C 191 - FALL 2014
Homework 9: due in lecture Dec. 3

1. *Entropy as a measure of information*

   Suppose we wish to quantify just how much information is provided by an event E which may occur in a probabilistic experiment, or alternatively, how much information do we acquire by witnessing the event E. We do this using an information function $f(E)$ whose value is determined by the event E. We make the following reasonable assumptions about this function:

   (a) $f(E)$ is a function only of the probability of the event E, so we may write $f = f(p)$, where p is a probability in the range 0 to 1.

   (b) f is a smooth function of probability.

   (c) When two independent events occur with individual probabilities $p > 0$ and $q > 0$, the information gained is given by the sum of the information gained from each event alone, i.e., $f(pq) = f(p) + f(q)$, i.e, information from independent events is additive.

   Show that these relationships imply that $f(p) = klogp$, for some constant, k. Hence show that the average information gain when one of a mutually exclusive set of events with probabilities $p_1, p_2 ..., p_n$ occurs is $k \sum_i p_i logp_i$. Disregarding the constant factor k, this is the Shannon (classical) entropy, which is usually written as $H(p) = -\sum_i p_i logp_i$. How would you determine the sign of k?

2. *Entanglement and robustness of three-qubit states*

   Consider the two 3-qubit states:

   - GHZ states: $\frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle\right)$
   - W states: $\frac{1}{\sqrt{3}}\left(|100\rangle + |010\rangle + |001\rangle\right)$

   (a) Compare the robustness of the GHZ state and the W state to loss of information about a single qubit (corresponding to either non-recorded (unreferred) measurement or physical loss of the qubit).

   (b) Find the entanglement entropy of the GHZ state and of the W state.

   (c) Compare your answers for the two states with regard to robustness and degree of entanglement.