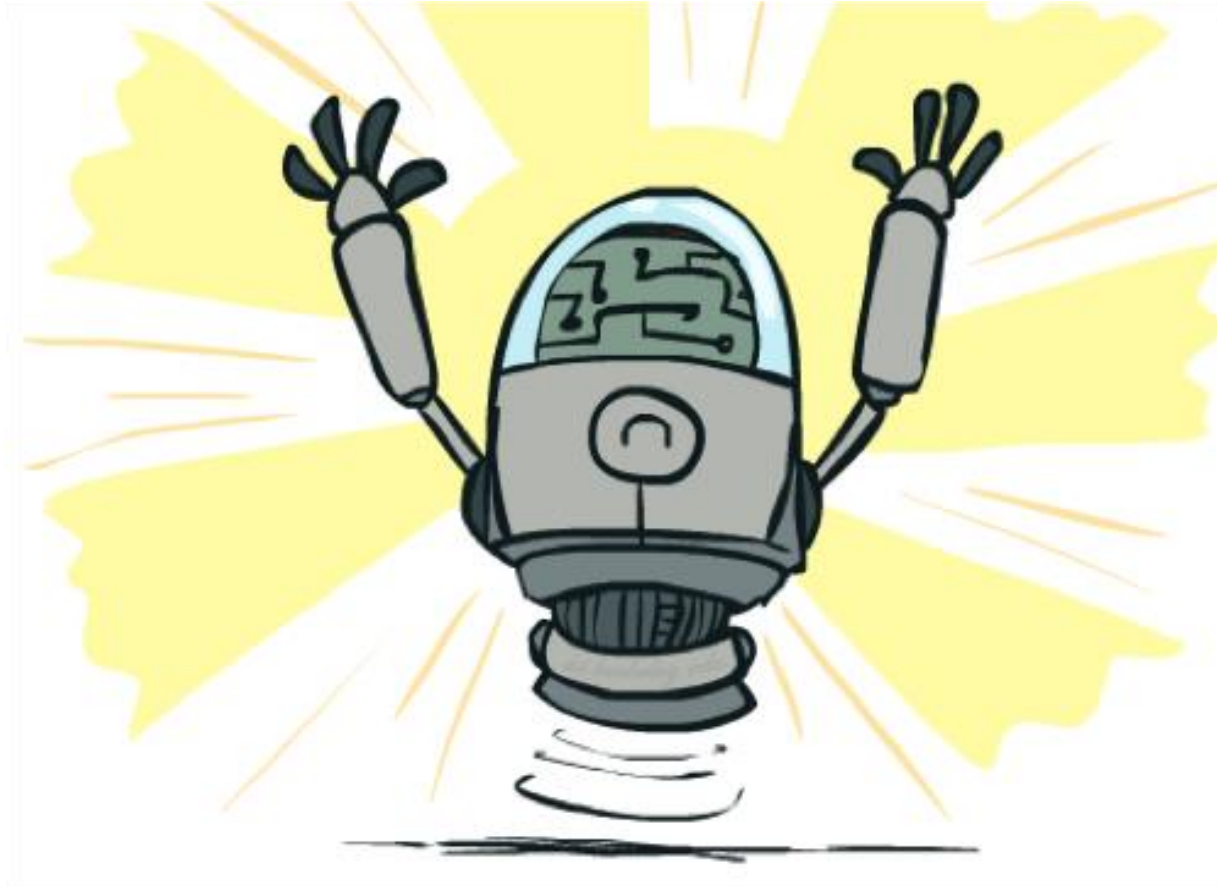


4



- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A* Tree Search



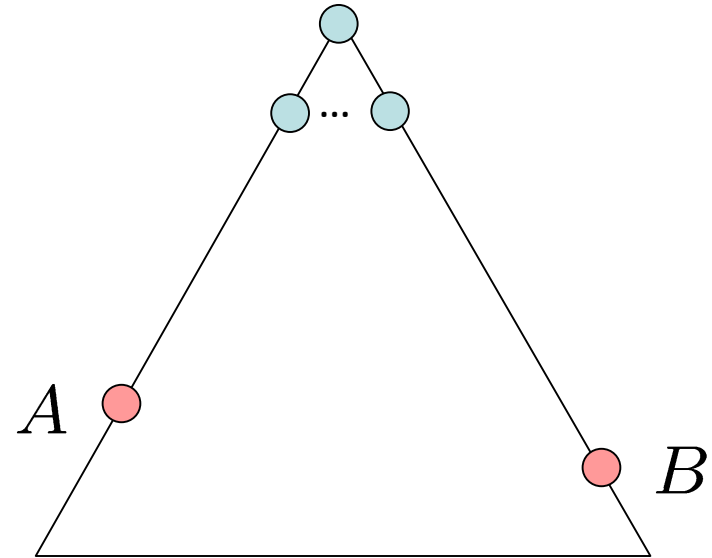
Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:

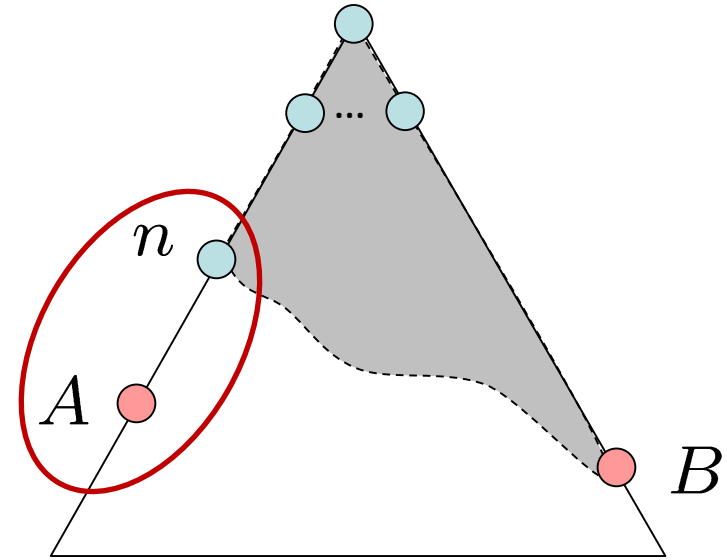
- A will exit the fringe before B



Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
 1. $f(n)$ is less or equal to $f(A)$



$$f(n) = g(n) + h(n)$$

$$f(n) \leq g(A)$$

$$g(A) = f(A)$$

Definition of f-cost

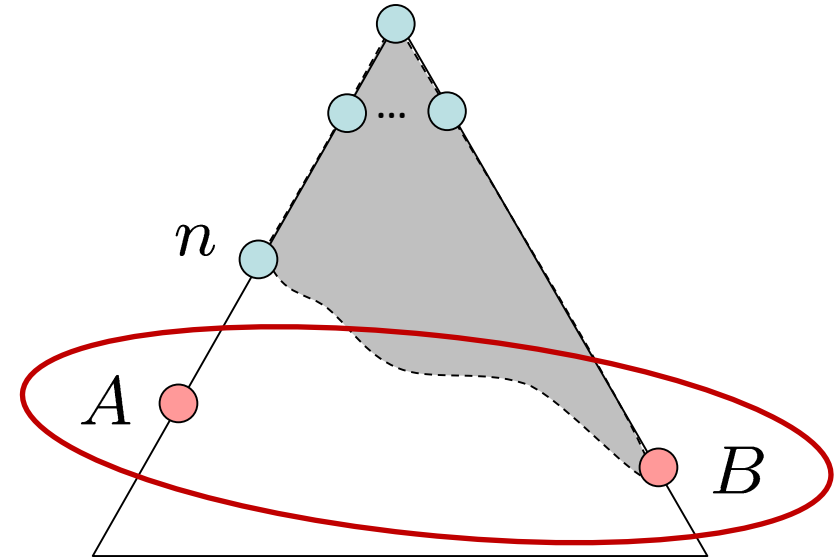
Admissibility of h

$h = 0$ at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
 1. $f(n)$ is less or equal to $f(A)$
 2. $f(A)$ is less than $f(B)$



$$g(A) < g(B)$$

$$f(A) < f(B)$$

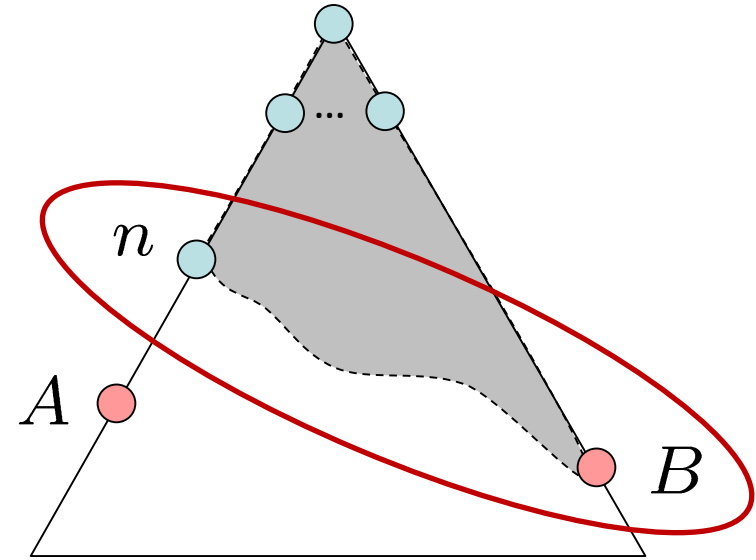
B is suboptimal

$h = 0$ at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
 1. $f(n)$ is less or equal to $f(A)$
 2. $f(A)$ is less than $f(B)$
 3. n expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal

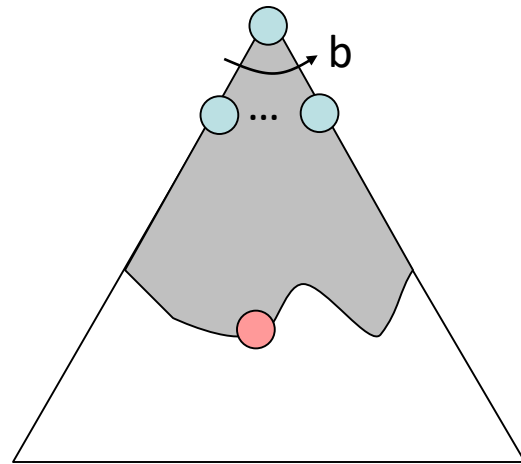


$$f(n) \leq f(A) < f(B)$$

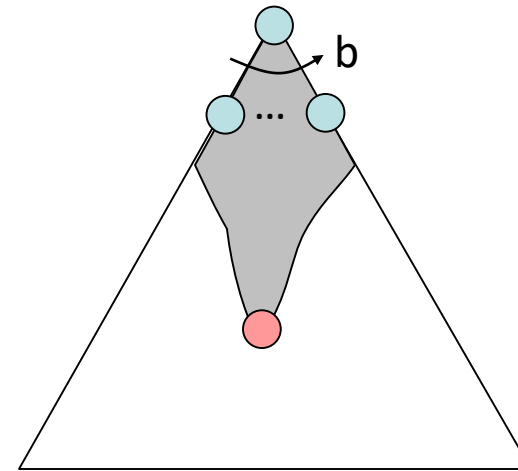
Properties of A^*

Properties of A*

Uniform-Cost

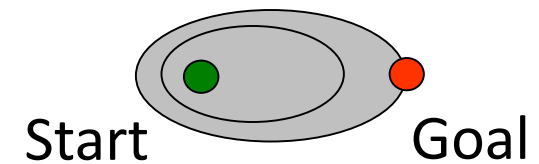
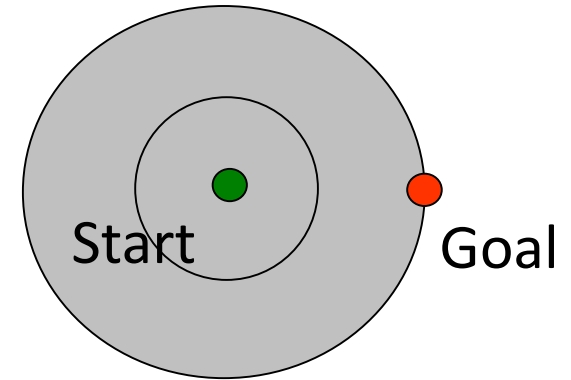


A*



UCS vs A* Contours

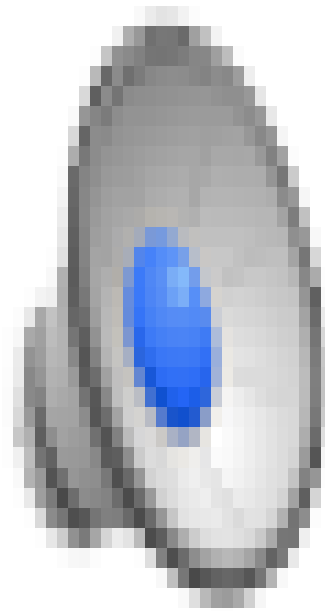
- Uniform-cost expands equally in all “directions”
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



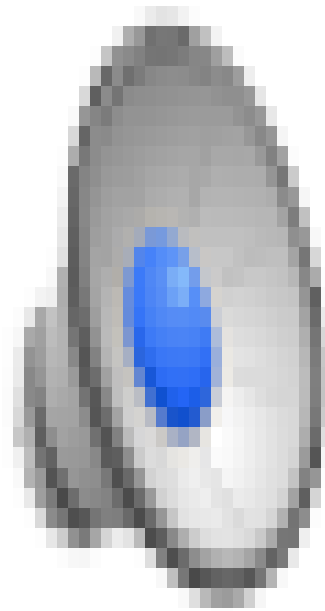
[Demo: contours UCS / greedy / A* empty (L3D1)]

[Demo: contours A* pacman small maze (L3D5)]

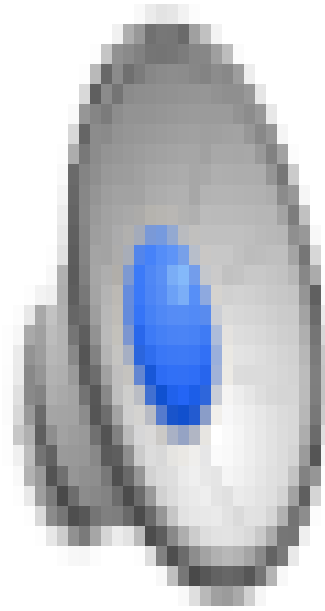
Video of Demo Contours (Empty) -- UCS



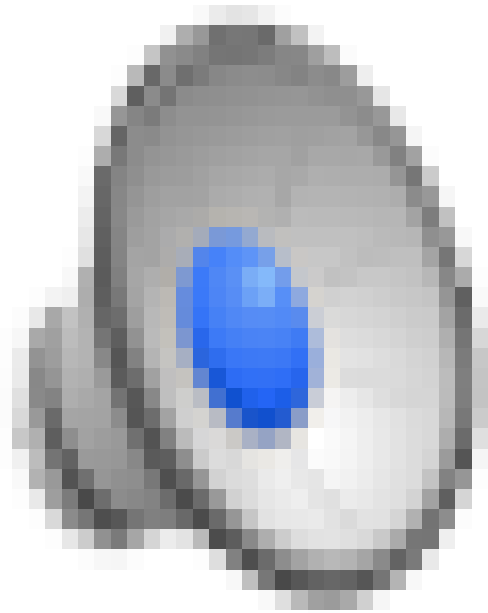
Video of Demo Contours (Empty) -- Greedy



Video of Demo Contours (Empty) – A*



Video of Demo Contours (Pacman Small Maze) – A*



Comparison



Greedy



Uniform Cost



A*

A* Applications



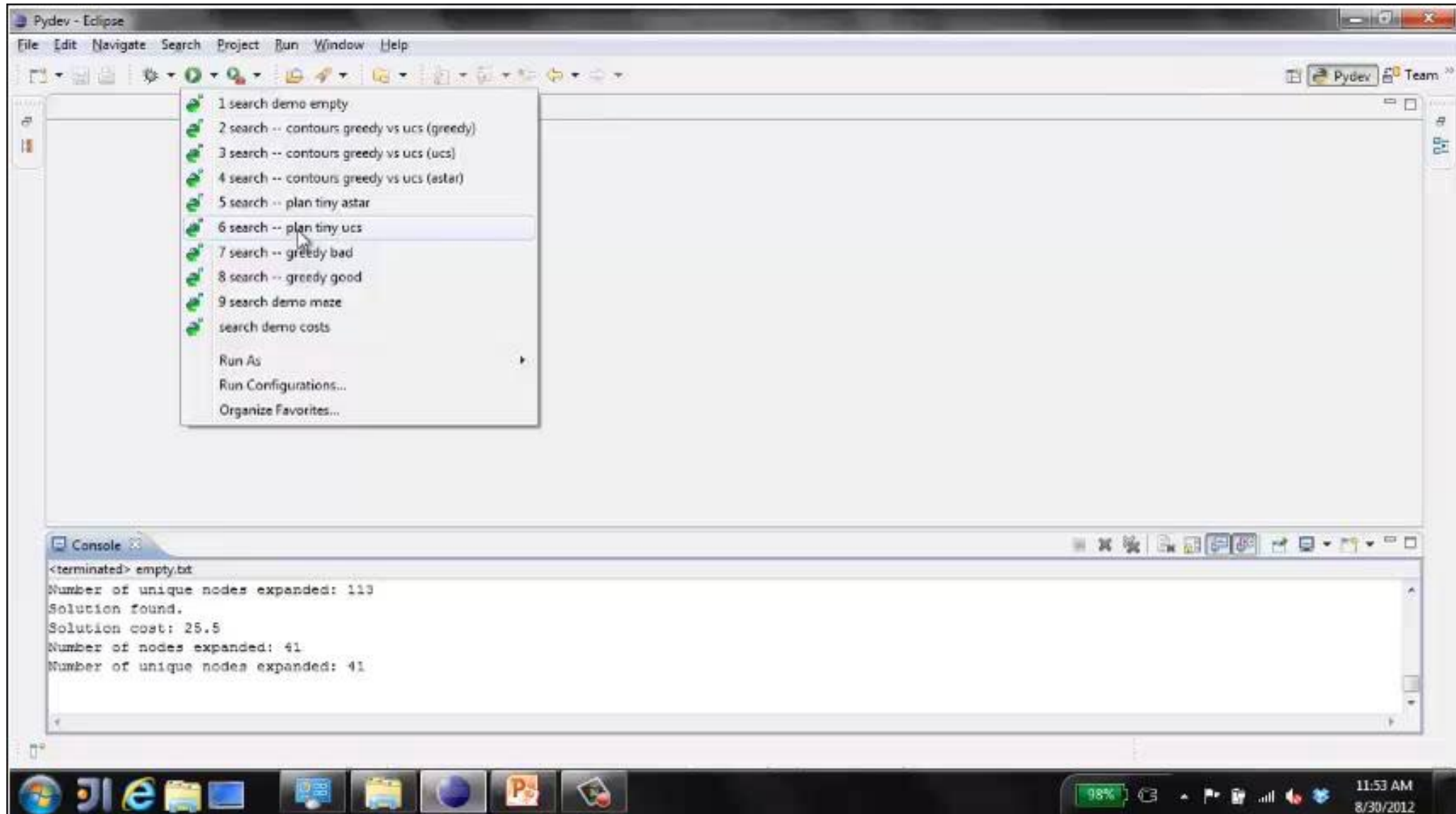
A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- ...

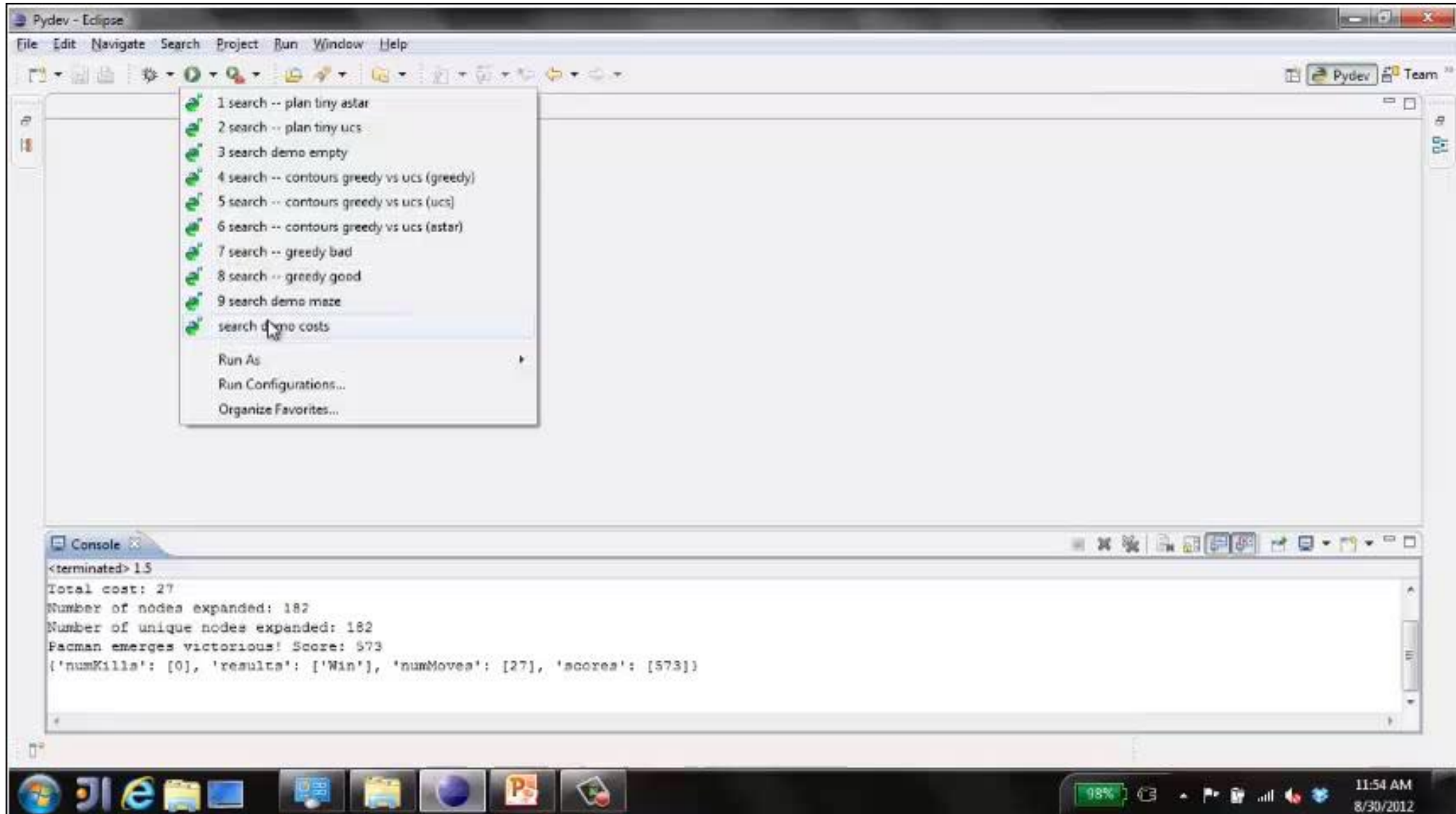


[Demo: UCS / A* pacman tiny maze (L3D6,L3D7)]
[Demo: guess algorithm Empty Shallow/Deep (L3D8)]

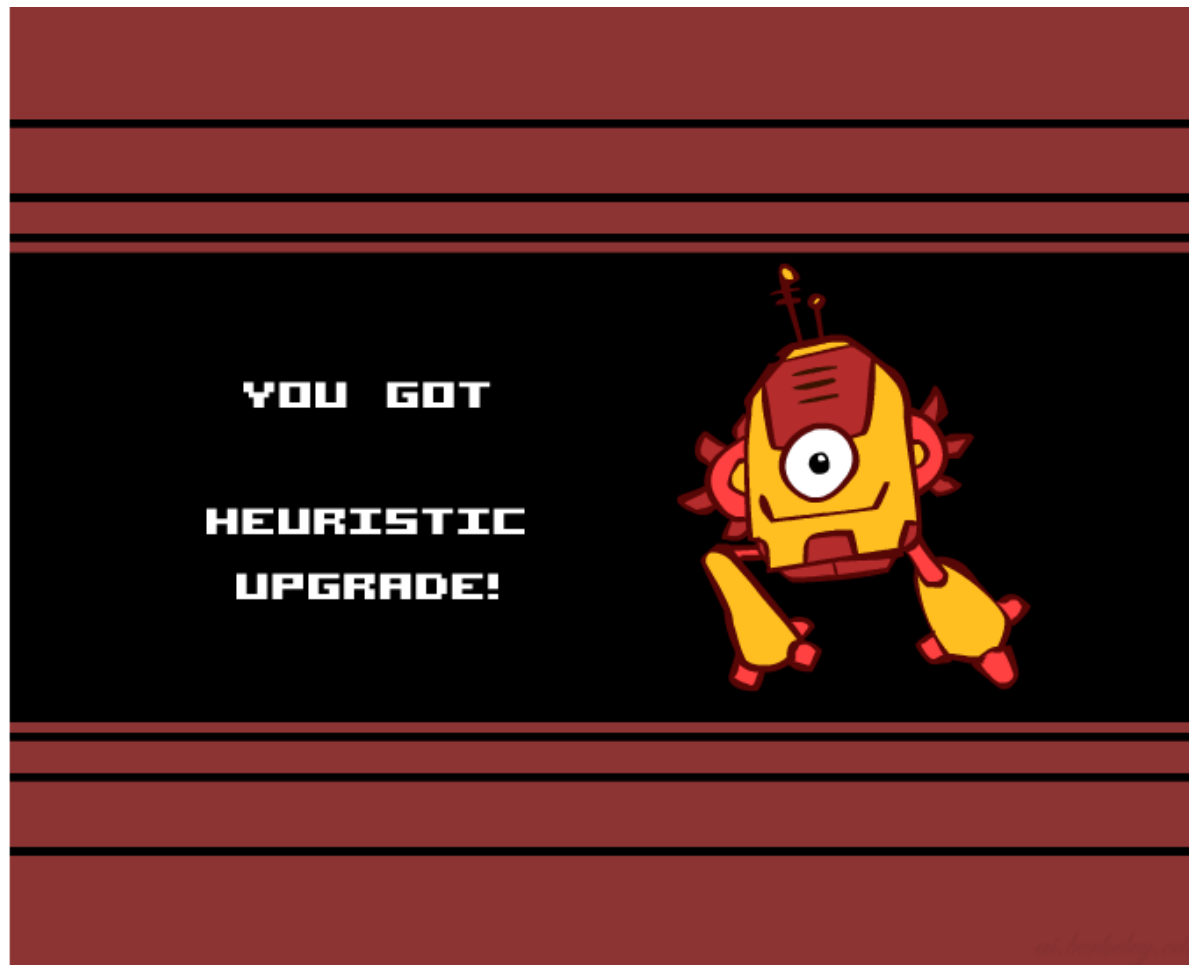
Video of Demo Pacman (Tiny Maze) – UCS / A*



Video of Demo Empty Water Shallow/Deep – Guess Algorithm

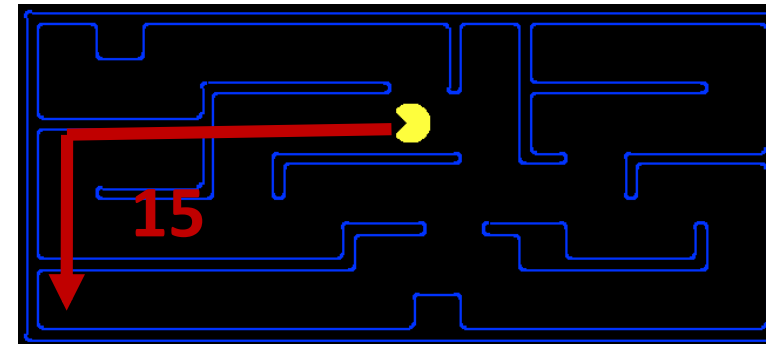
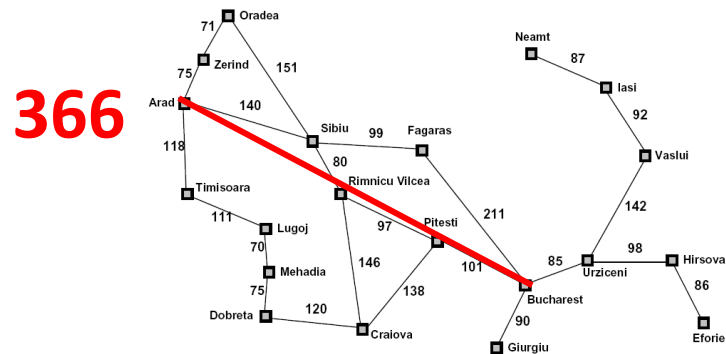


Creating Heuristics



Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

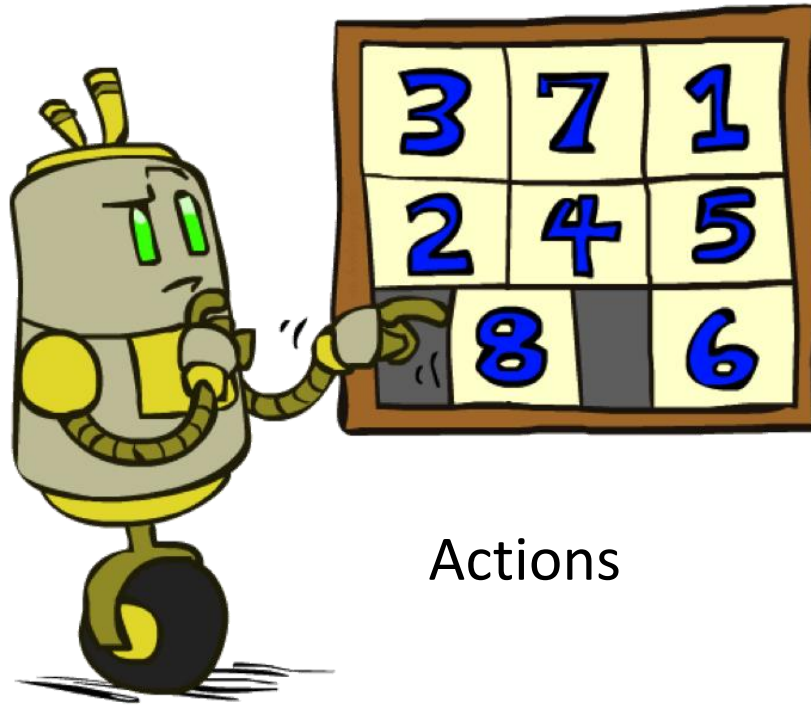


- Inadmissible heuristics are often useful too

Example: 8 Puzzle

| | | |
|---|---|---|
| 7 | 2 | 4 |
| 5 | | 6 |
| 8 | 3 | 1 |

Start State



Actions

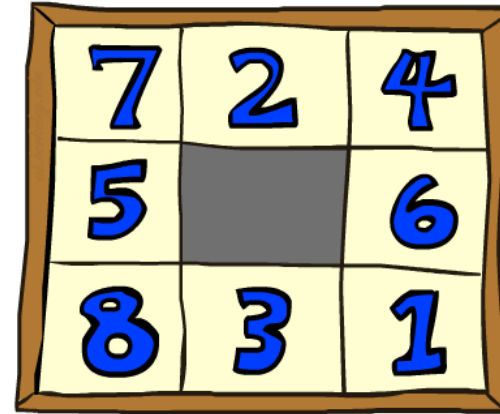
| | | |
|---|---|---|
| | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |

Goal State

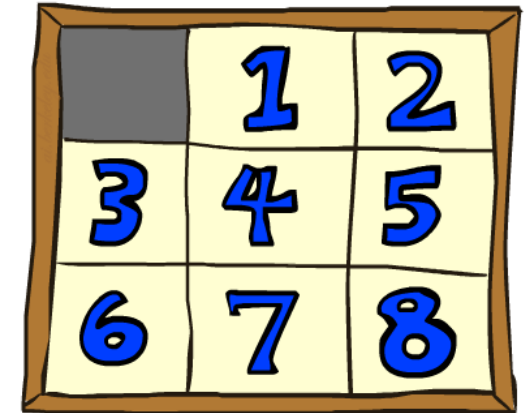
- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

8 Puzzle I

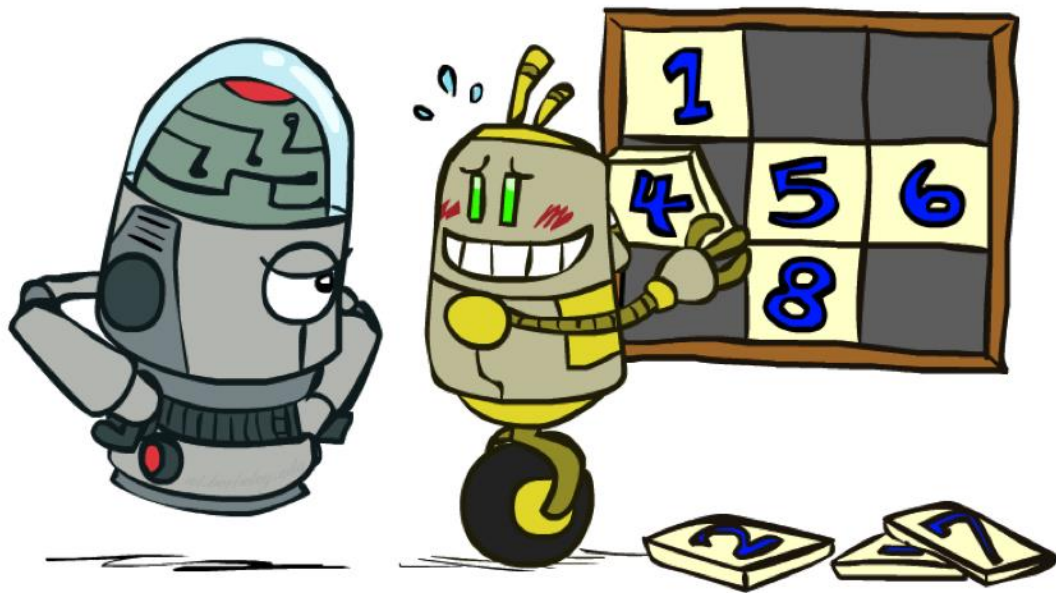
- Heuristic: Number of tiles misplaced
- Why is it admissible?
- $h(\text{start}) = 8$
- This is a *relaxed-problem* heuristic



Start State



Goal State

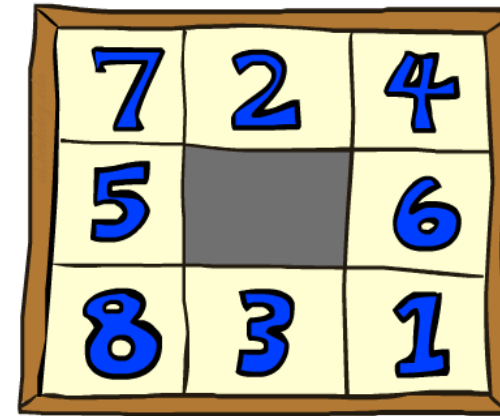


Average nodes expanded
when the optimal path has...

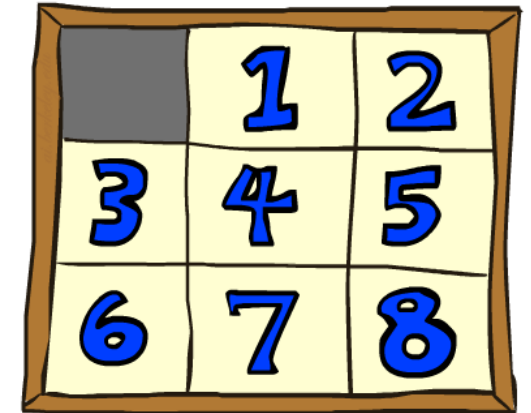
| | ...4 steps | ...8 steps | ...12 steps |
|-------|------------|------------|-------------------|
| UCS | 112 | 6,300 | 3.6×10^6 |
| TILES | 13 | 39 | 227 |

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?
- $h(\text{start}) = 3 + 1 + 2 + \dots = 18$



Start State

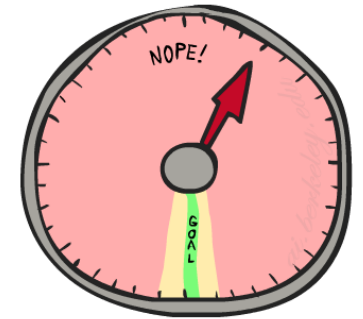
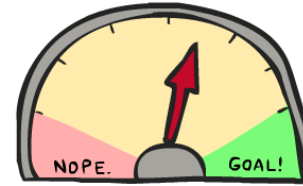


Goal State

| | Average nodes expanded when the optimal path has... | | |
|-----------|---|------------|-------------|
| | ...4 steps | ...8 steps | ...12 steps |
| TILES | 13 | 39 | 227 |
| MANHATTAN | 12 | 25 | 73 |

8 Puzzle III

- How about using the *actual cost* as a heuristic?
 - Would it be admissible?
 - Would we save on nodes expanded?
 - What's wrong with it?



- With A^* : a trade-off between quality of estimate and work per node
 - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Semi-Lattice of Heuristics

Trivial Heuristics, Dominance

- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

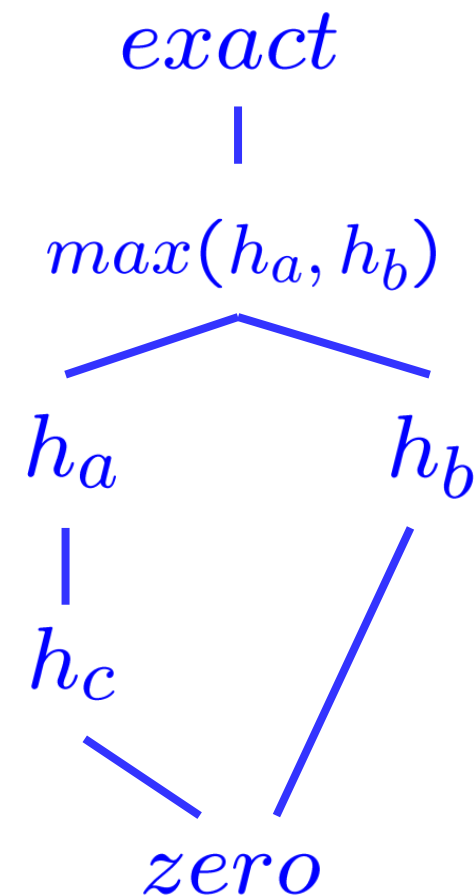
- Heuristics form a semi-lattice:

- Max of admissible heuristics is admissible

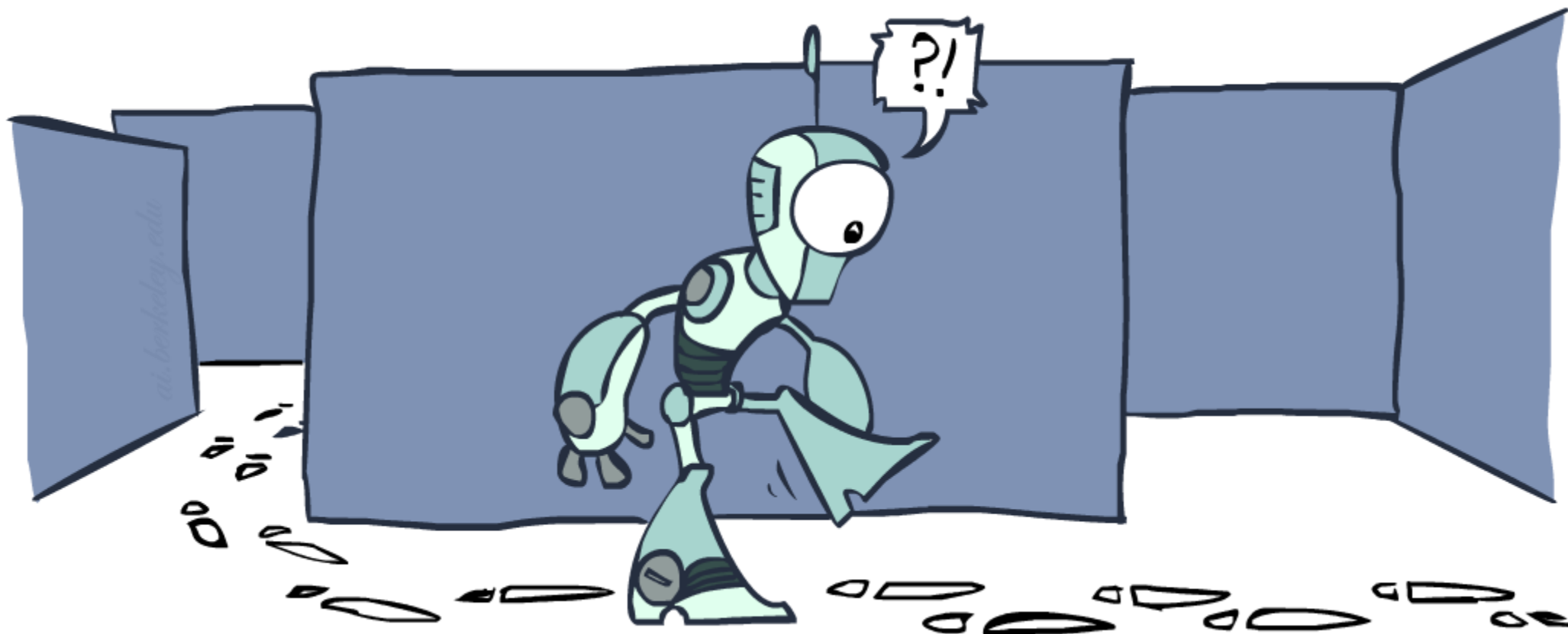
$$h(n) = \max(h_a(n), h_b(n))$$

- Trivial heuristics

- Bottom of lattice is the zero heuristic (what does this give us?)
- Top of lattice is the exact heuristic

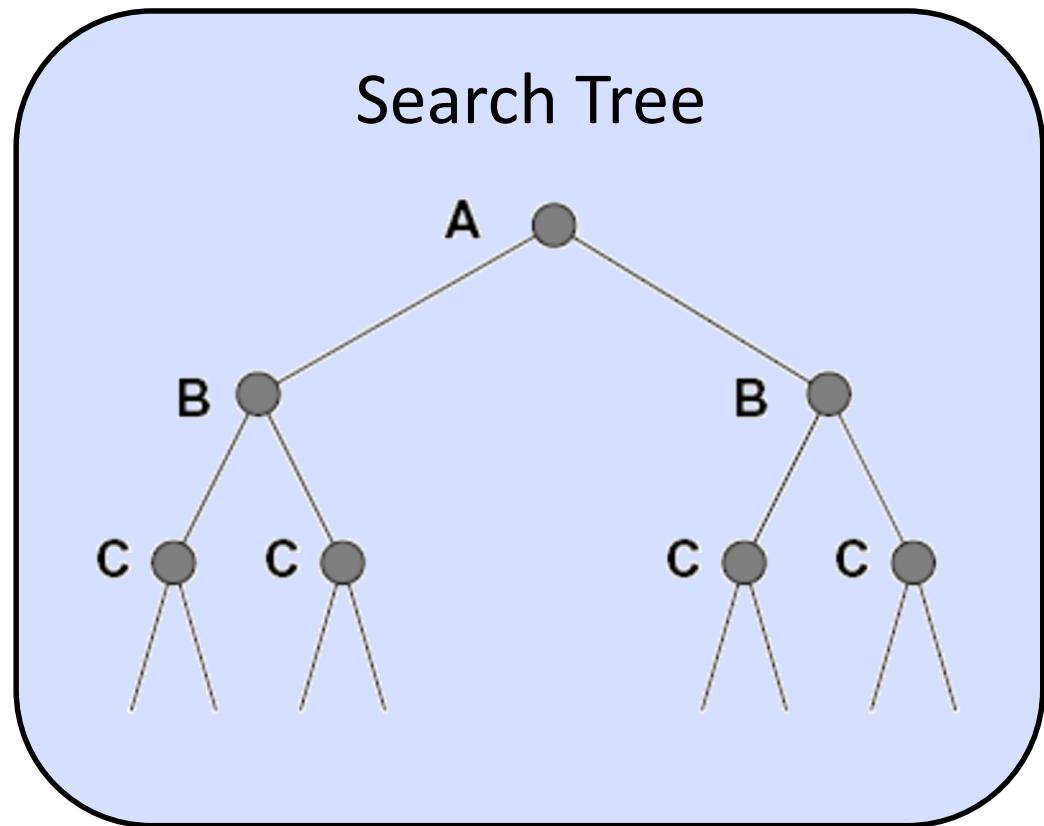
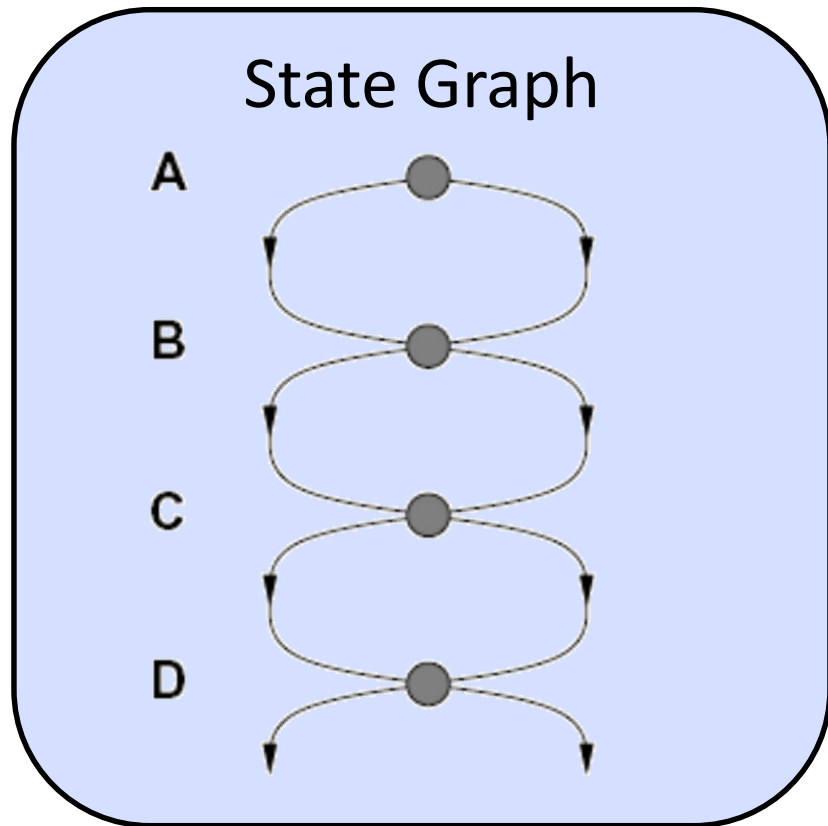


Graph Search



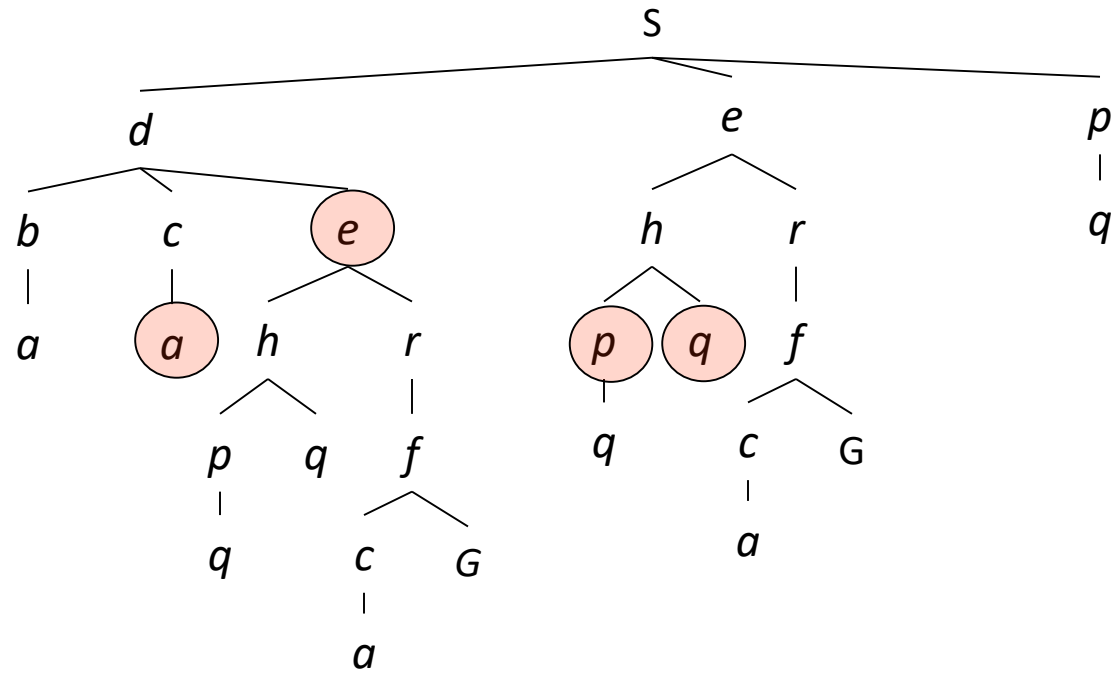
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.



Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)

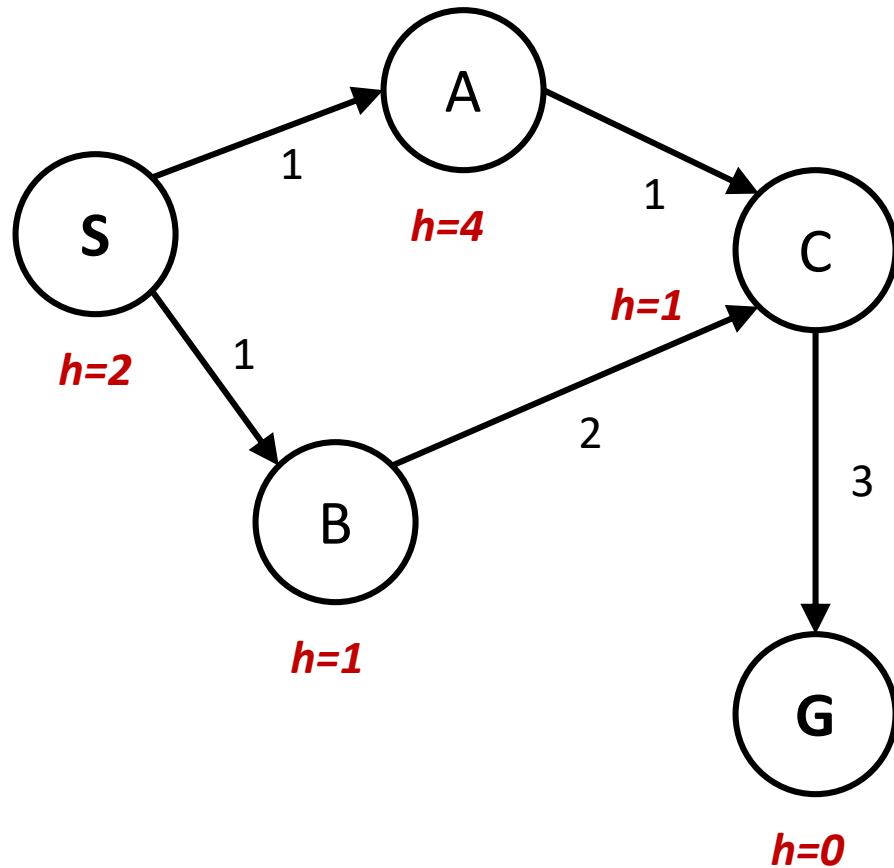


Graph Search

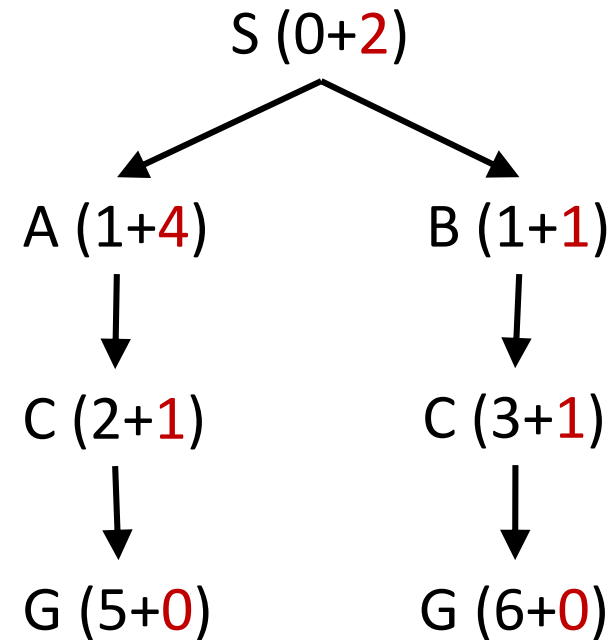
- Idea: never **expand** a state twice
- How to implement:
 - Tree search + set of expanded states (“closed set”)
 - Expand the search tree node-by-node, but...
 - Before expanding a node, check to make sure its state has never been expanded before
 - If not new, skip it, if new add to closed set
- Important: **store the closed set as a set**, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong?

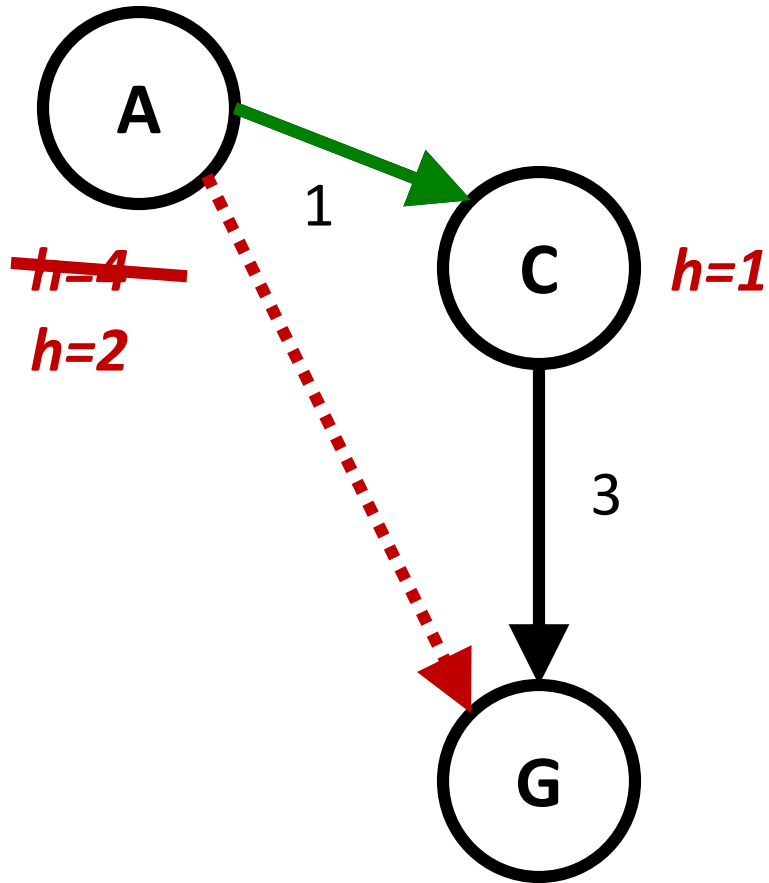
State space graph



Search tree

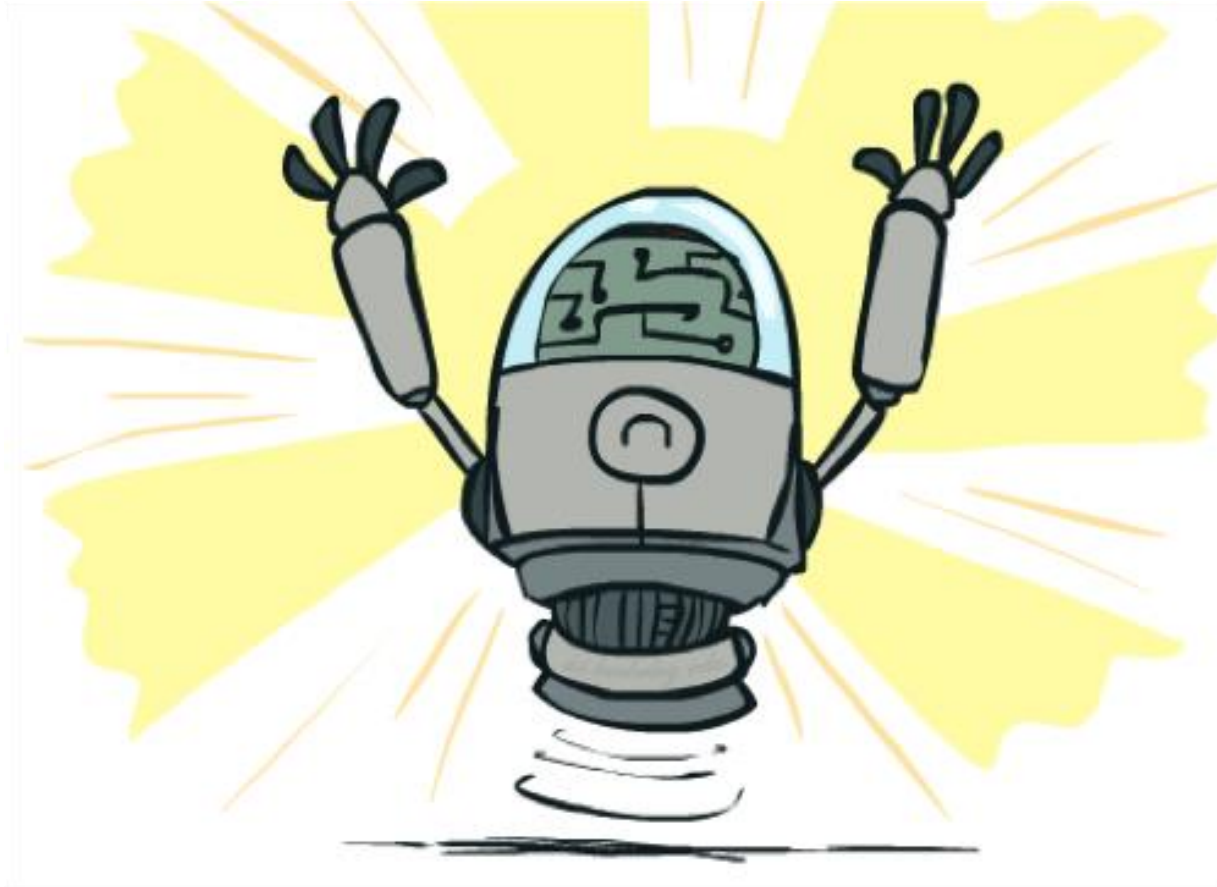


Consistency of Heuristics



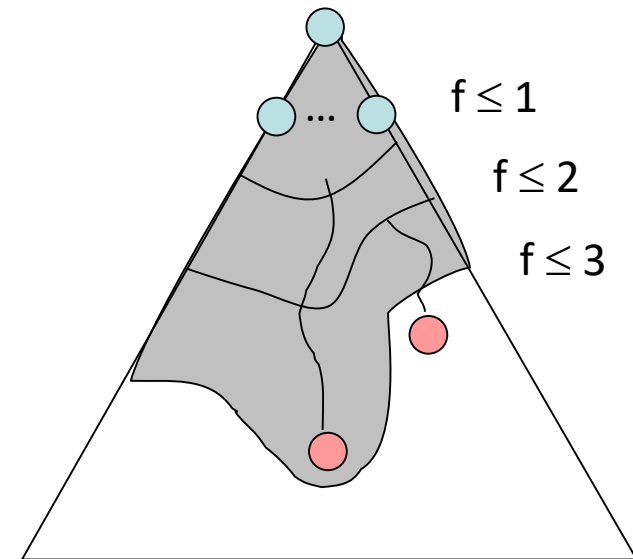
- Main idea: estimated heuristic costs \leq actual costs
 - Admissibility: heuristic cost \leq actual cost to goal
$$h(A) \leq \text{actual cost from A to G}$$
 - Consistency: heuristic “arc” cost \leq actual cost for each arc
$$h(A) - h(C) \leq \text{cost}(A \text{ to } C)$$
- Consequences of consistency:
 - The f value along a path never decreases
$$h(A) \leq \text{cost}(A \text{ to } C) + h(C)$$
 - A* graph search is optimal

Optimality of A* Graph Search



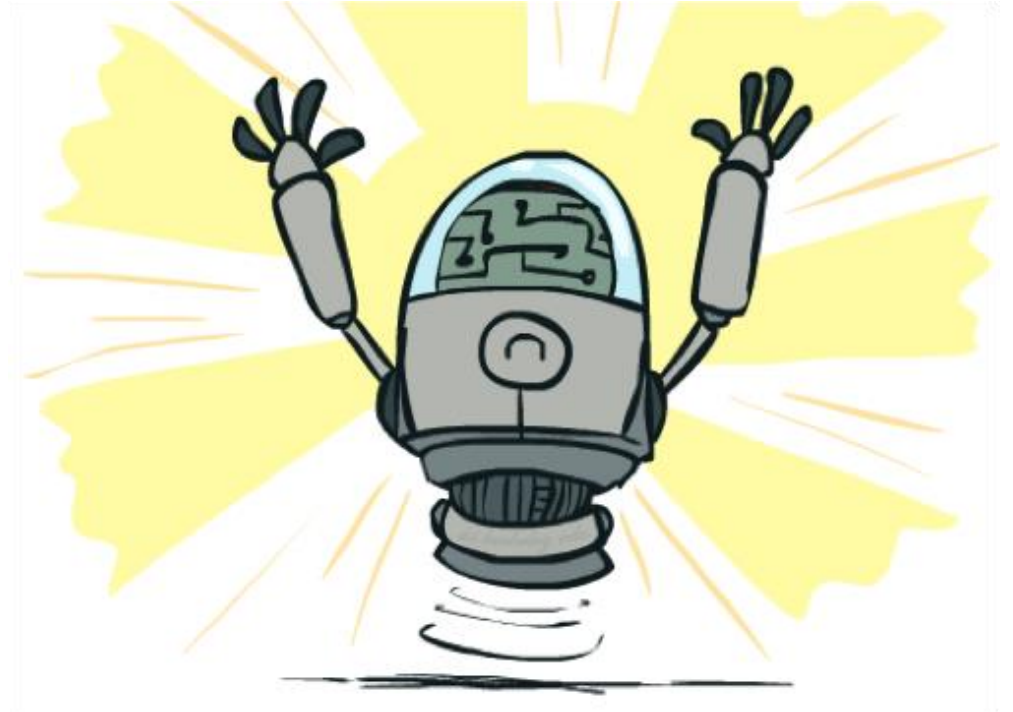
Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
 - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
 - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
- Result: A* graph search is optimal



Optimality

- Tree search:
 - A* is optimal if heuristic is admissible
 - UCS is a special case ($h = 0$)
- Graph search:
 - A* optimal if heuristic is consistent
 - UCS optimal ($h = 0$ is consistent)
- Consistency implies admissibility
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

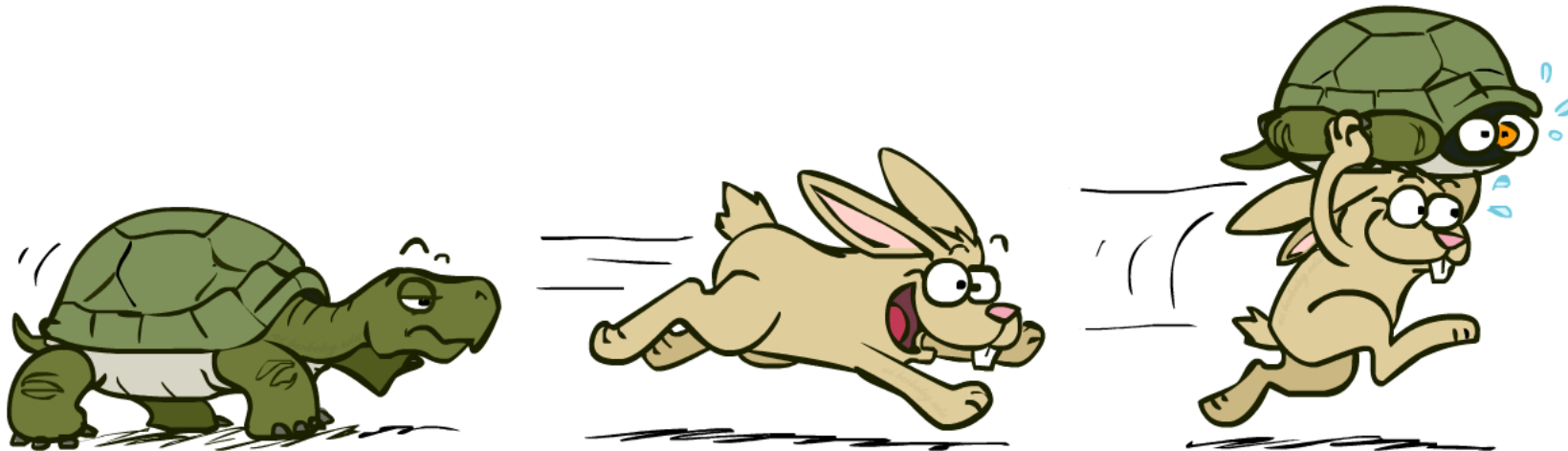


A*: Summary



A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    for child-node in EXPAND(STATE[node], problem) do
      fringe ← INSERT(child-node, fringe)
    end
  end
```

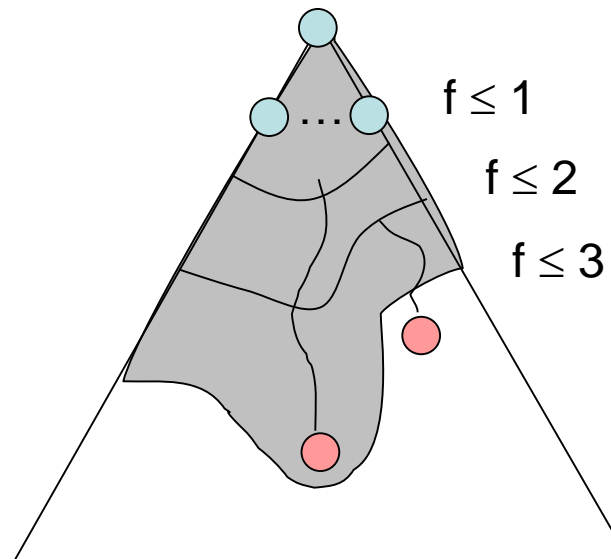
Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
  closed ← an empty set
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    if STATE[node] is not in closed then
      add STATE[node] to closed
      for child-node in EXPAND(STATE[node], problem) do
        fringe ← INSERT(child-node, fringe)
      end
    end
  end
```

Optimality of A* Graph Search

- Consider what A* does:
 - Expands nodes in increasing total f value (f-contours)
Reminder: $f(n) = g(n) + h(n) = \text{cost to } n + \text{heuristic}$
 - Proof idea: the optimal goal(s) have the lowest f value, so it must get expanded first

There's a problem with this argument. What are we assuming is true?



Optimality of A* Graph Search

Proof:

- New possible problem: some n on path to G^* isn't in queue when we need it, because some worse n' for the same state dequeued and expanded first (disaster!)
- Take the highest such n in tree
- Let p be the ancestor of n that was on the queue when n' was popped
- $f(p) < f(n)$ because of **consistency**
- $f(n) < f(n')$ because n' is suboptimal
- p would have been expanded before n'
- Contradiction!

