# CS191 – Fall 2014
# Homework 7: due in lecture Nov. 5th

1. ***Error correction for a mixture of errors.*** In lecture we showed that linear combinations of correctable errors are correctable by a quantum error correction code. In this problem you will show by example that convex mixtures of correctable errors are also correctable.

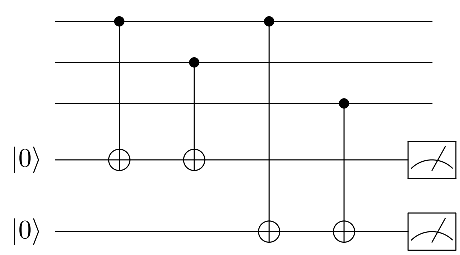
   Suppose $|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$ is a general single qubit state encoded in the bit flip code. Then, due to errors it is mapped to the following mixed state:

   $$\rho = (1-p)\rho_0 + \frac{p}{3}(X_1\rho_0 X_1 + X_2\rho_0 X_2 + X_3\rho_0 X_3),$$

   where $\rho_0 = |\psi\rangle\langle\psi|$, and $X_1 = \sigma_x \otimes I \otimes I$ and so on.

   (a) Write out the state $\rho$ in bra-ket form.

   (b) Compute the state that is produced when the bit-flip code error detection circuit

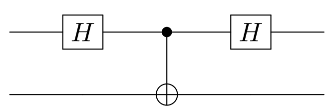
   is executed with the state of the first three qubits being $\rho$. What are the probabilities of getting the four possible measurement results (00, 01, 10, and 11) when the ancilla are measured?

   (c) Recall that the correction gates for each of the ancilla measurement results are:

   $$\begin{array}{lcl} 00 & \rightarrow & \text{no correction} \\ 01 & \rightarrow & X_3 \\ 10 & \rightarrow & X_2 \\ 11 & \rightarrow & X_1 \end{array}$$

   Confirm that when the correct correction gate is applied, you recover the original state $|\psi\rangle$.

2. ***Bit flip code encoding and logical evolution.*** Suppose we want to execute the following circuit

   where the two input states are arbitrary single qubit states. But we are worried about bit flip errors occurring during the circuit and so we encode each qubit in the bit flip code.

   (a) Draw the circuit that executes the above logical transformation when the two qubits are encoded in the bit flip code. Use the transversal encoded CNOT gate shown in lecture.

   (b) Suppose we now want to compute the output of this circuit on the initial state $|0\rangle \otimes |0\rangle$. The encoded initial state is $|000\rangle \otimes |000\rangle$ (where we have only explicitly written the tensor product between blocks). However, suppose a bit flip error occurs on one of the qubits in the encoded block for the first qubit before the first Hadamard, and the erroneous encoded input state becomes $|100\rangle \otimes |000\rangle$. Compute the output of the encoded circuit (with transversal gates) on this erroneous encoded input state. The error before the circuit will result in error(s) after the circuit. How many errors are there (how many qubits values are flipped from the ideal encoded output state), and are they correctable using the bit flip code detection procedure?