Growth

Announcements

Measuring Efficiency

Recursive Computation of the Fibonacci Sequence

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fib(5)

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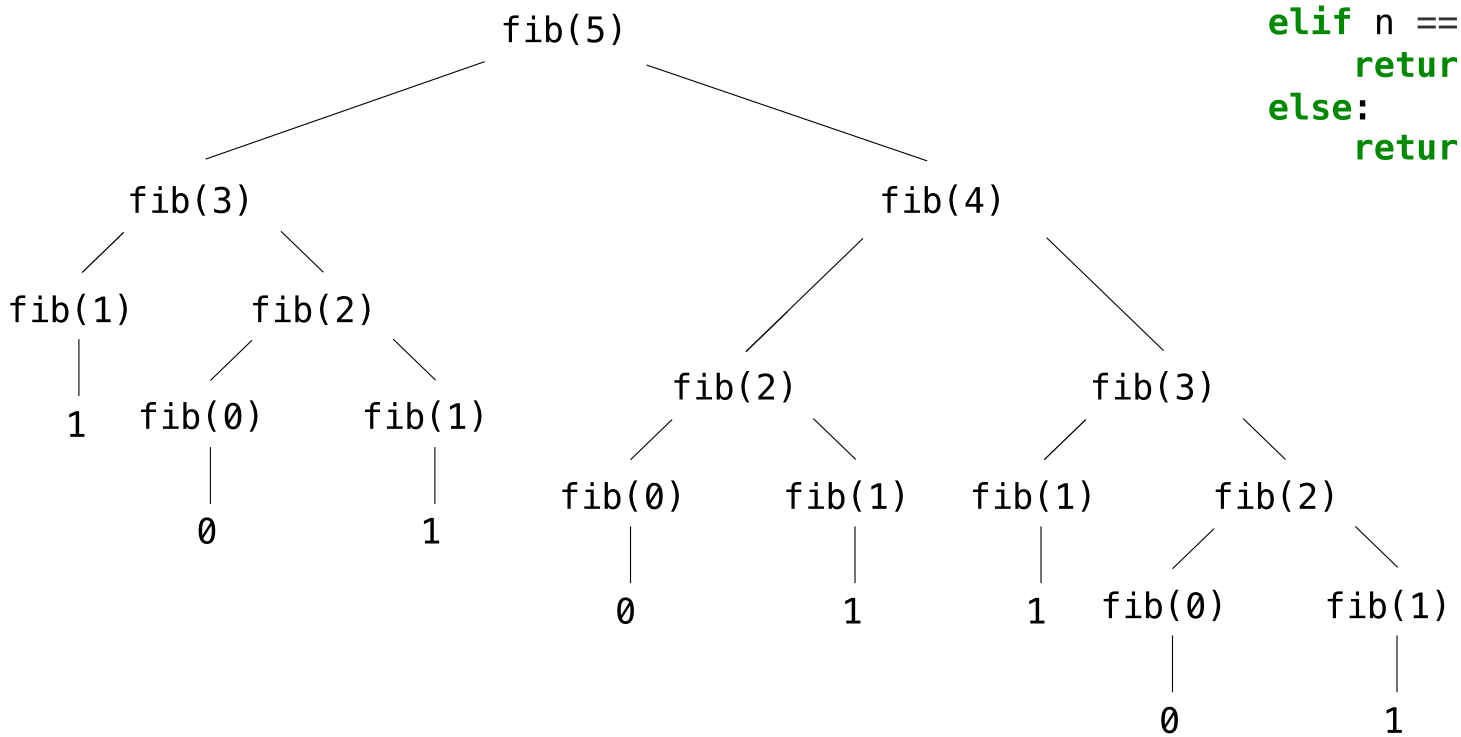

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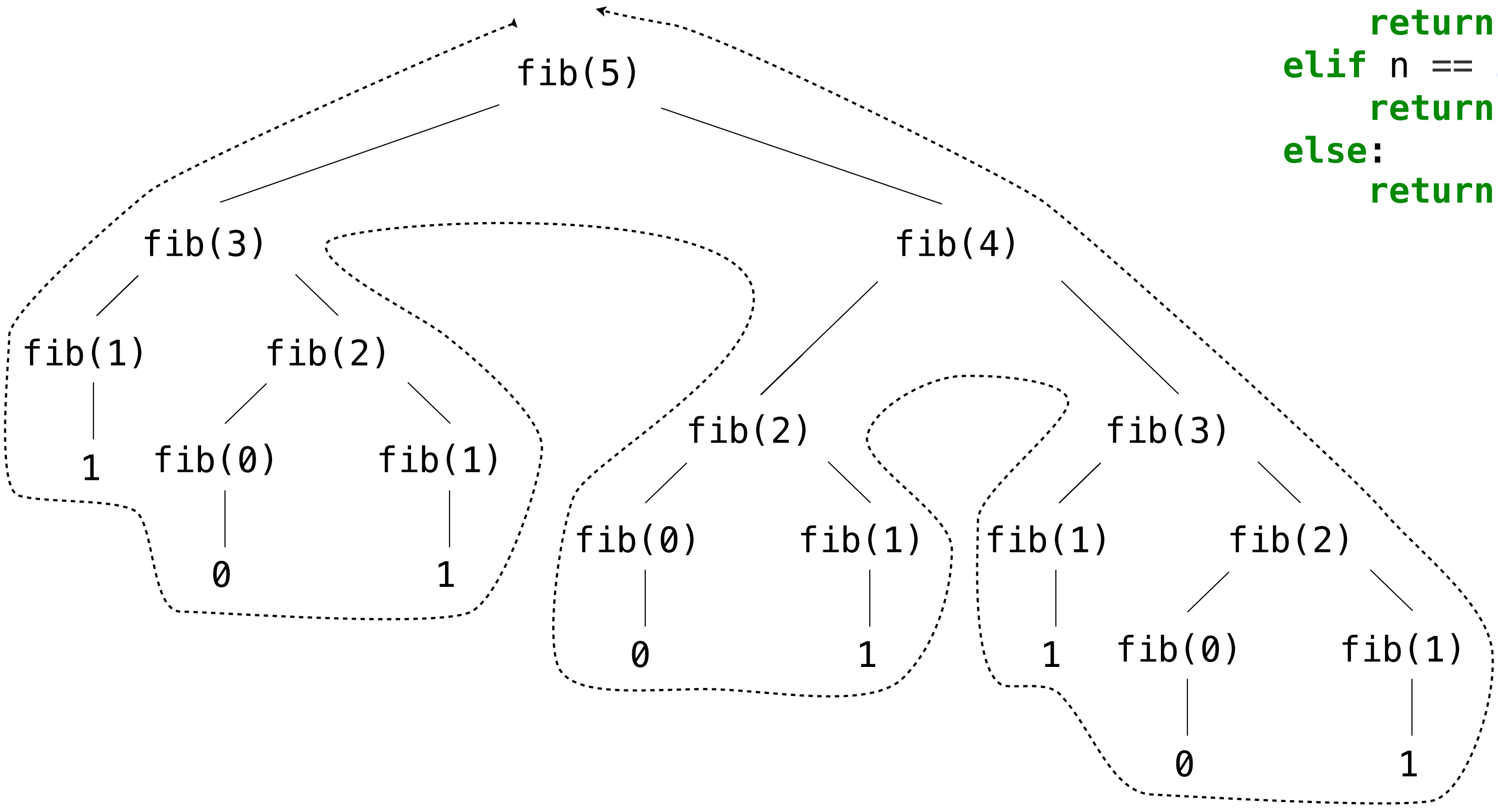

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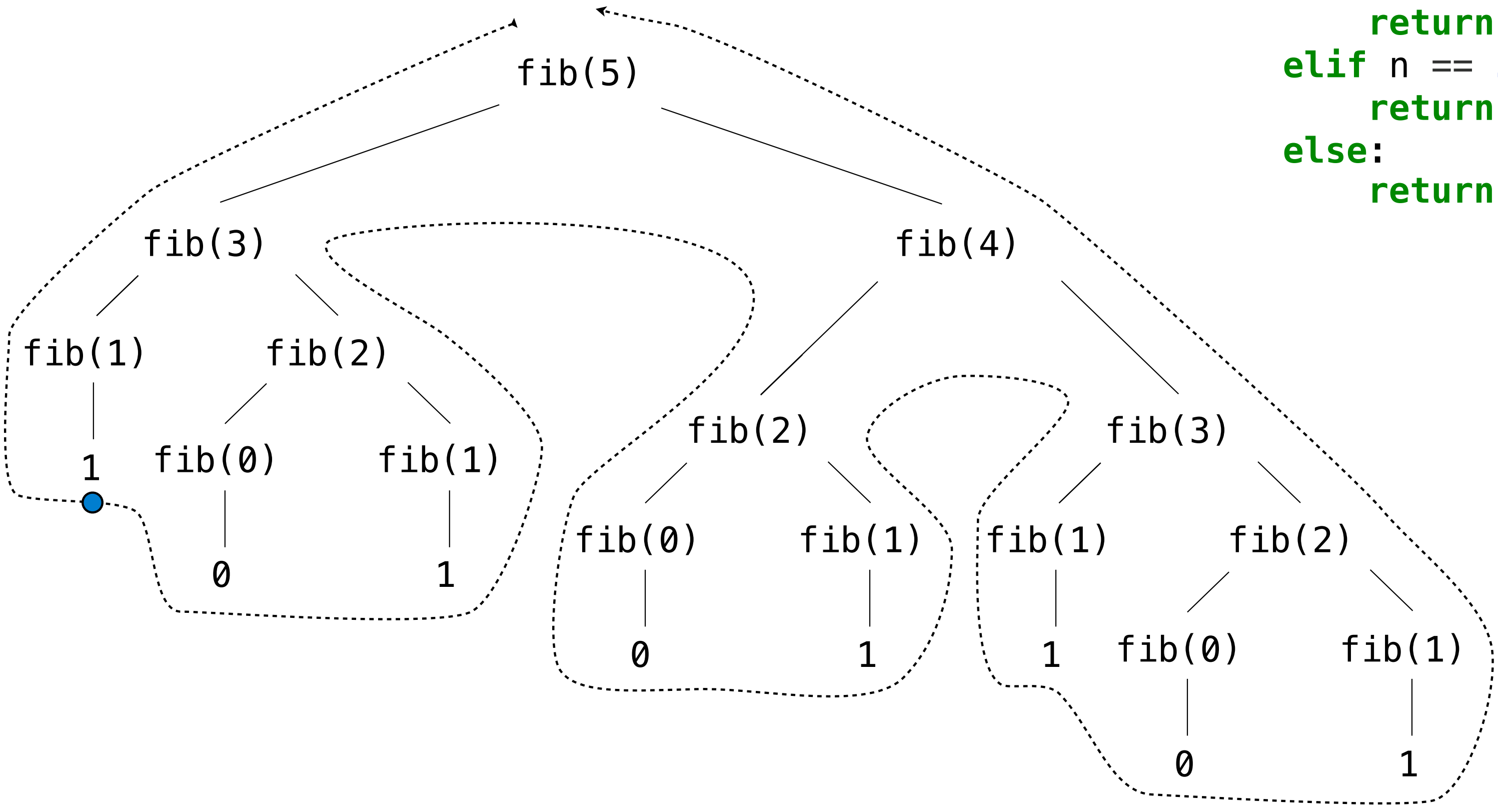

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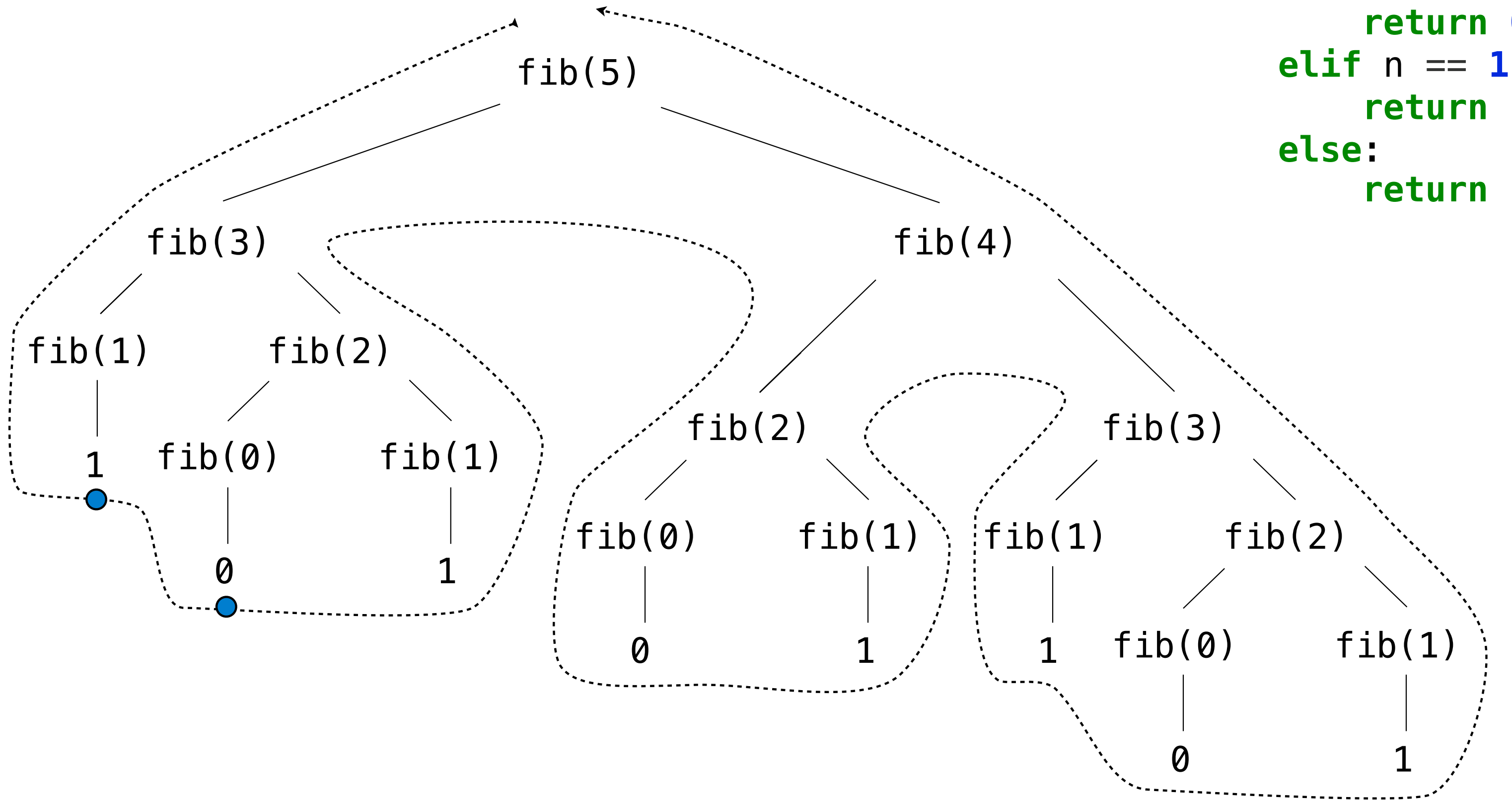


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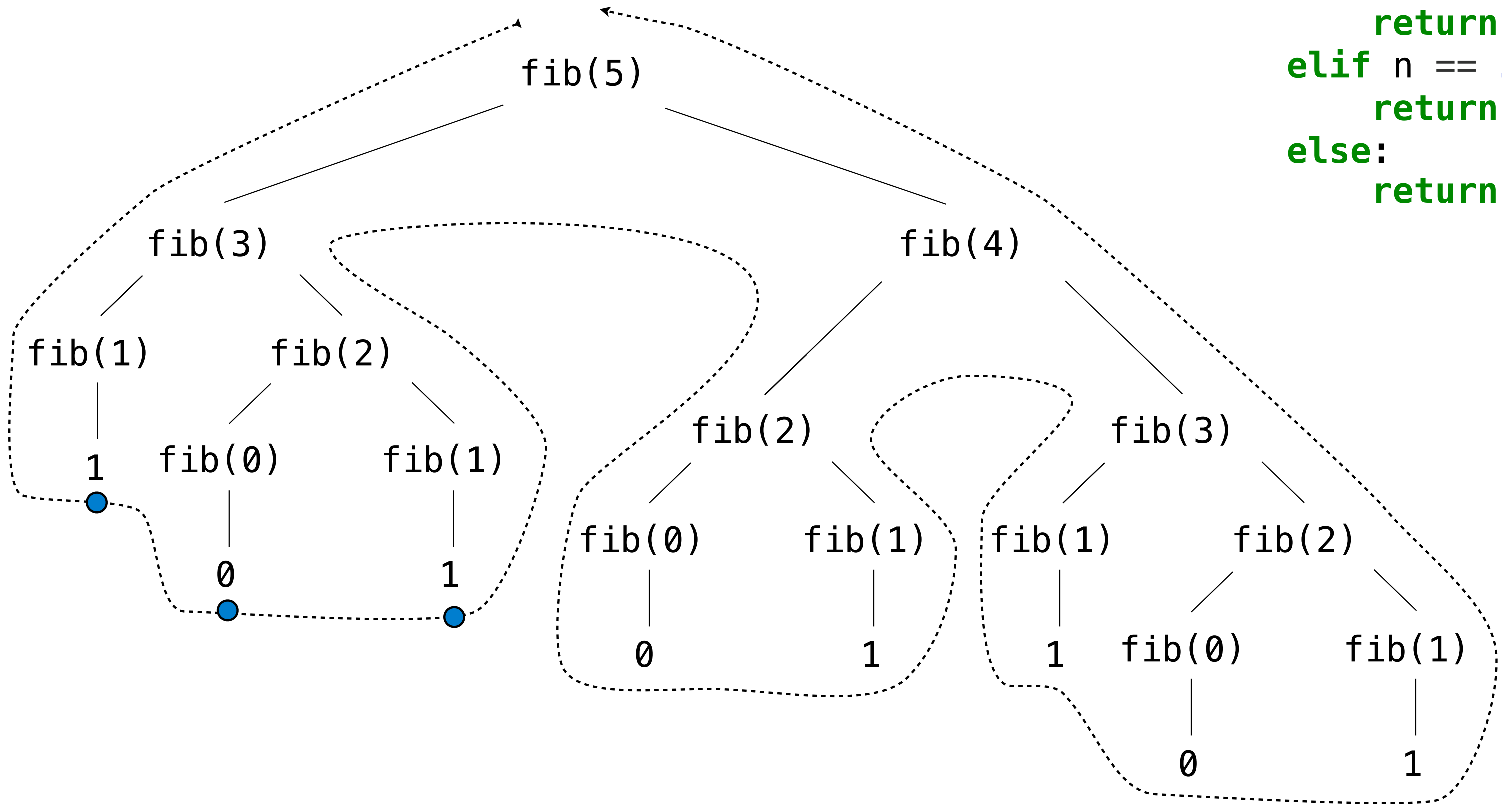


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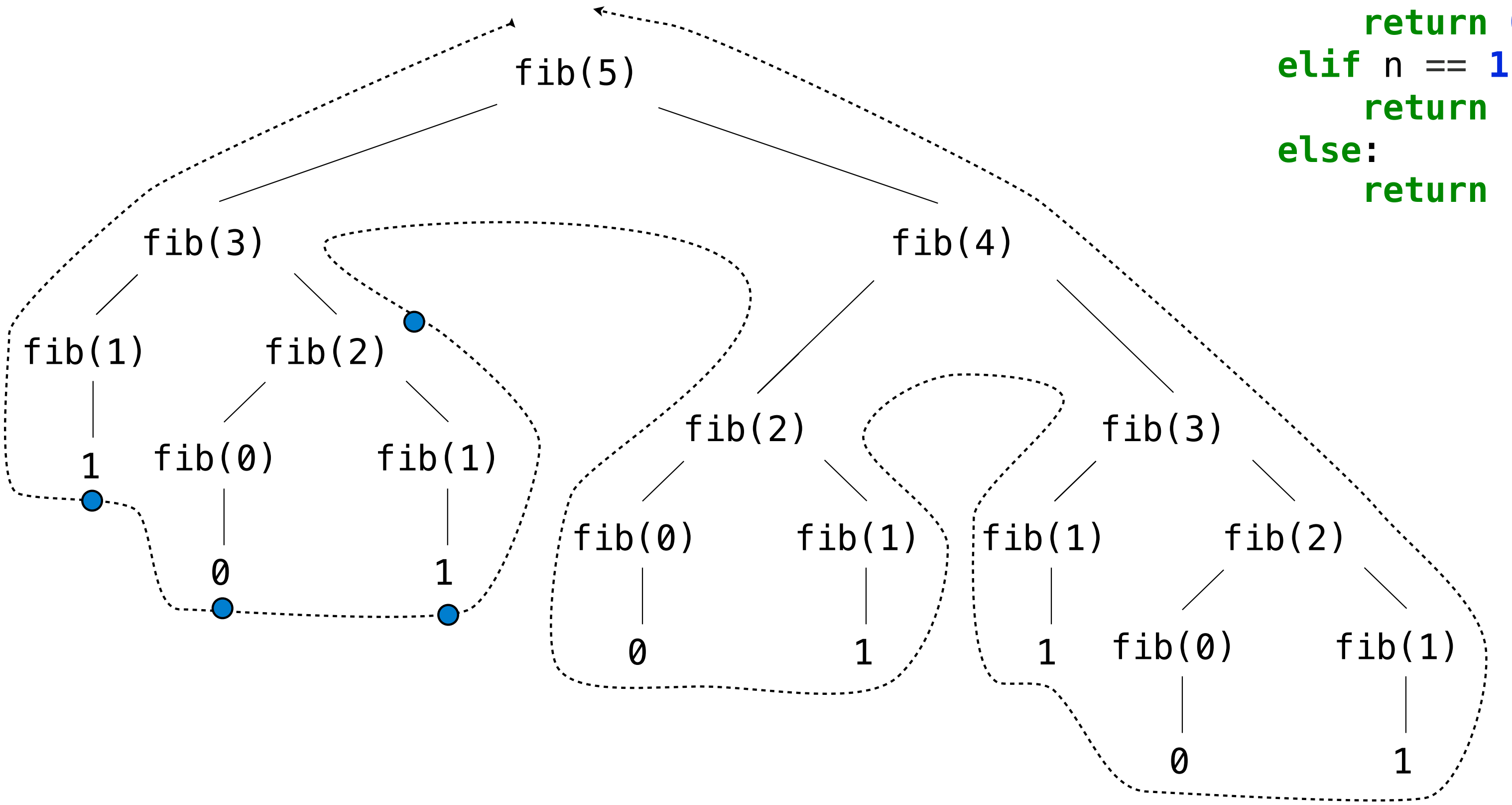


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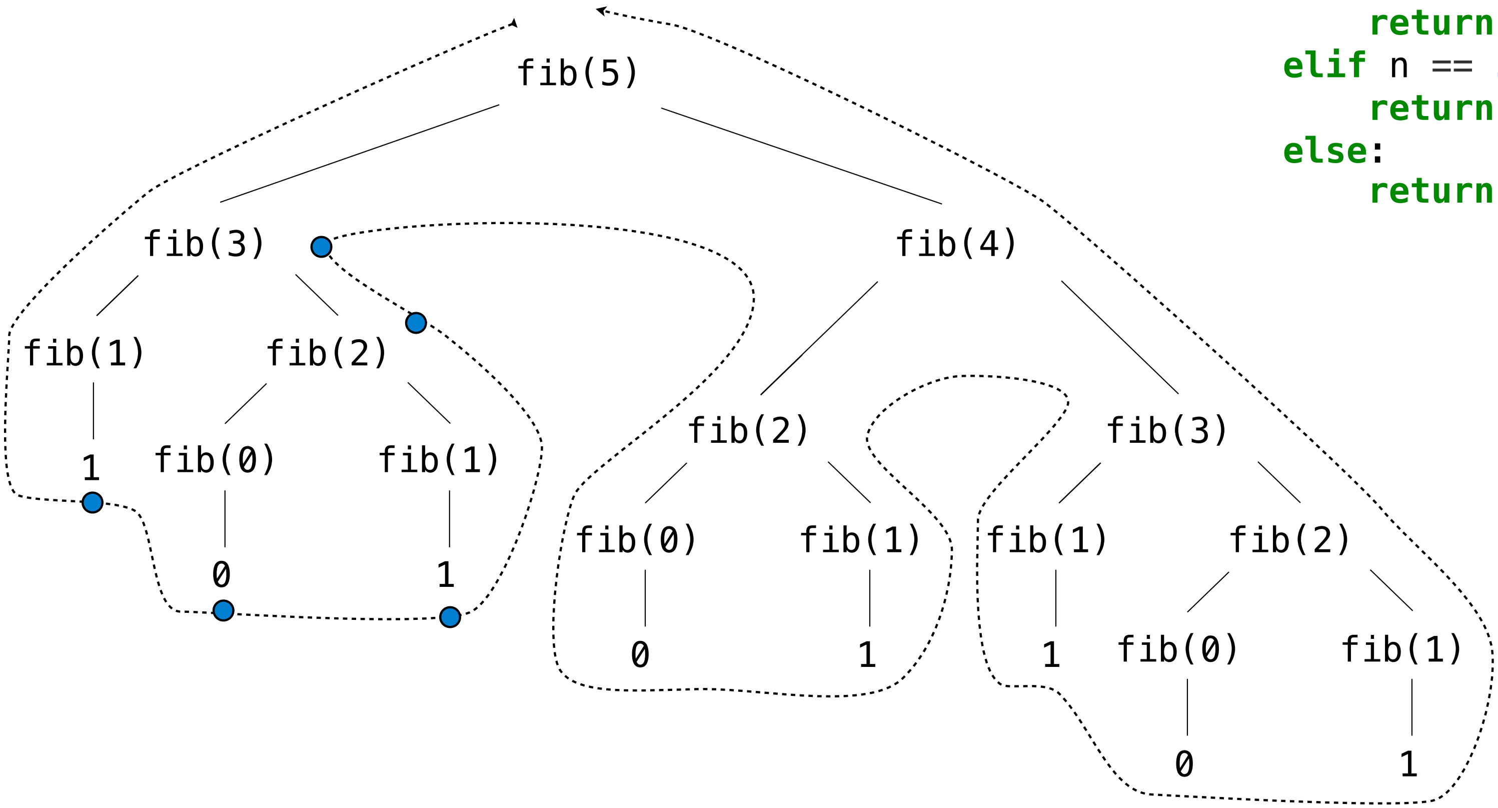

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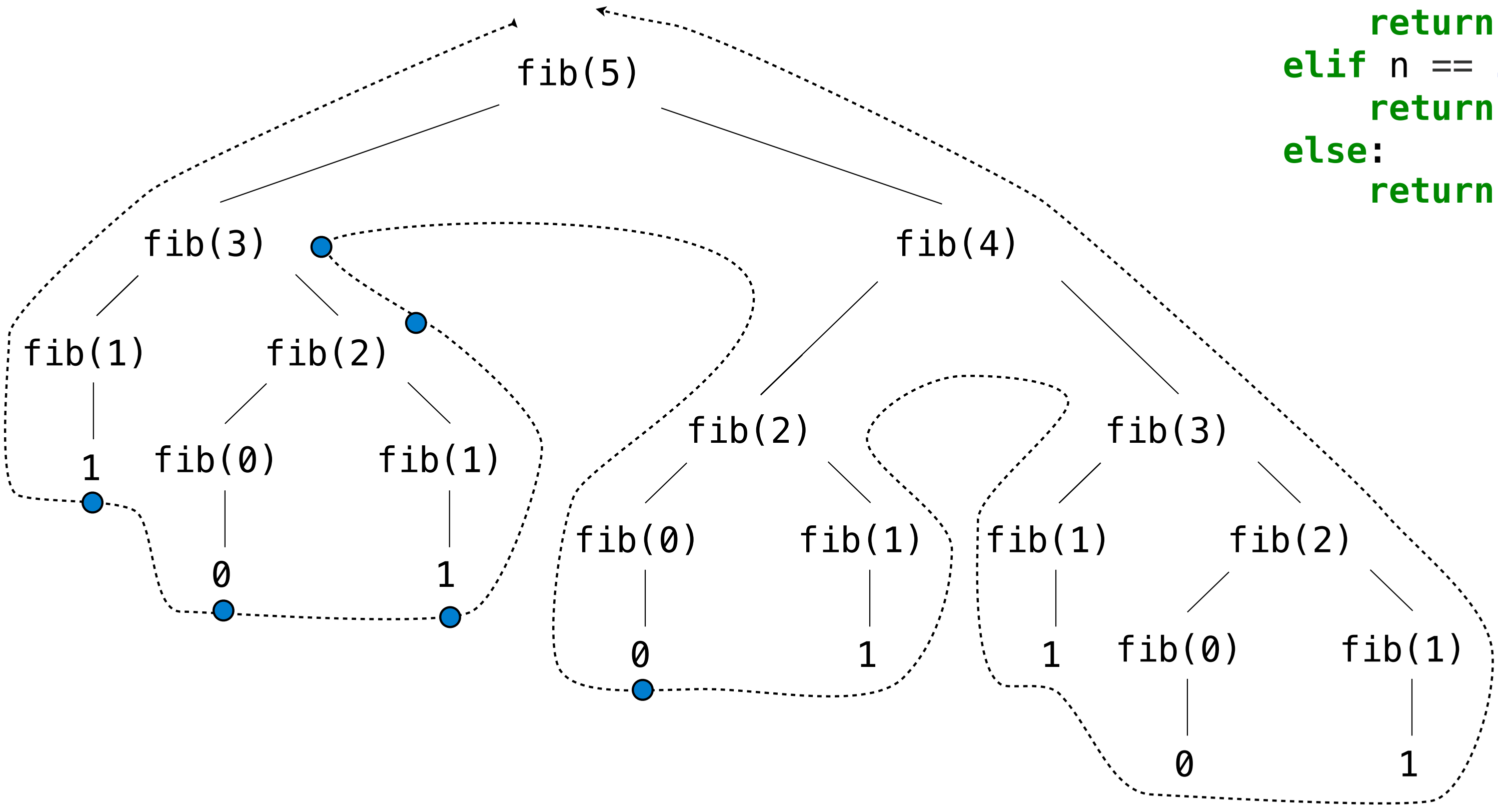

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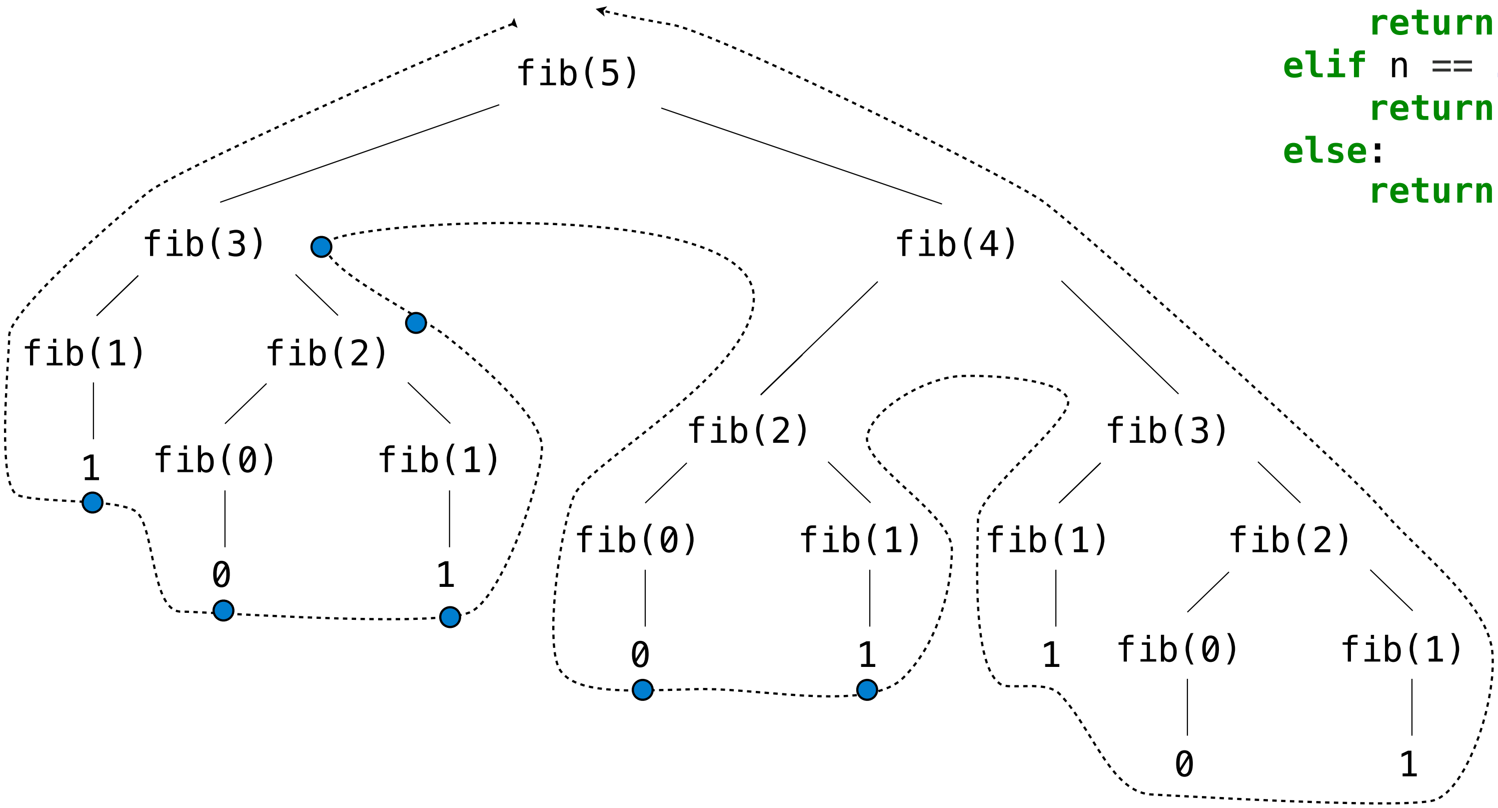

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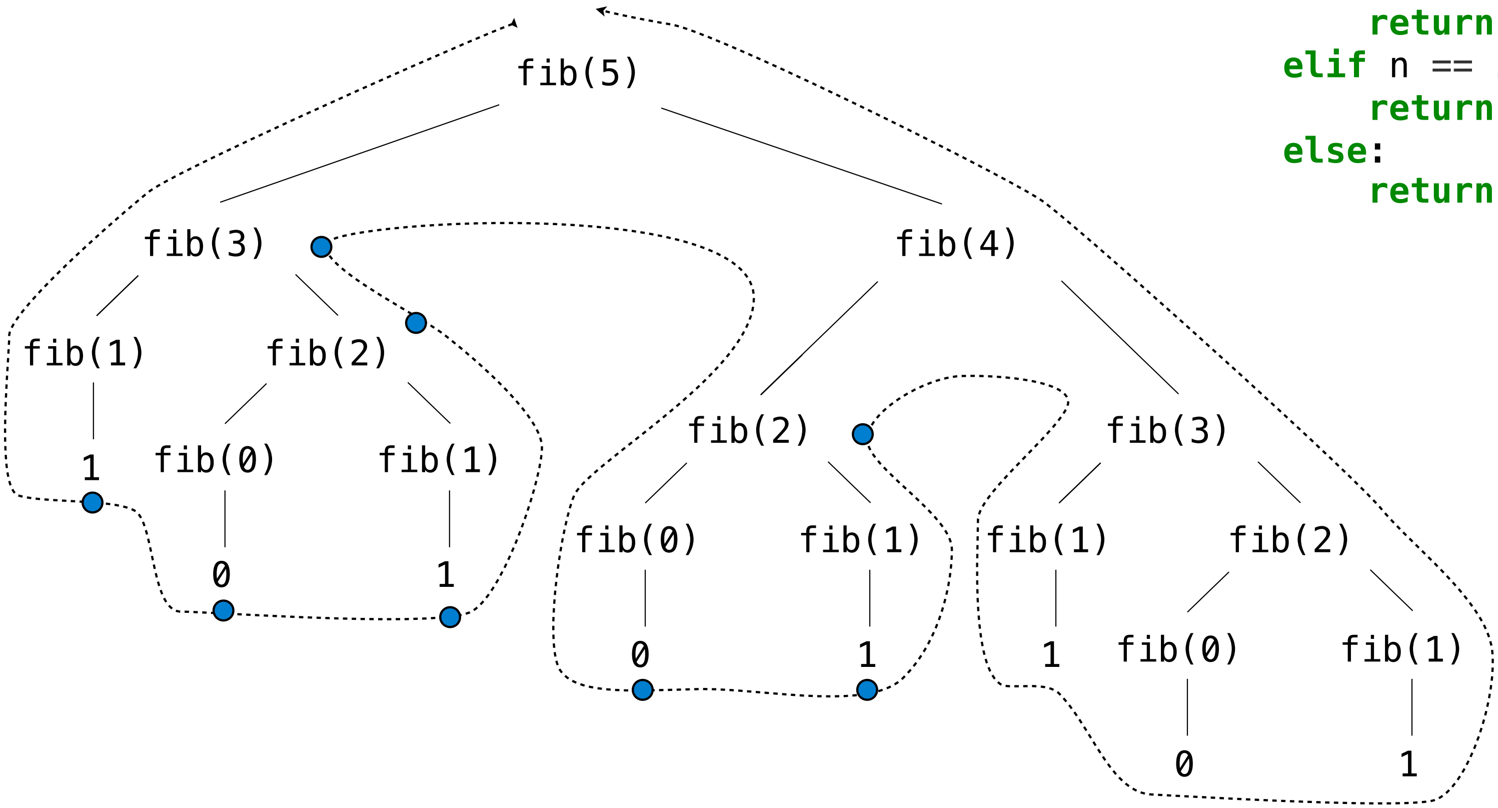

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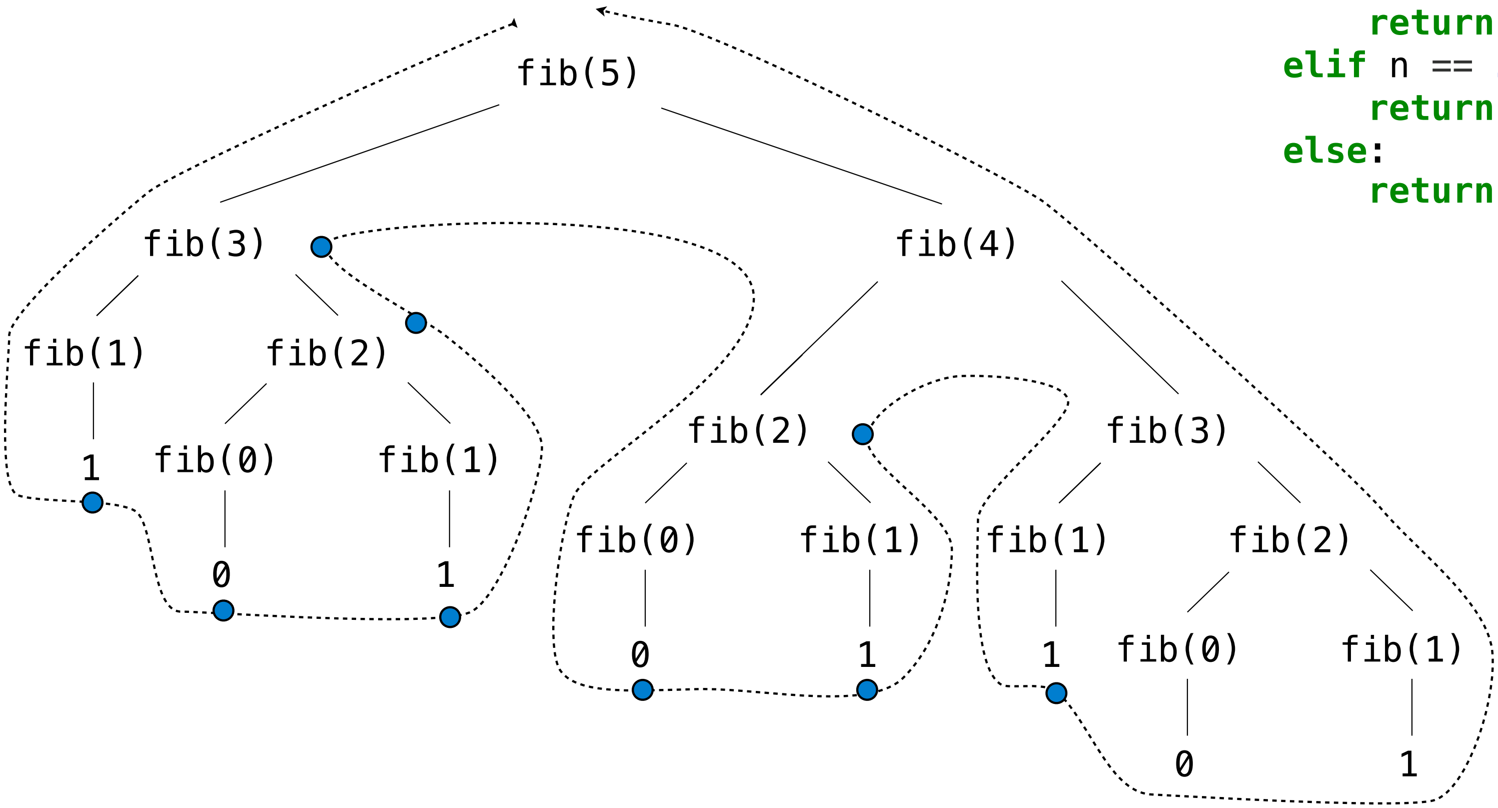


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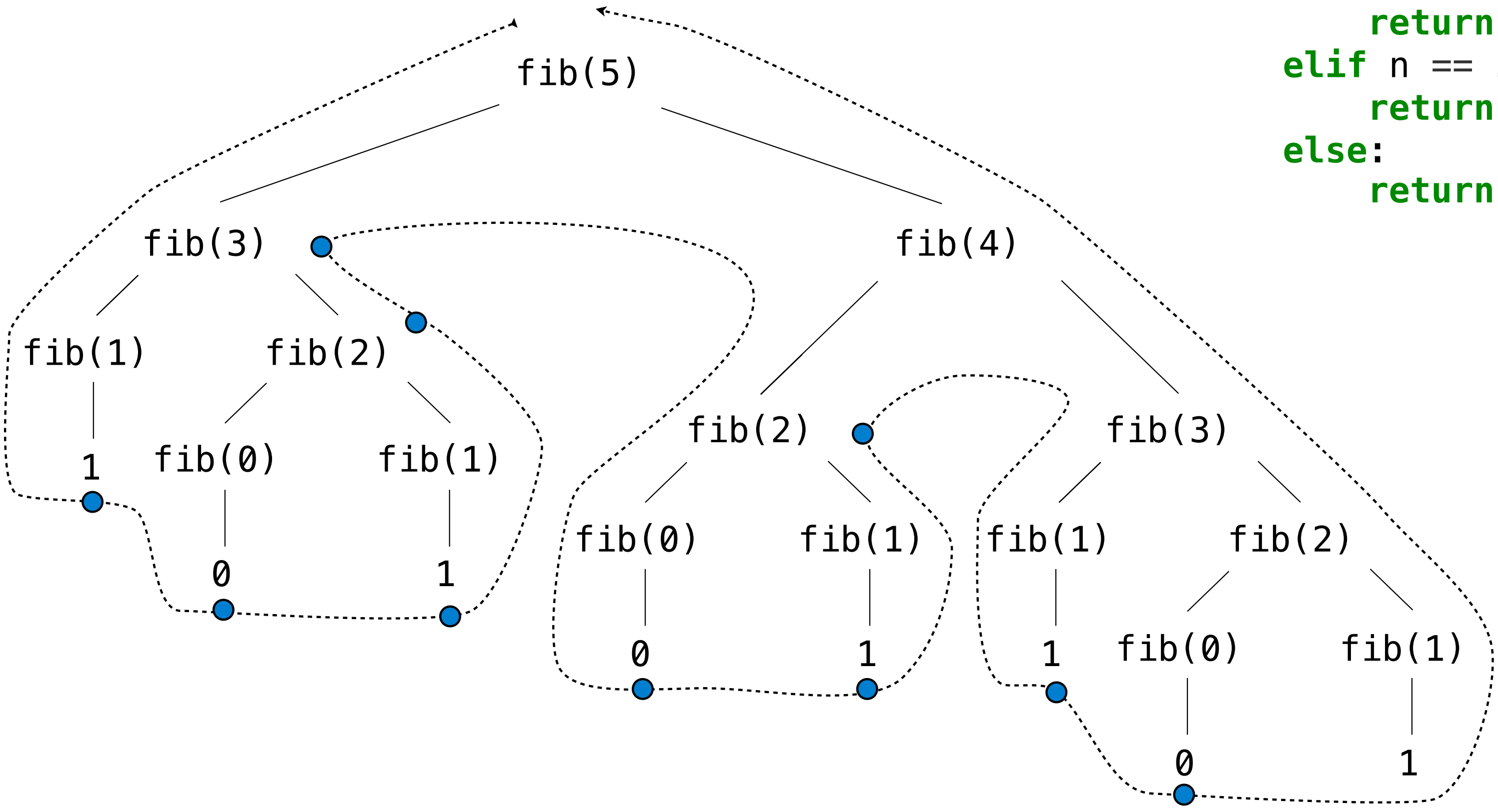

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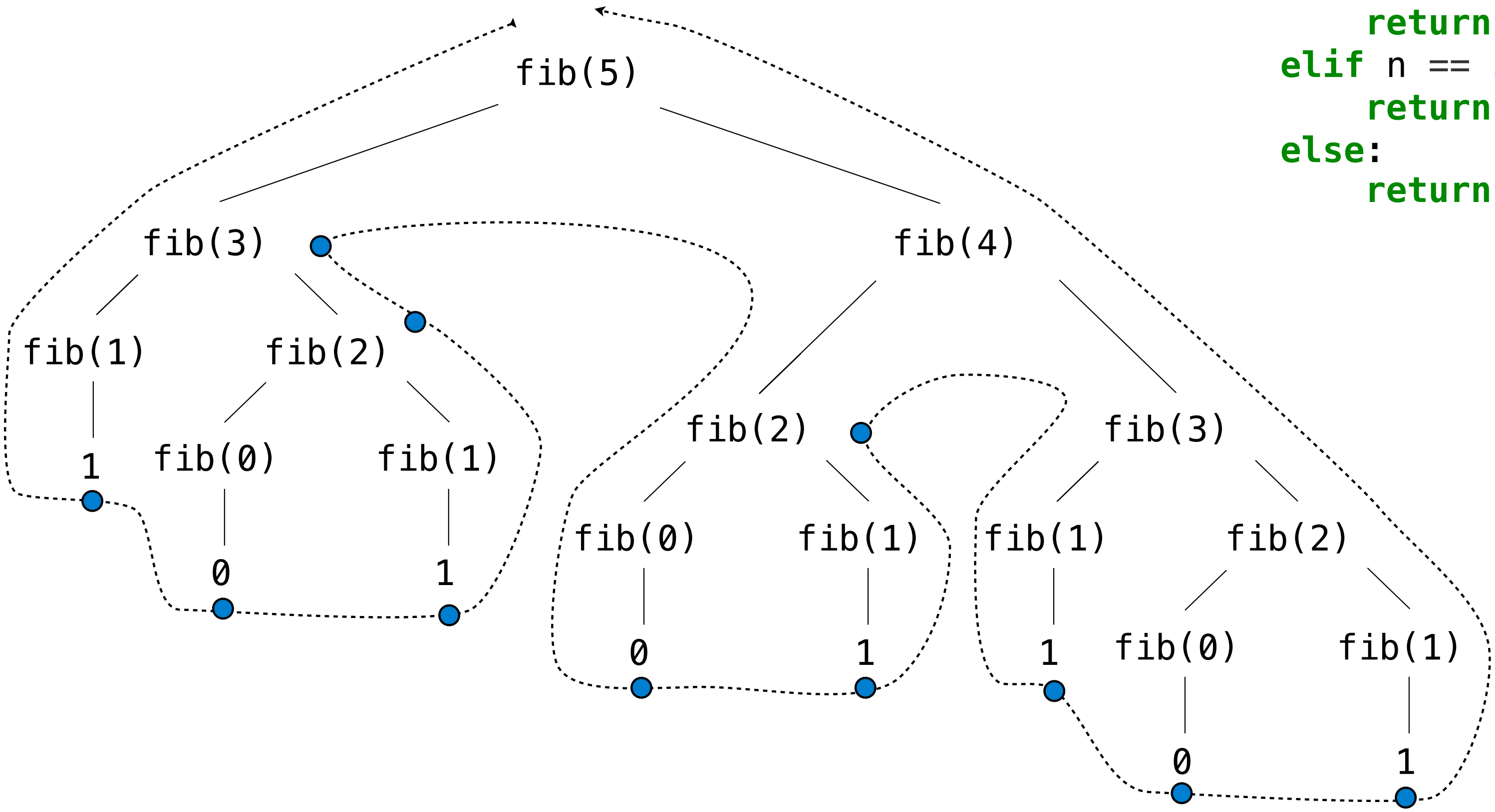

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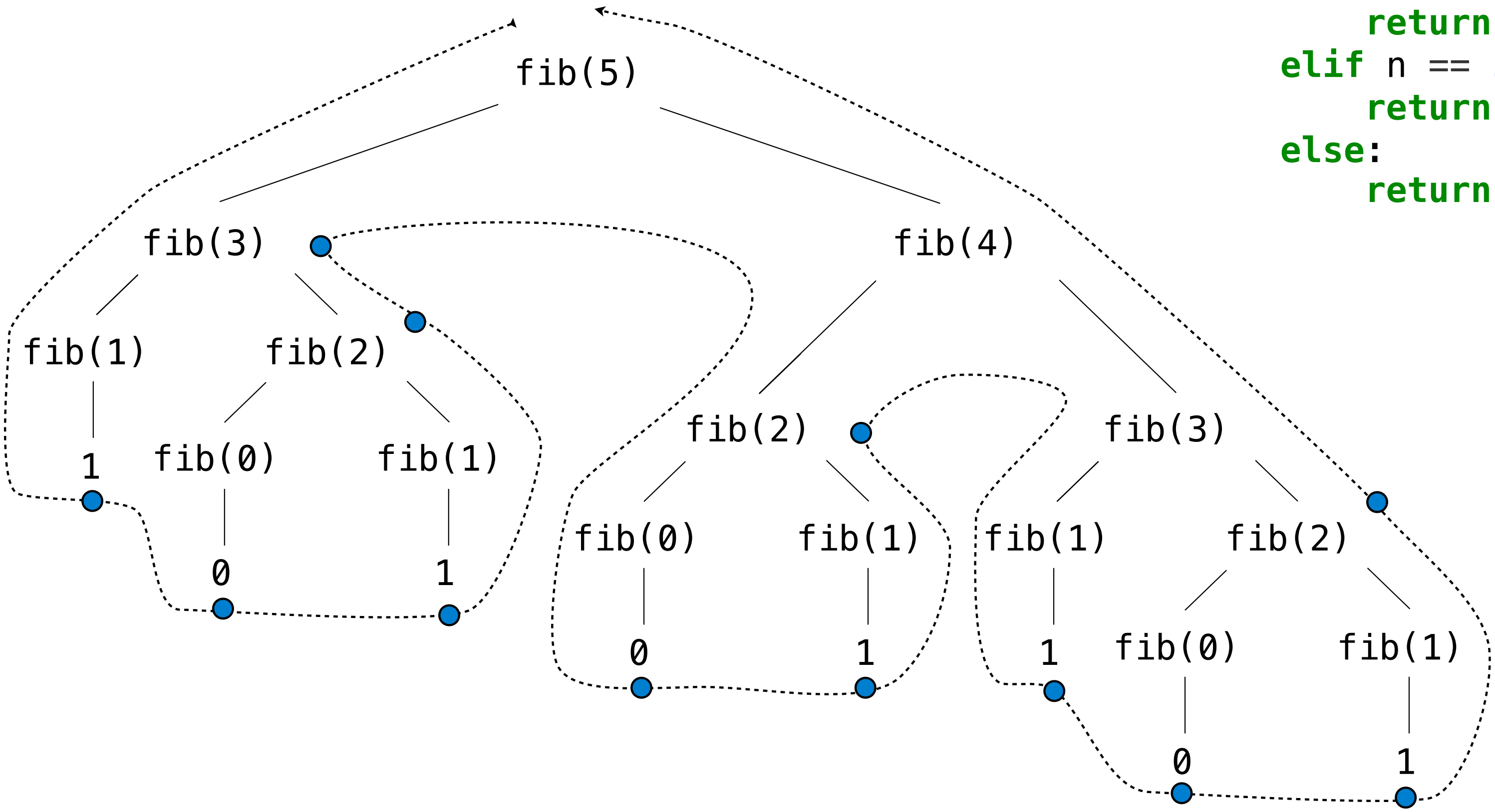


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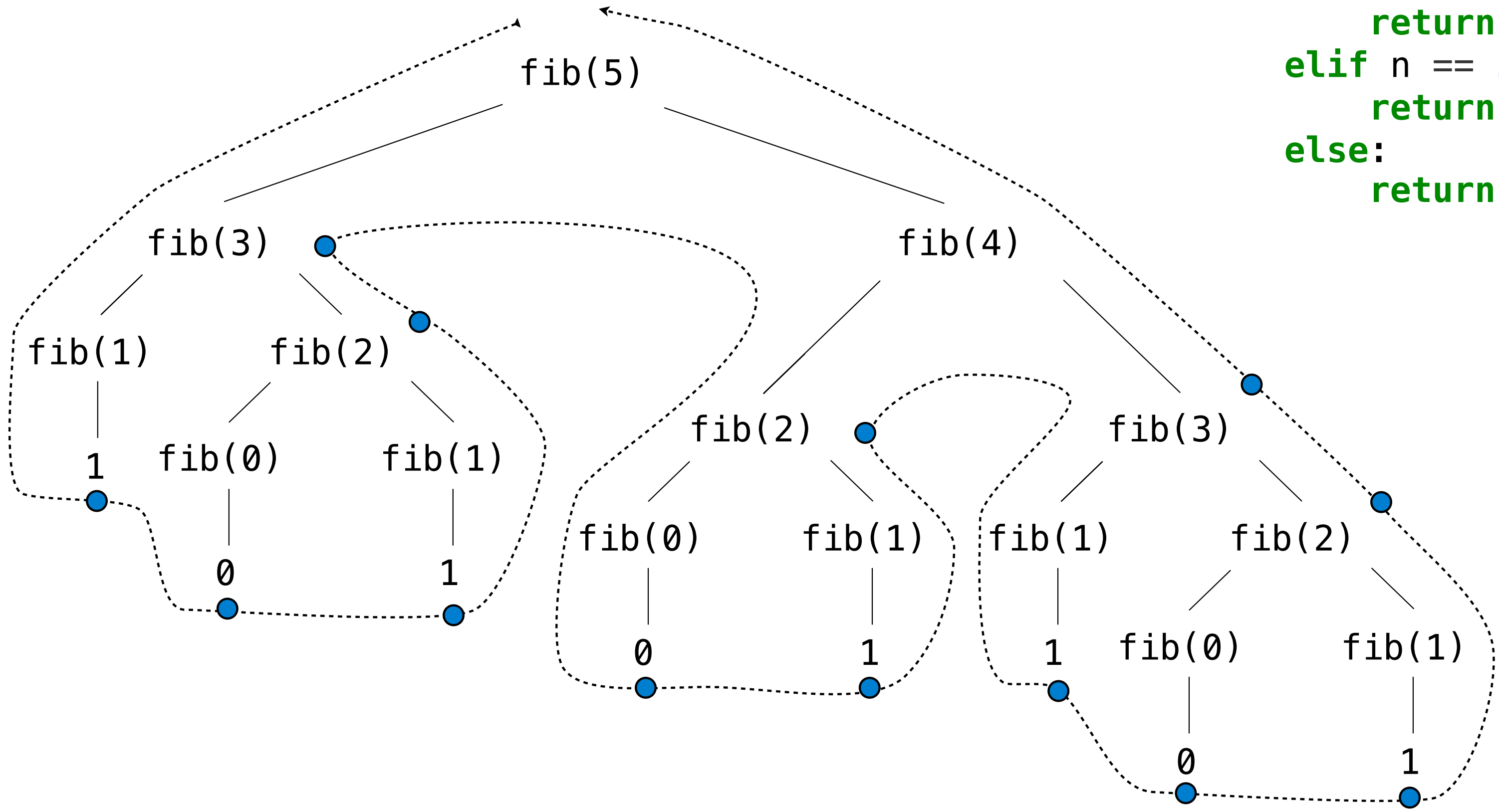

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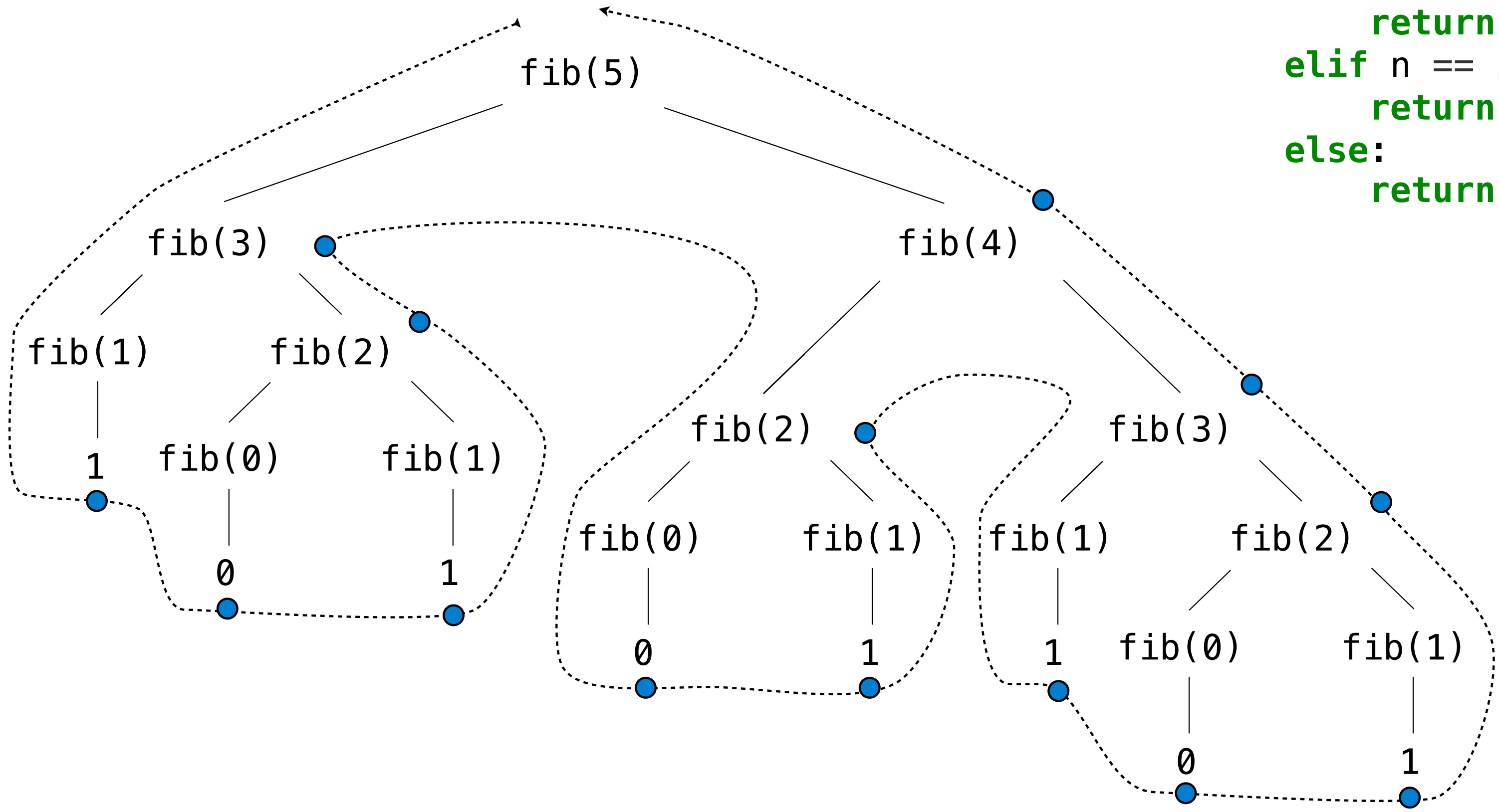

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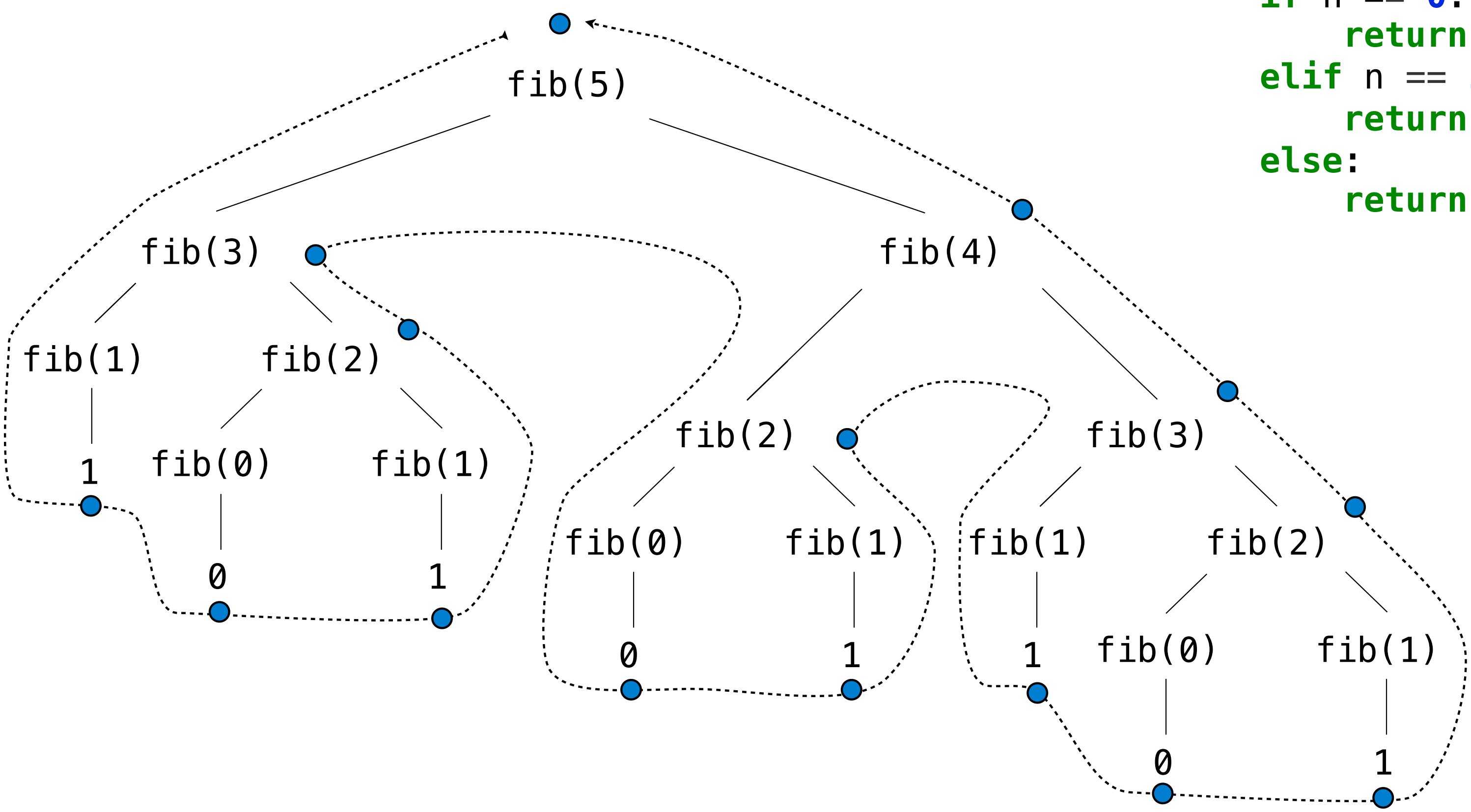


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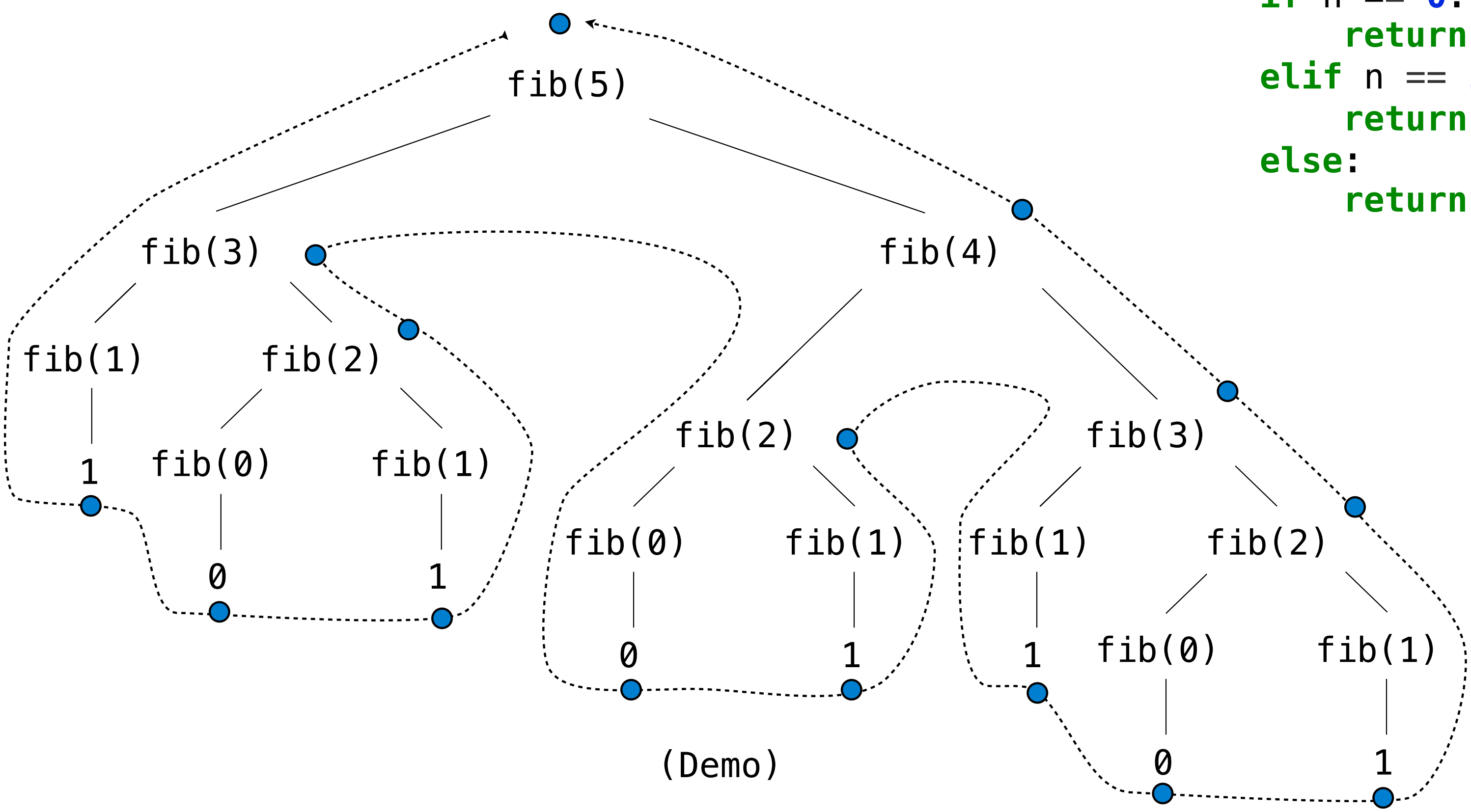


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(Demo)

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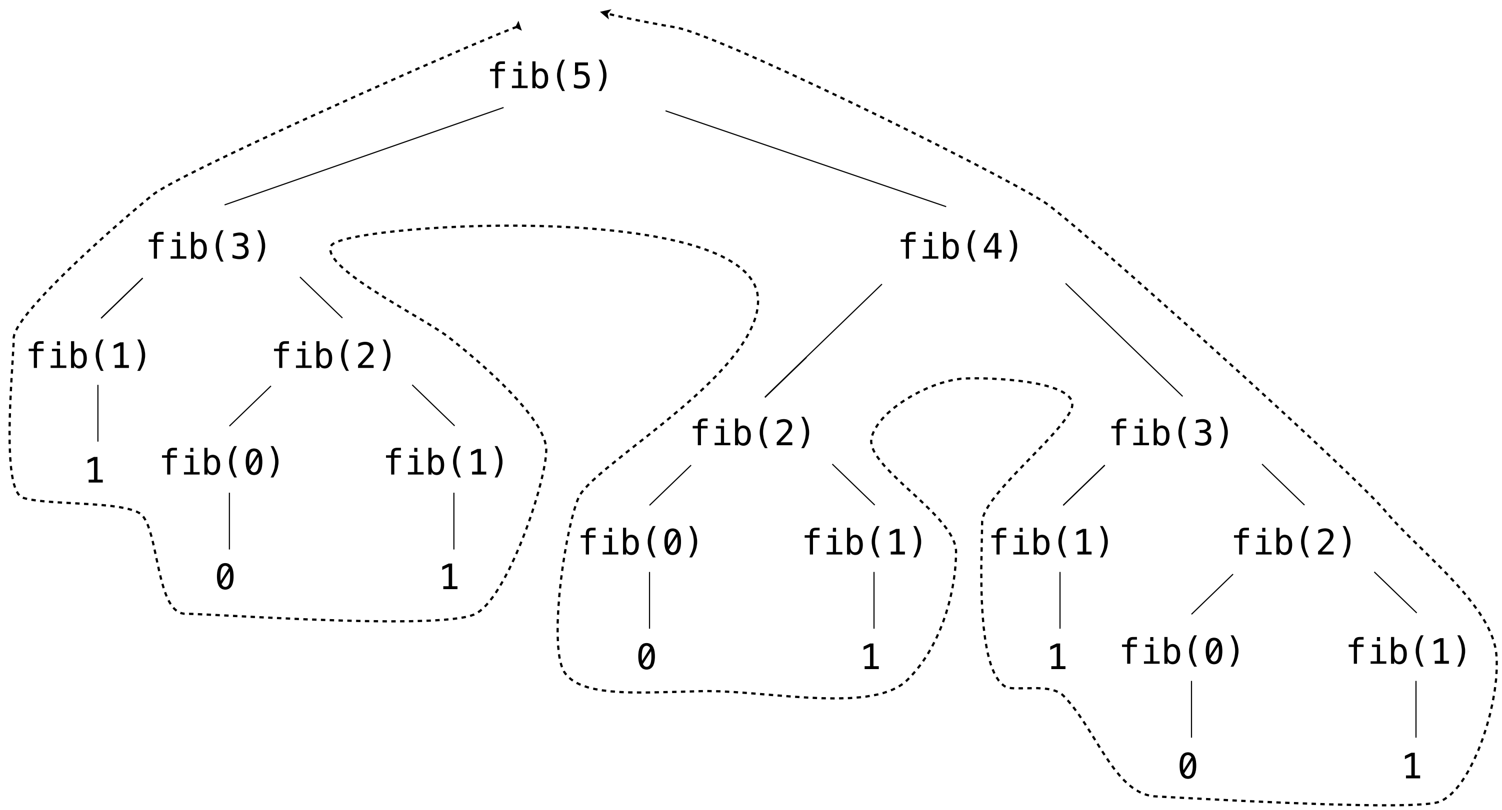

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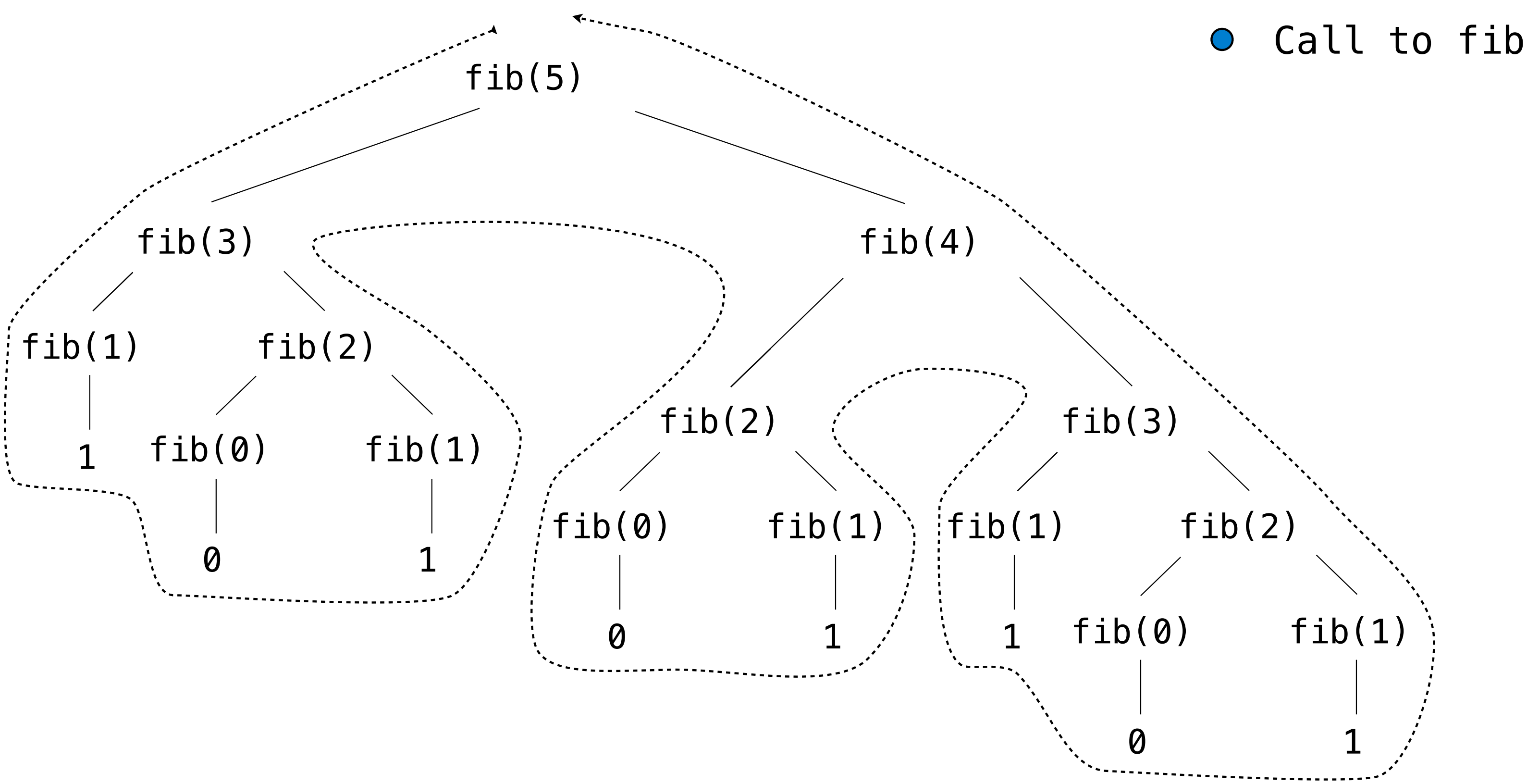

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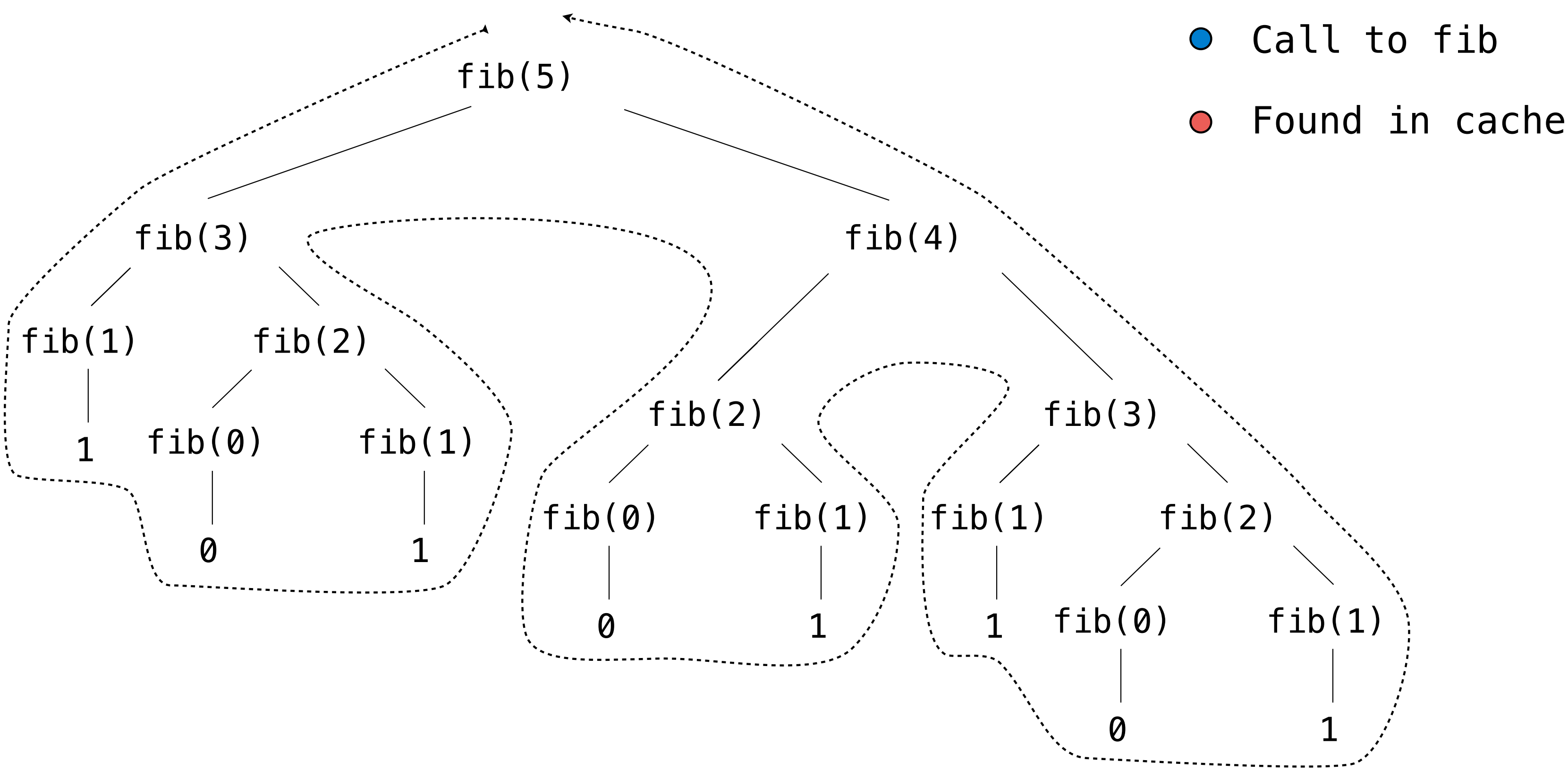

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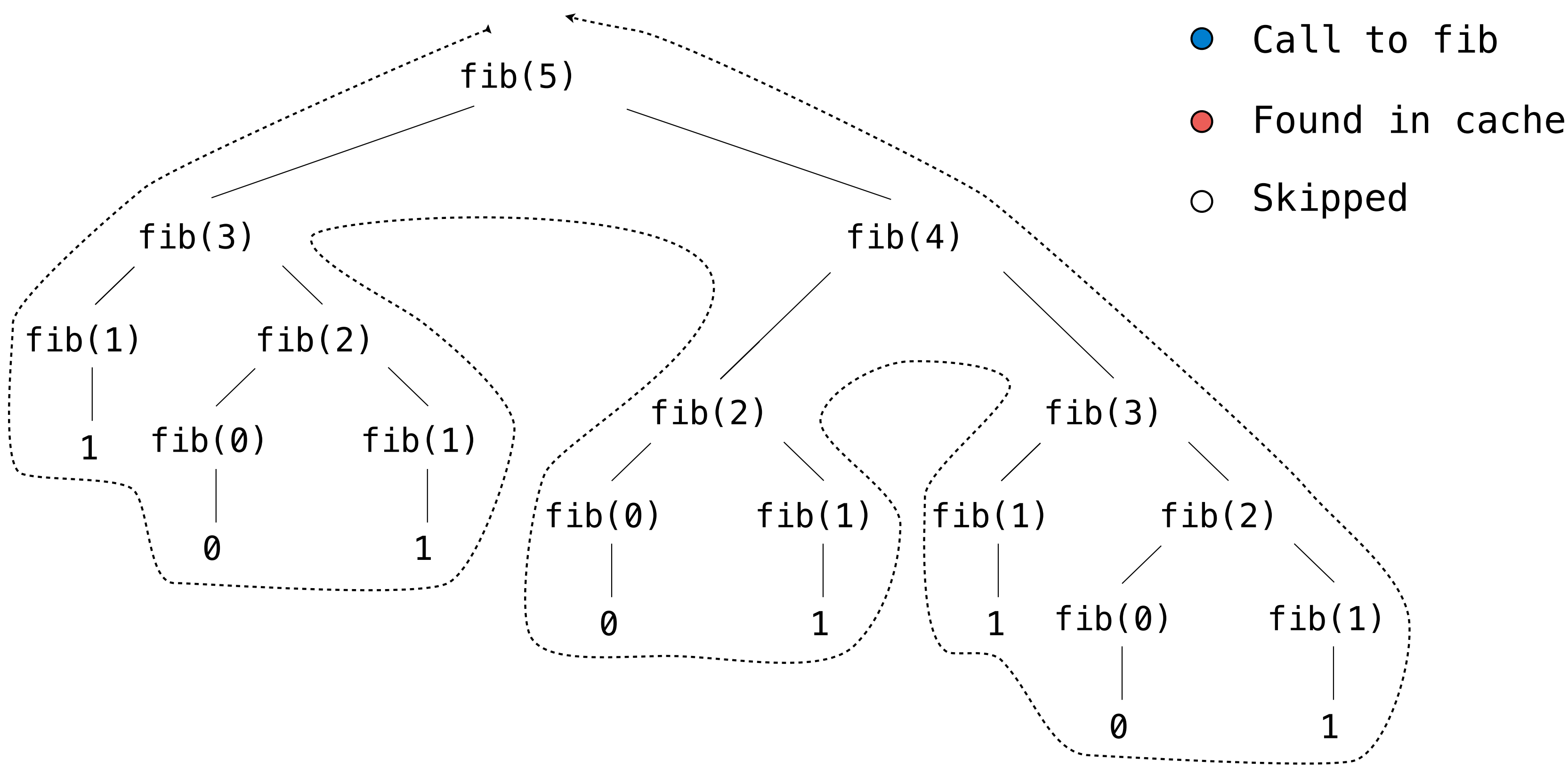

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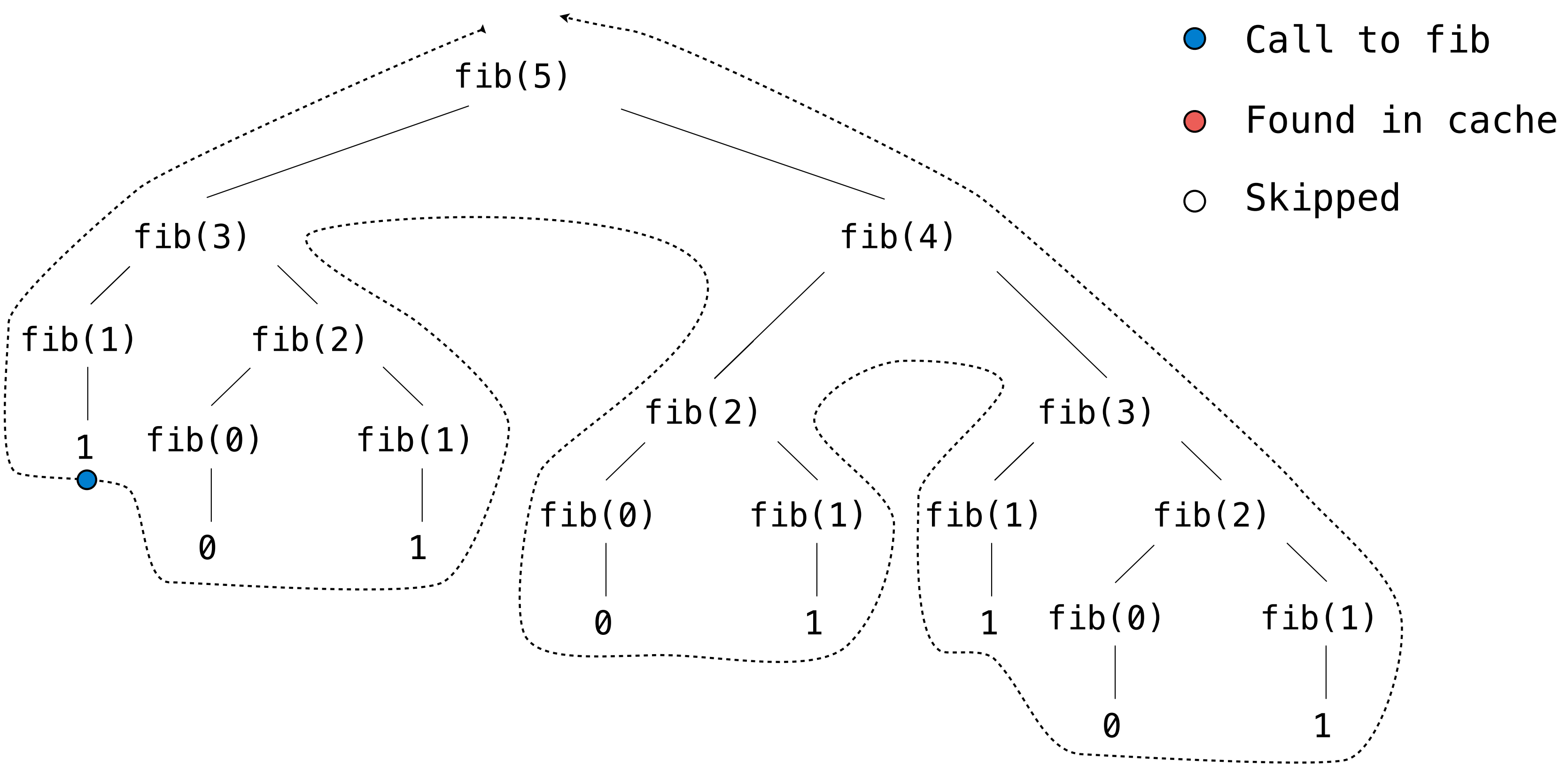

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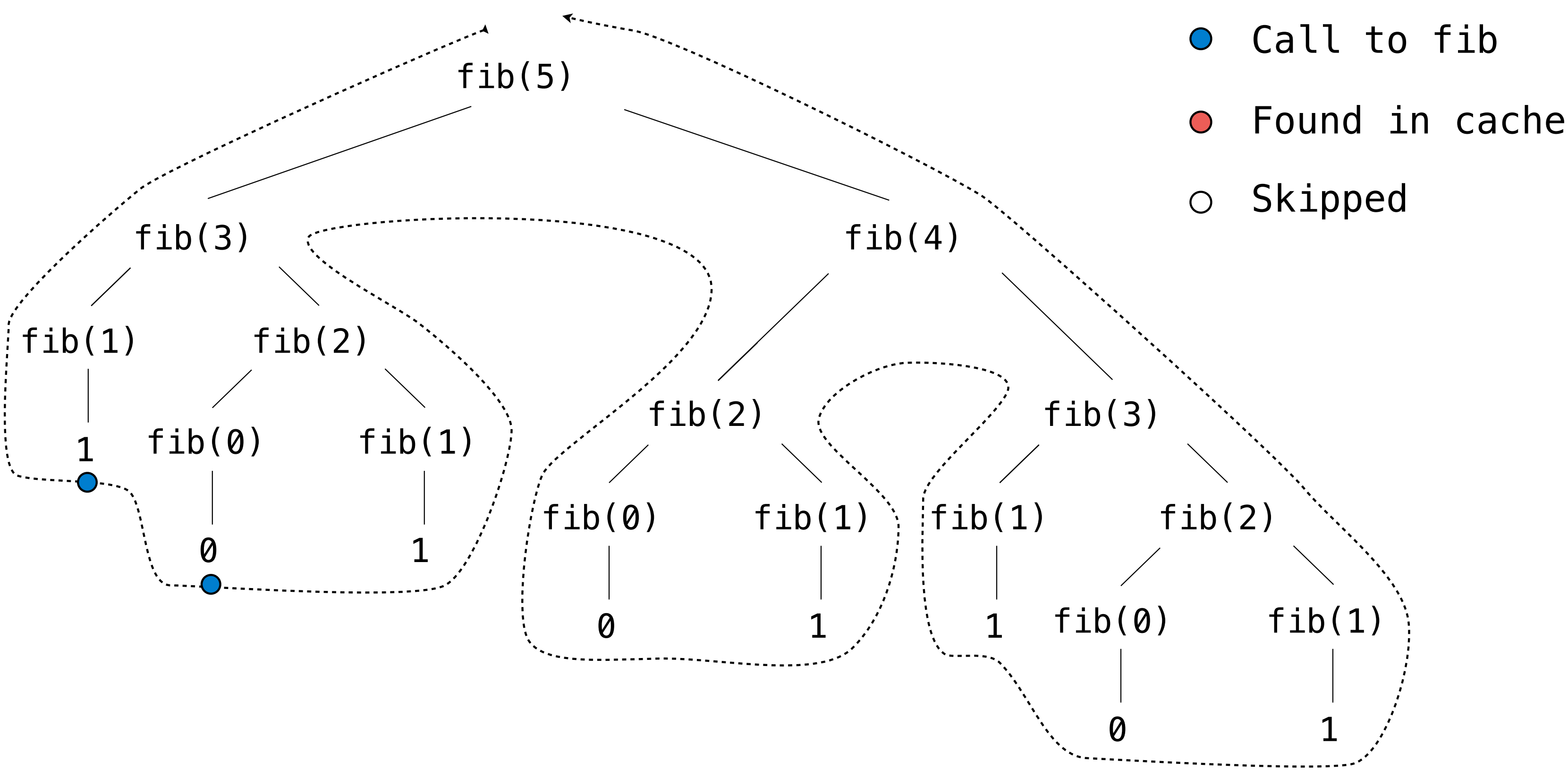

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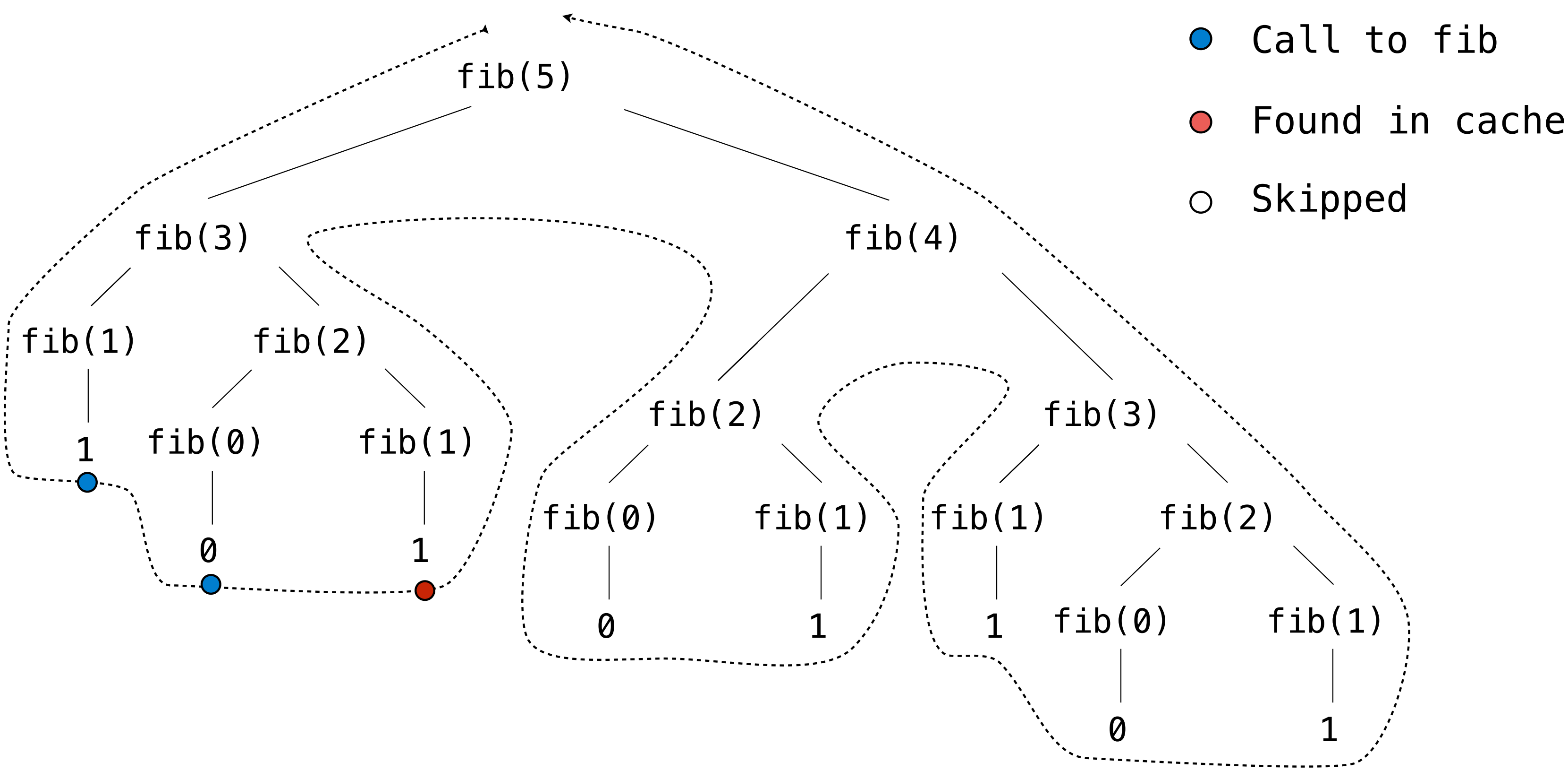

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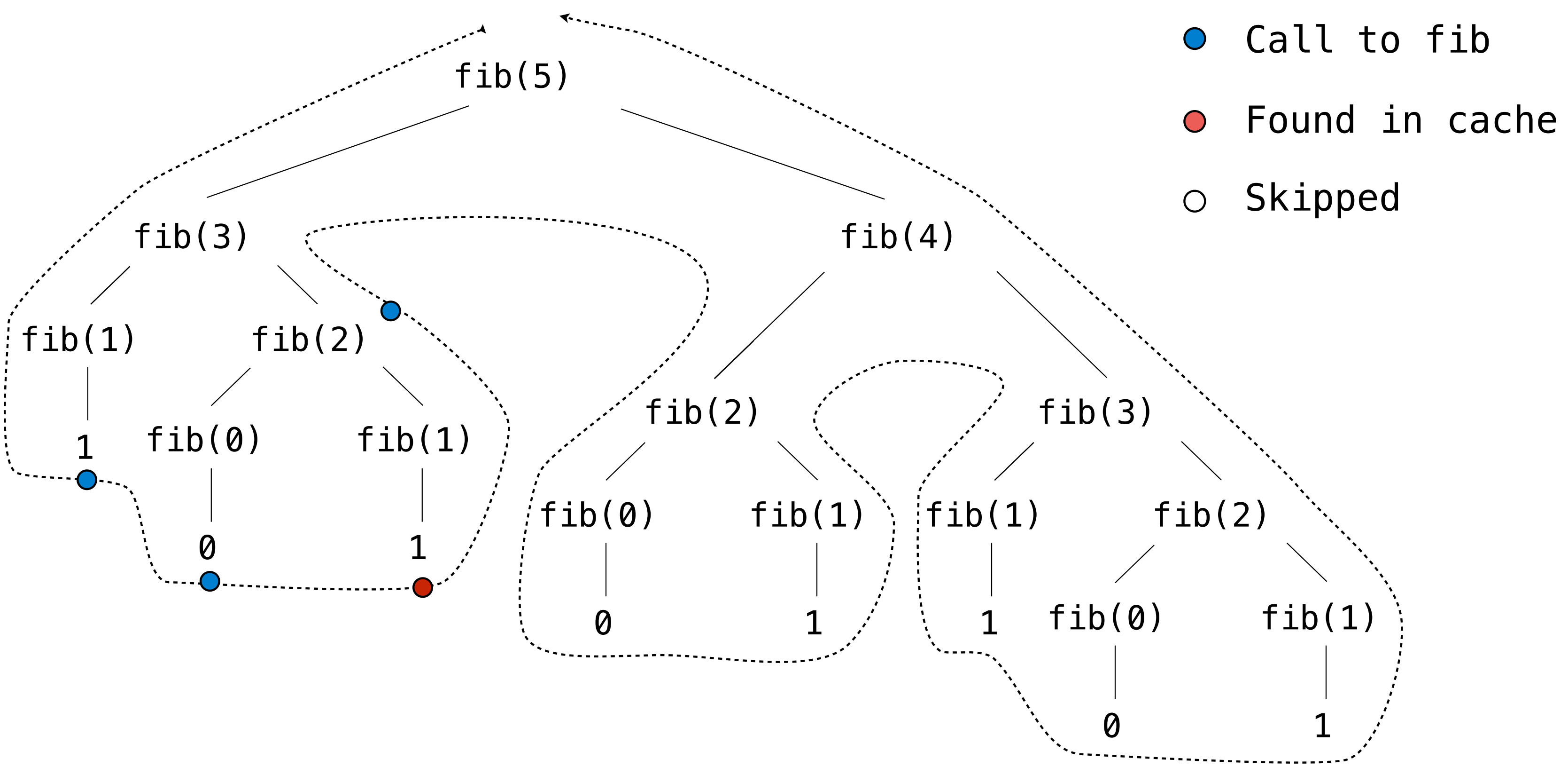

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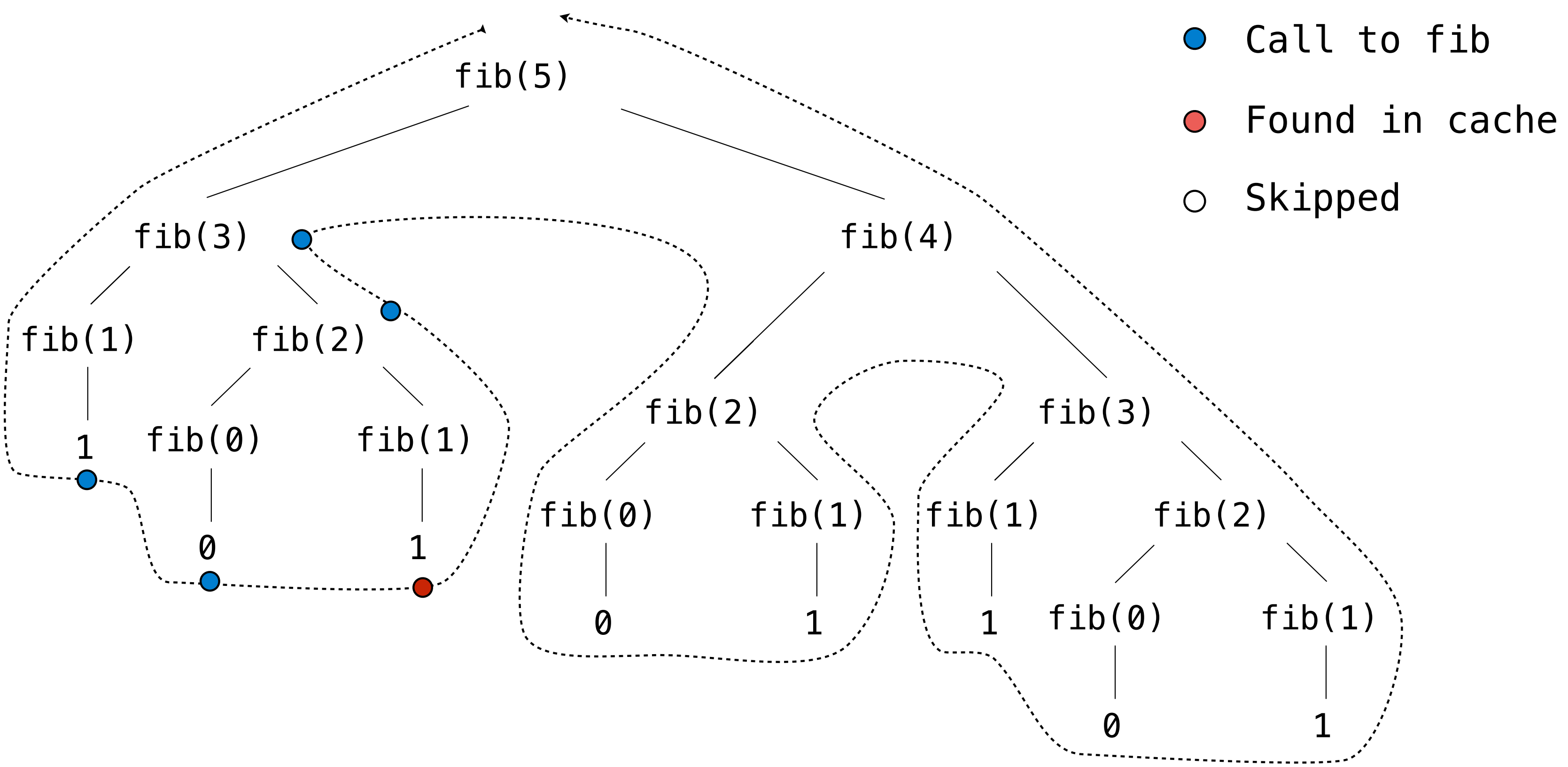

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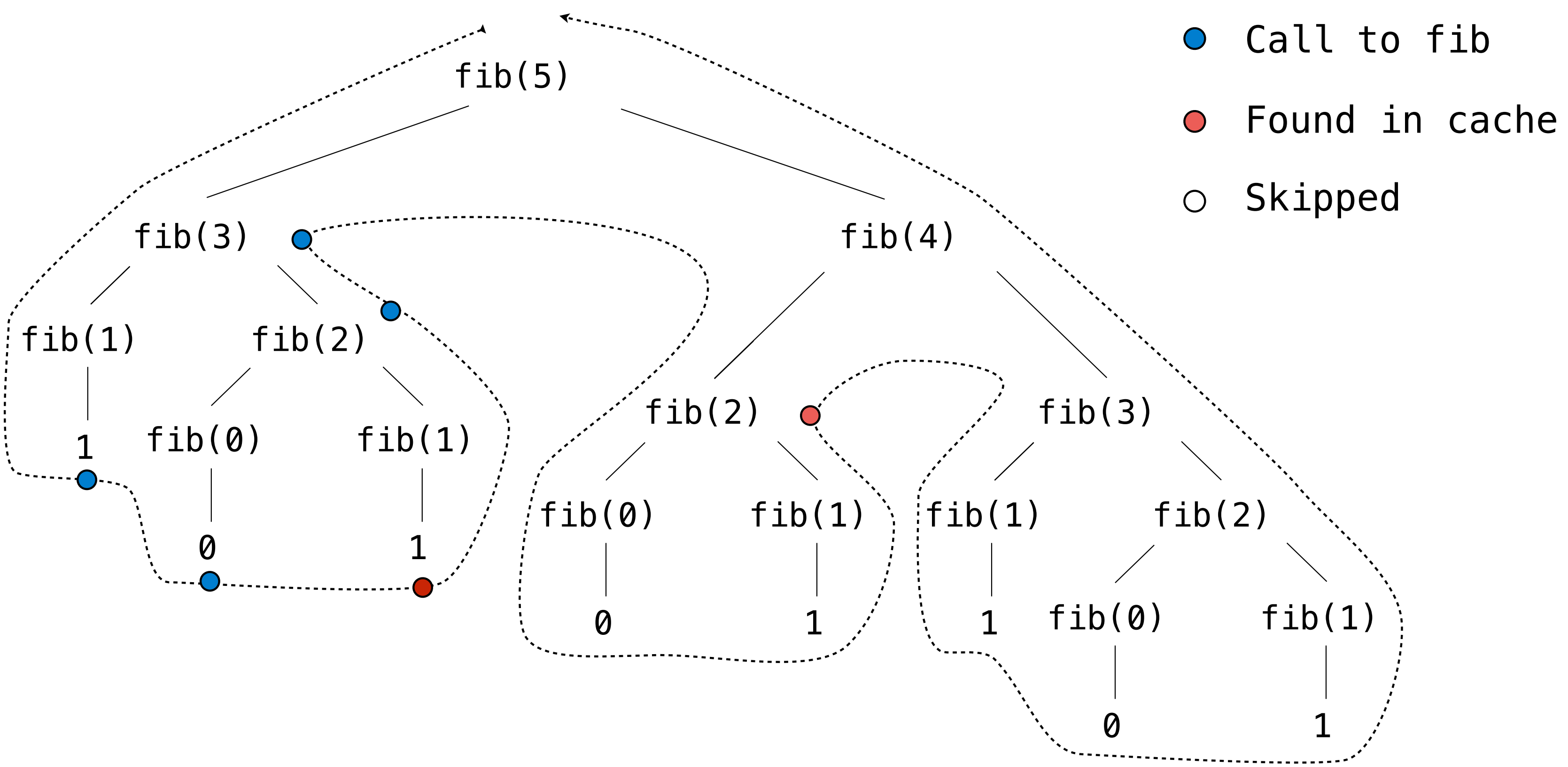

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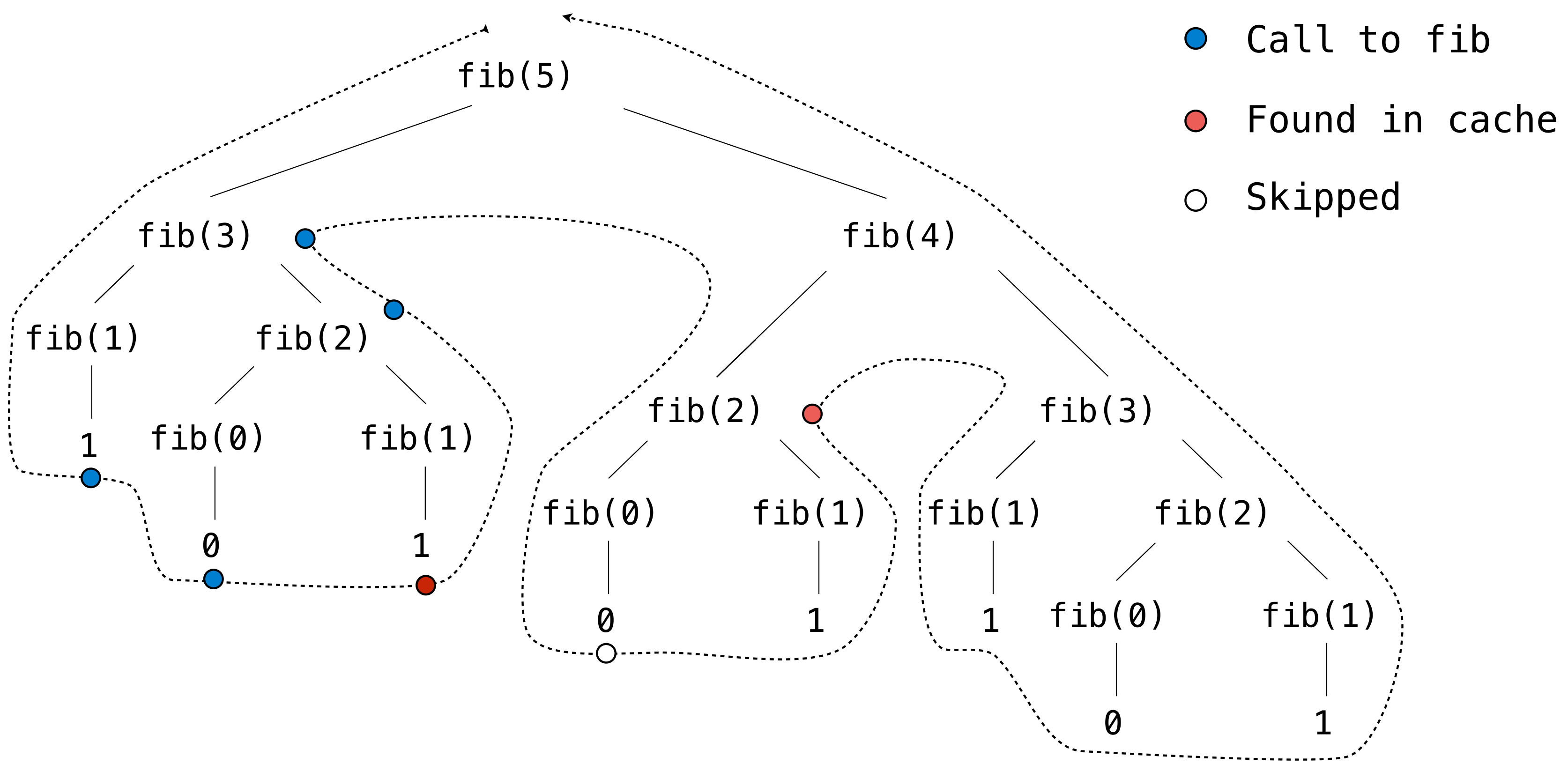

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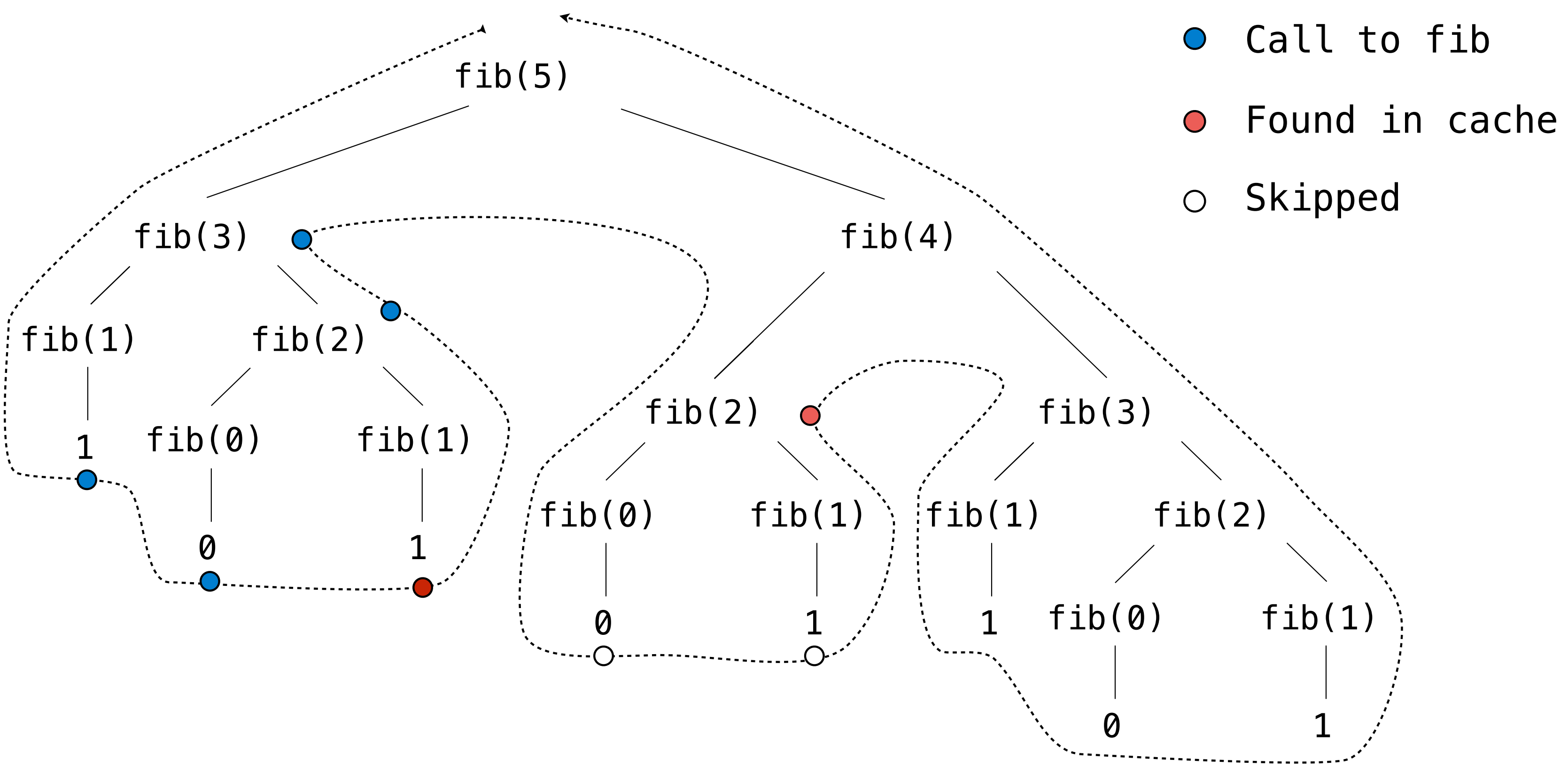

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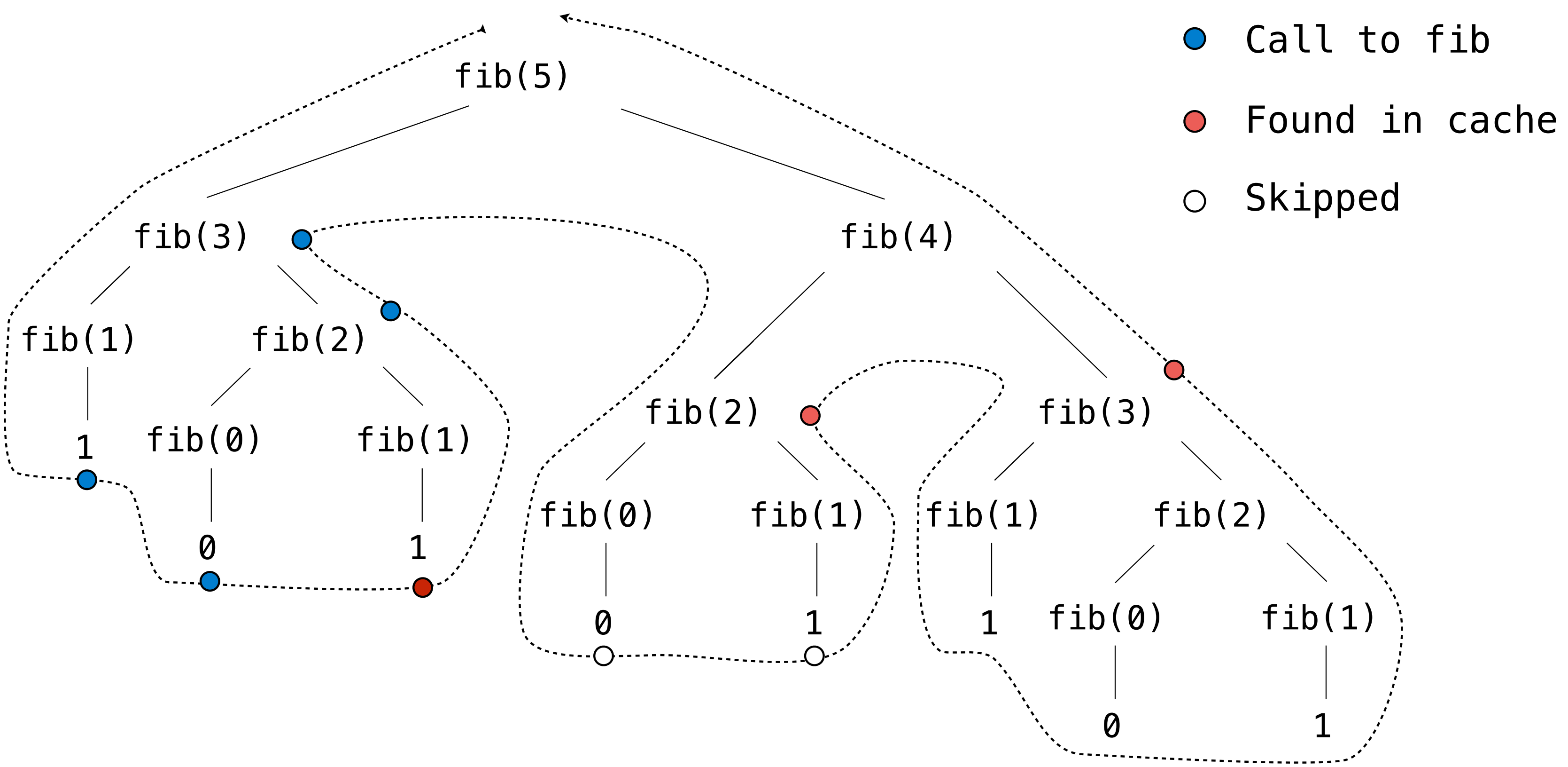

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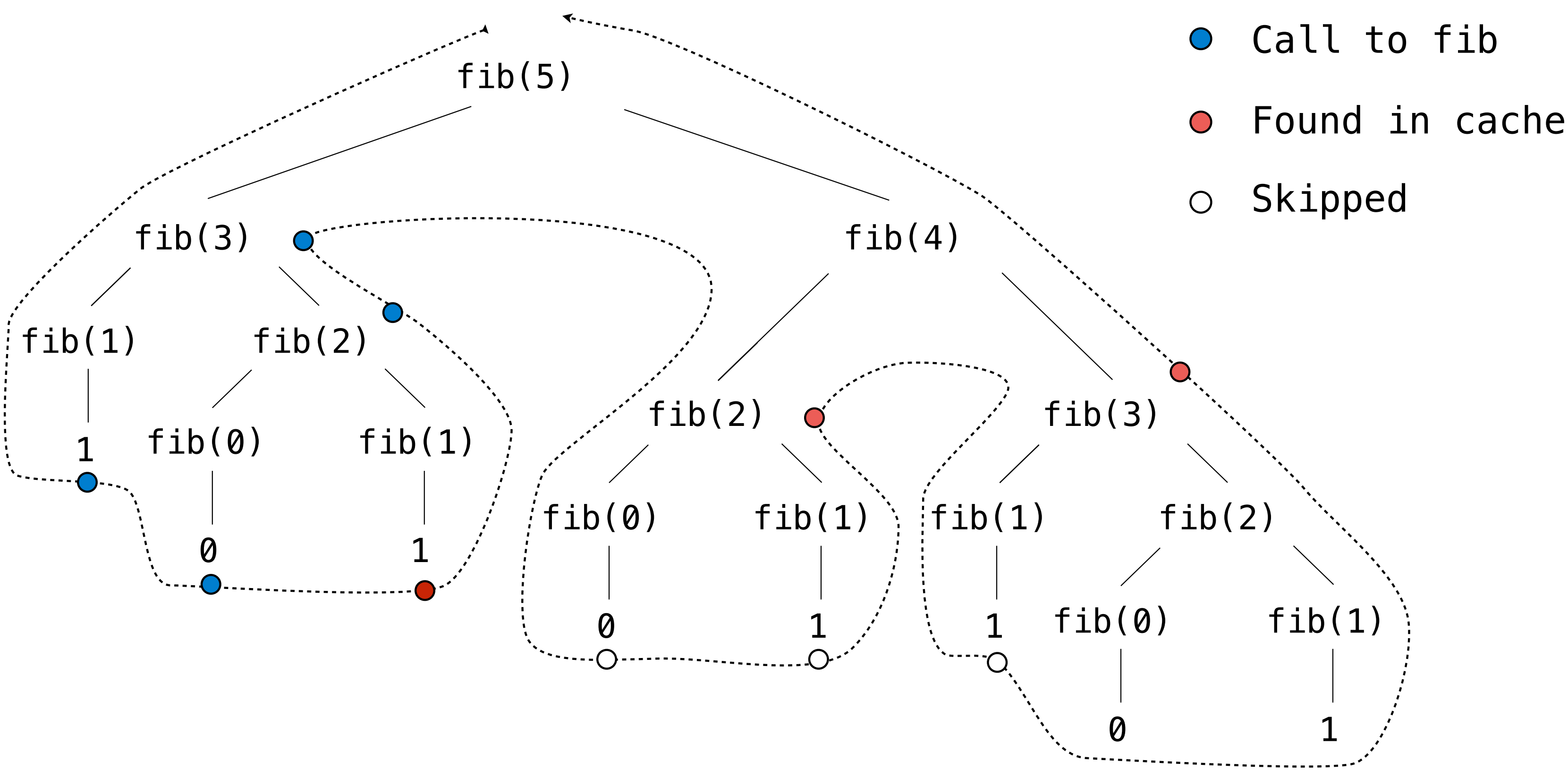

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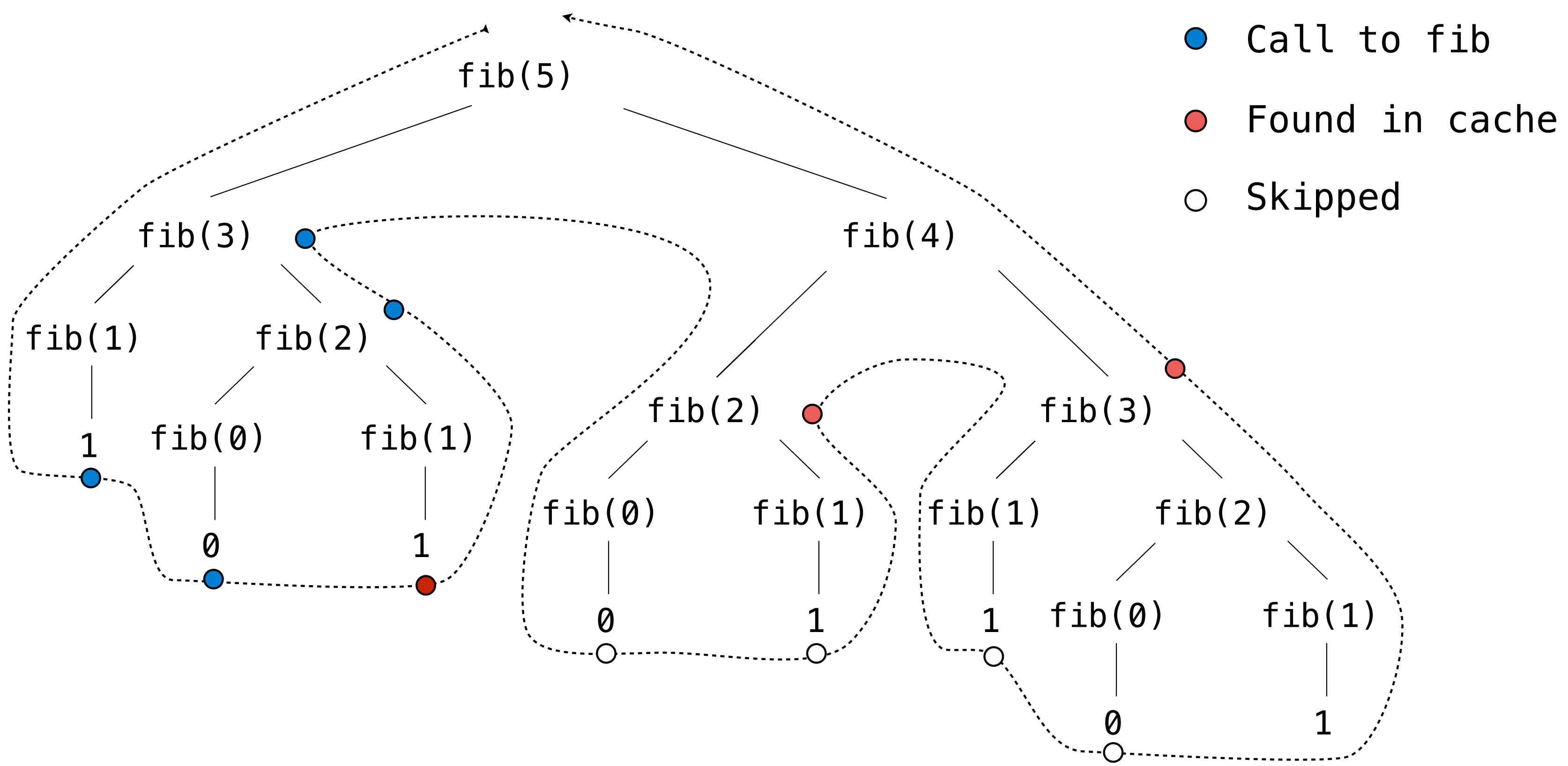

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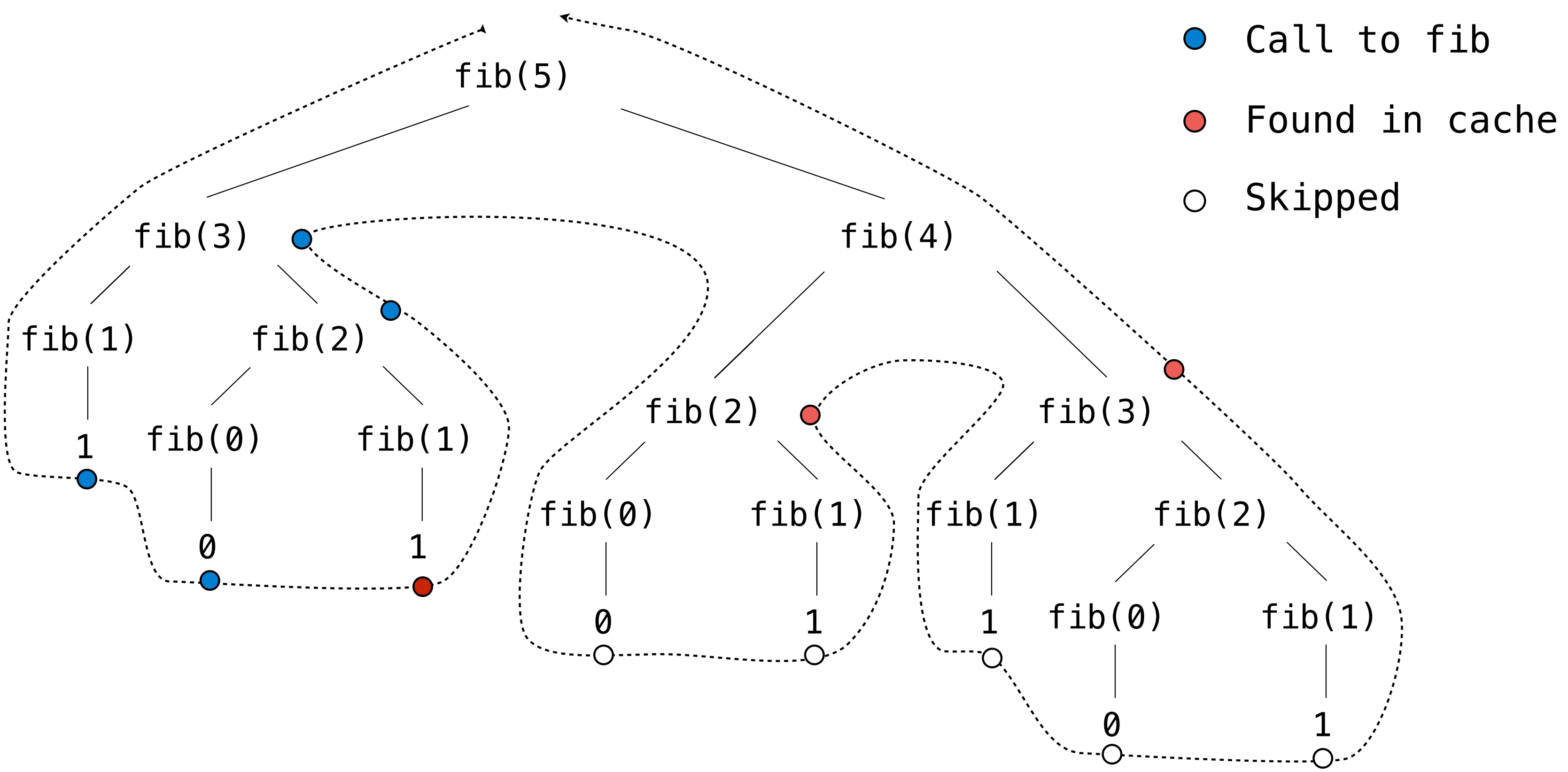

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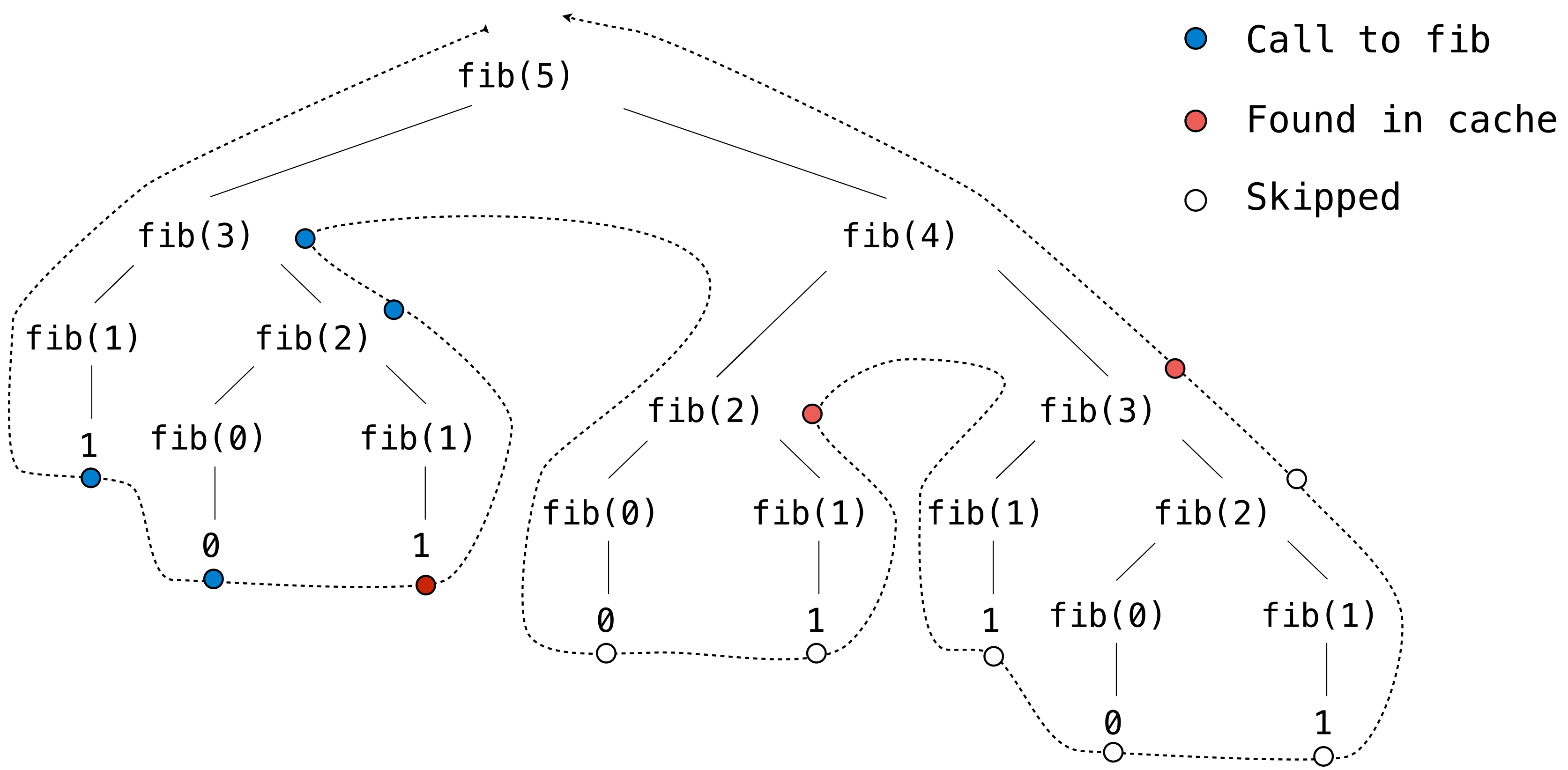

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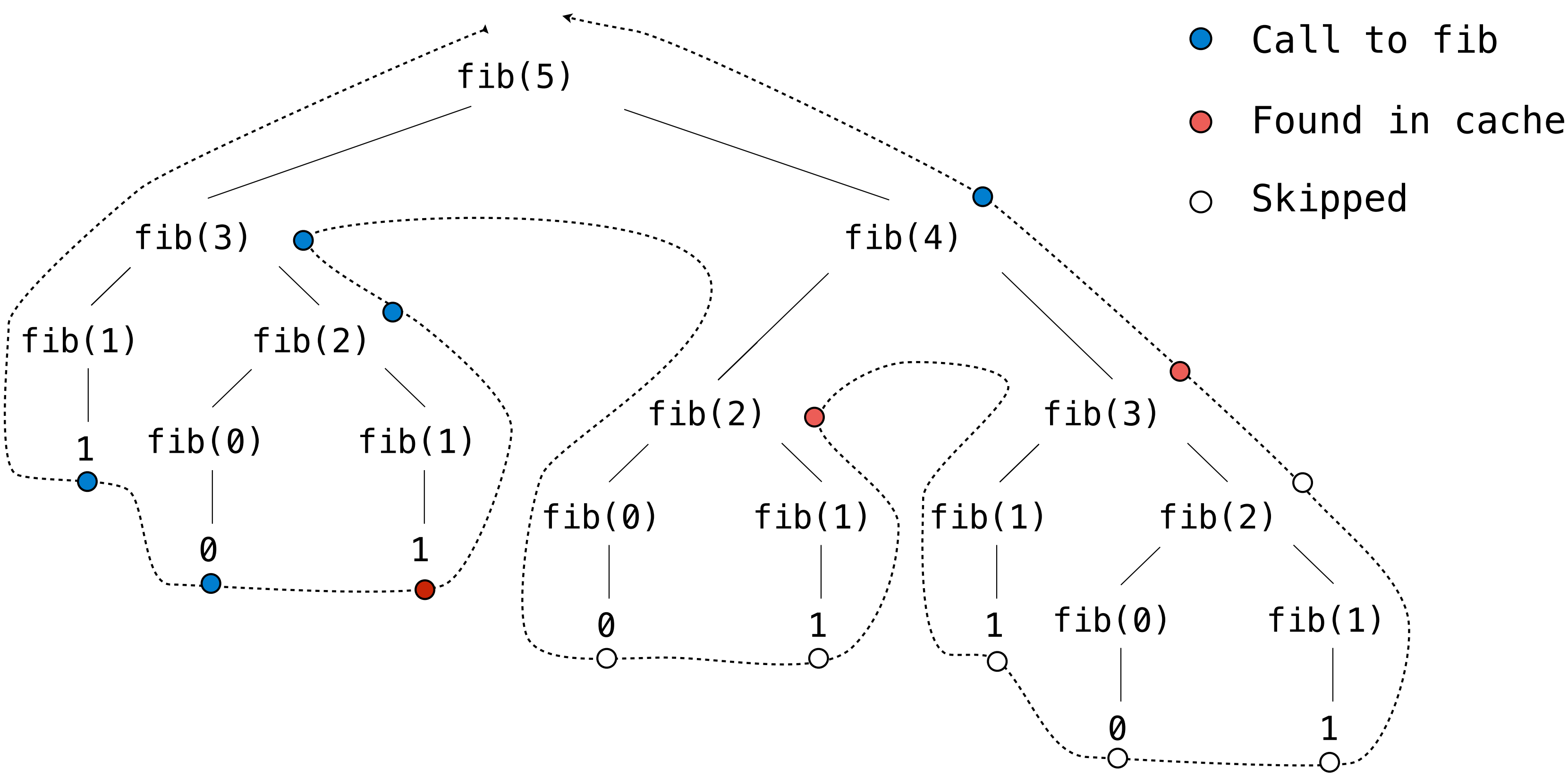

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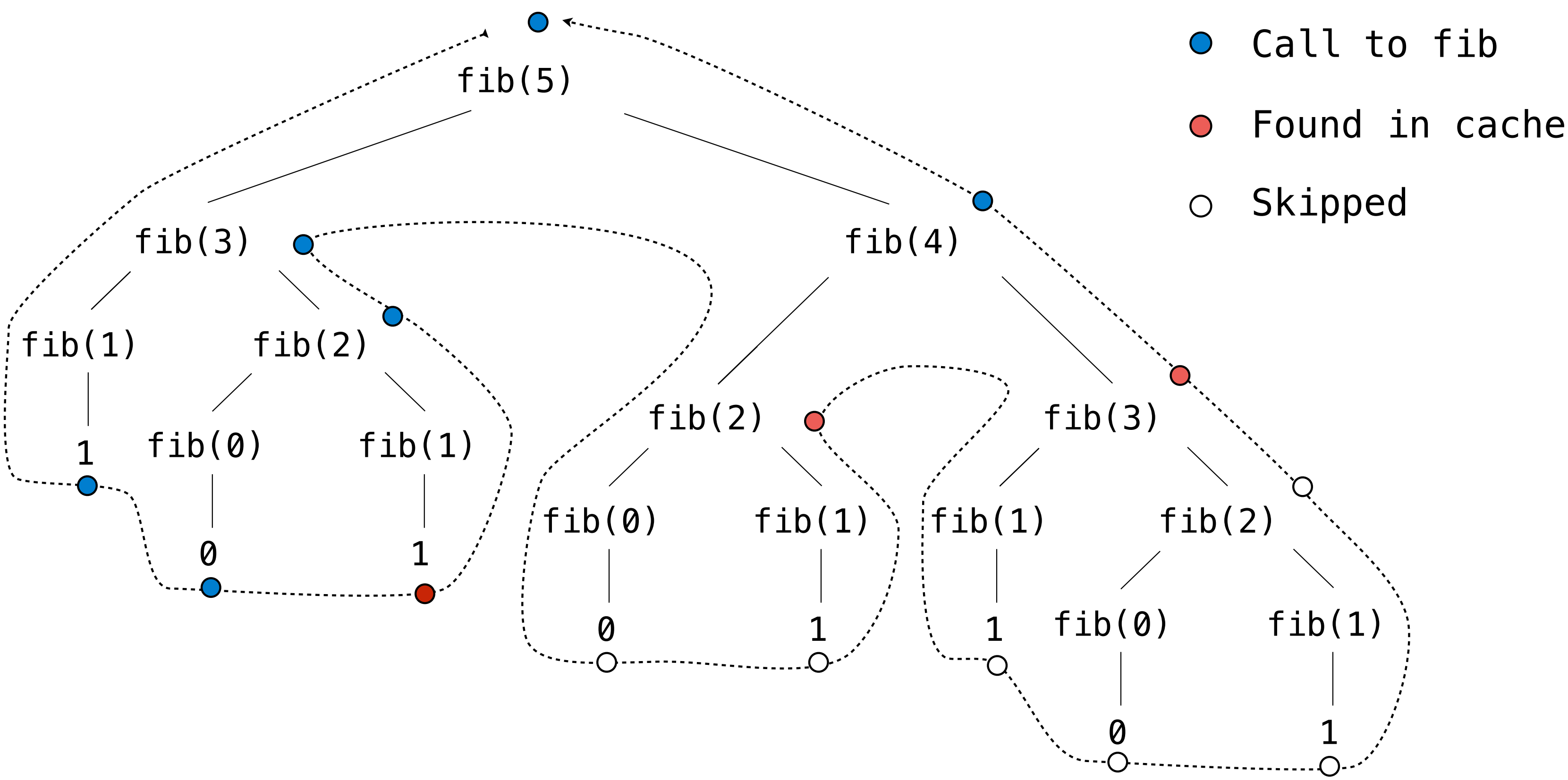

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Interactive Diagram

Fibonacci Space Consumption

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Fibonacci Space Consumption

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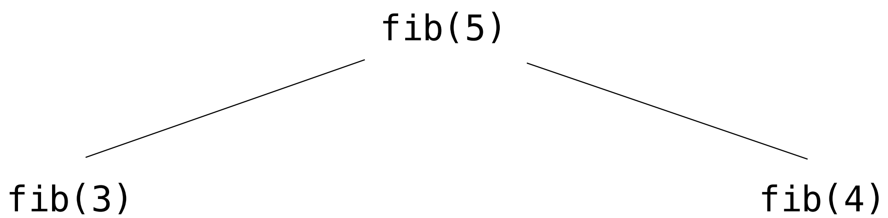

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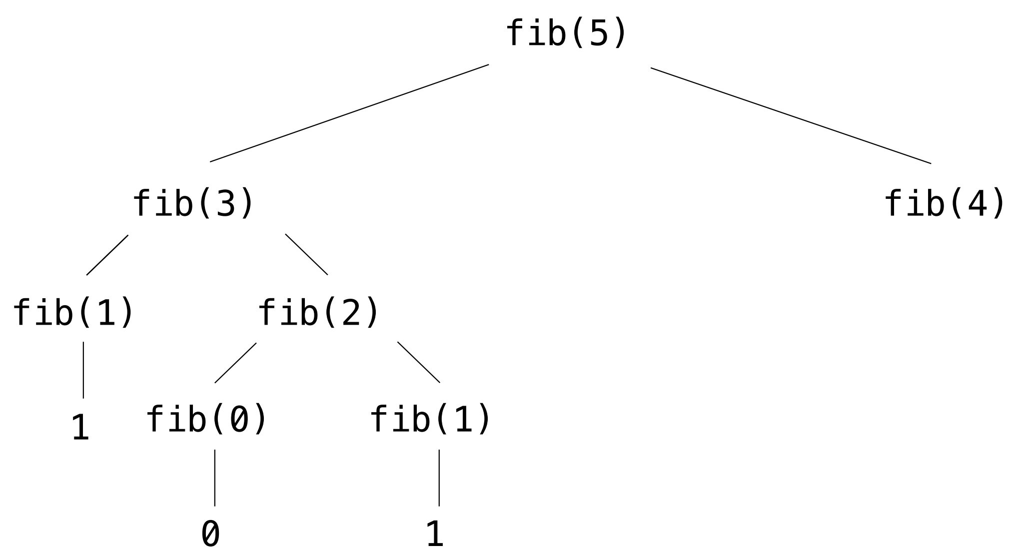
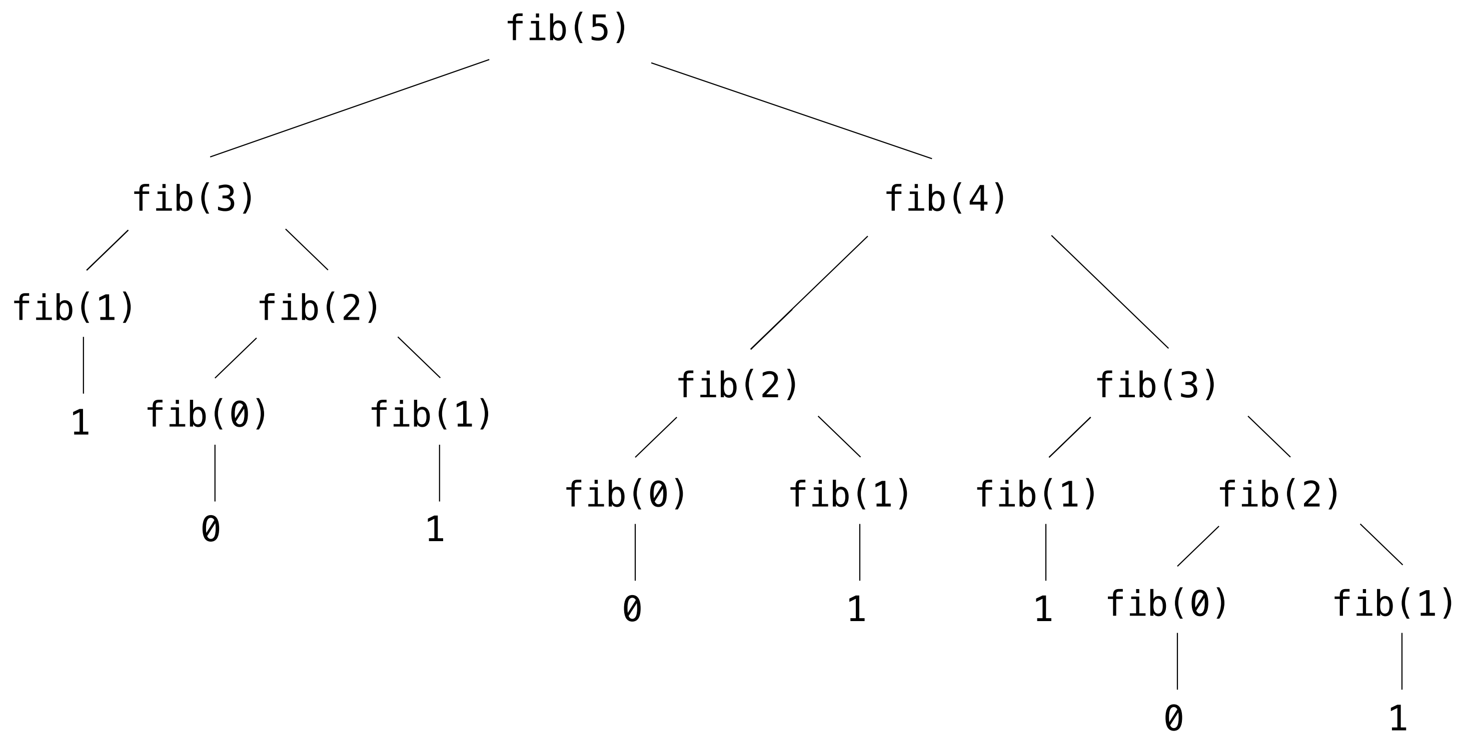


Fibonacci Space Consumption



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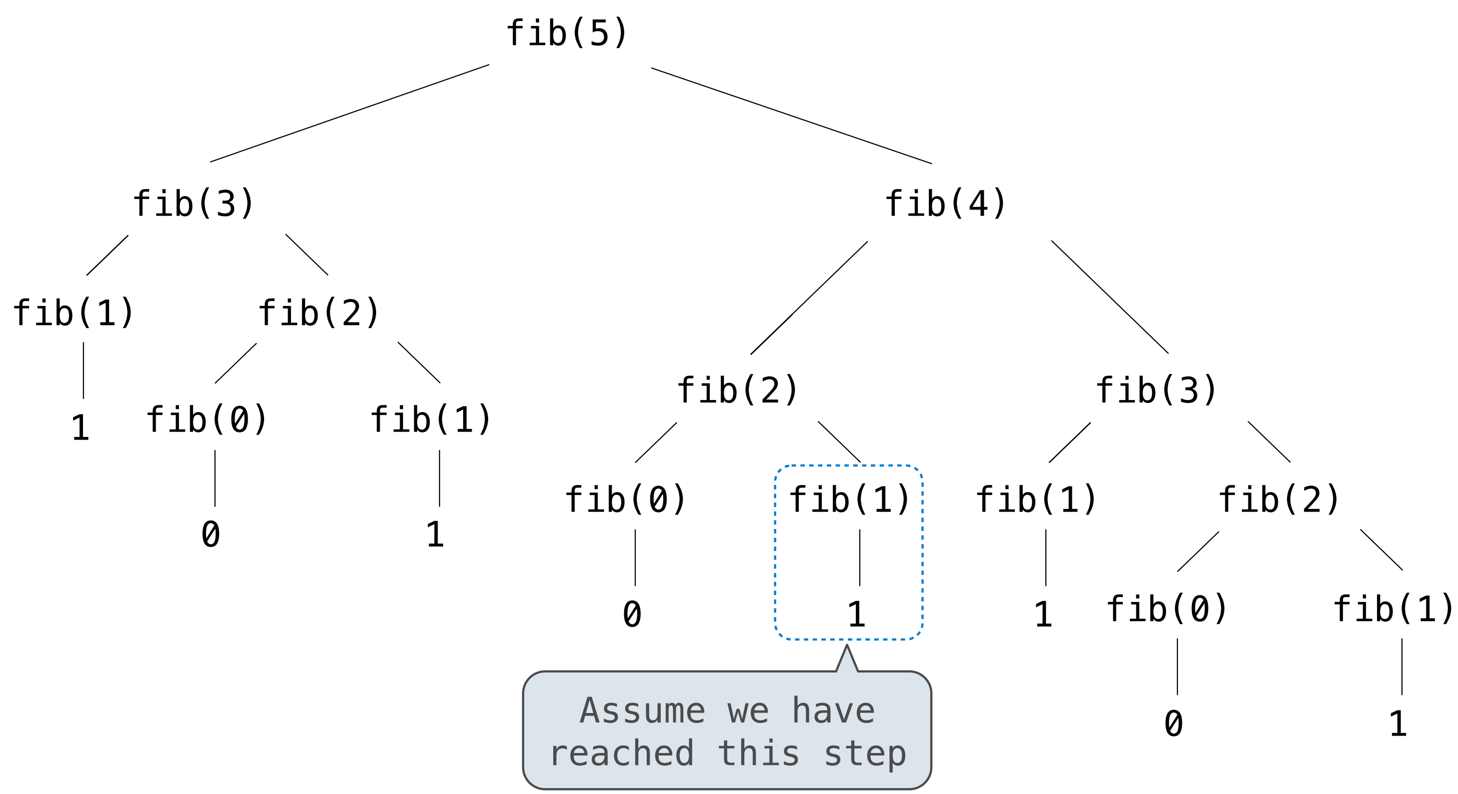

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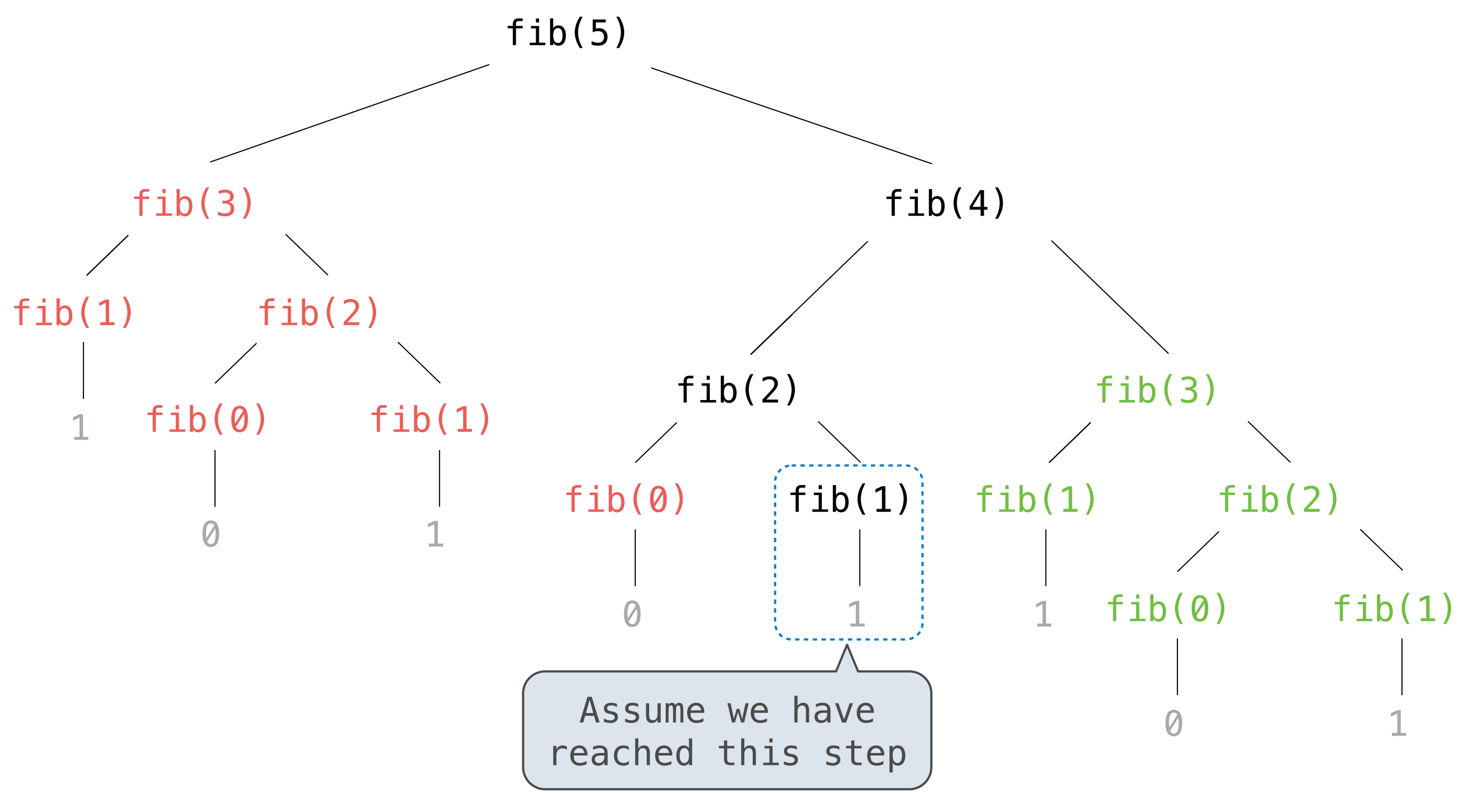

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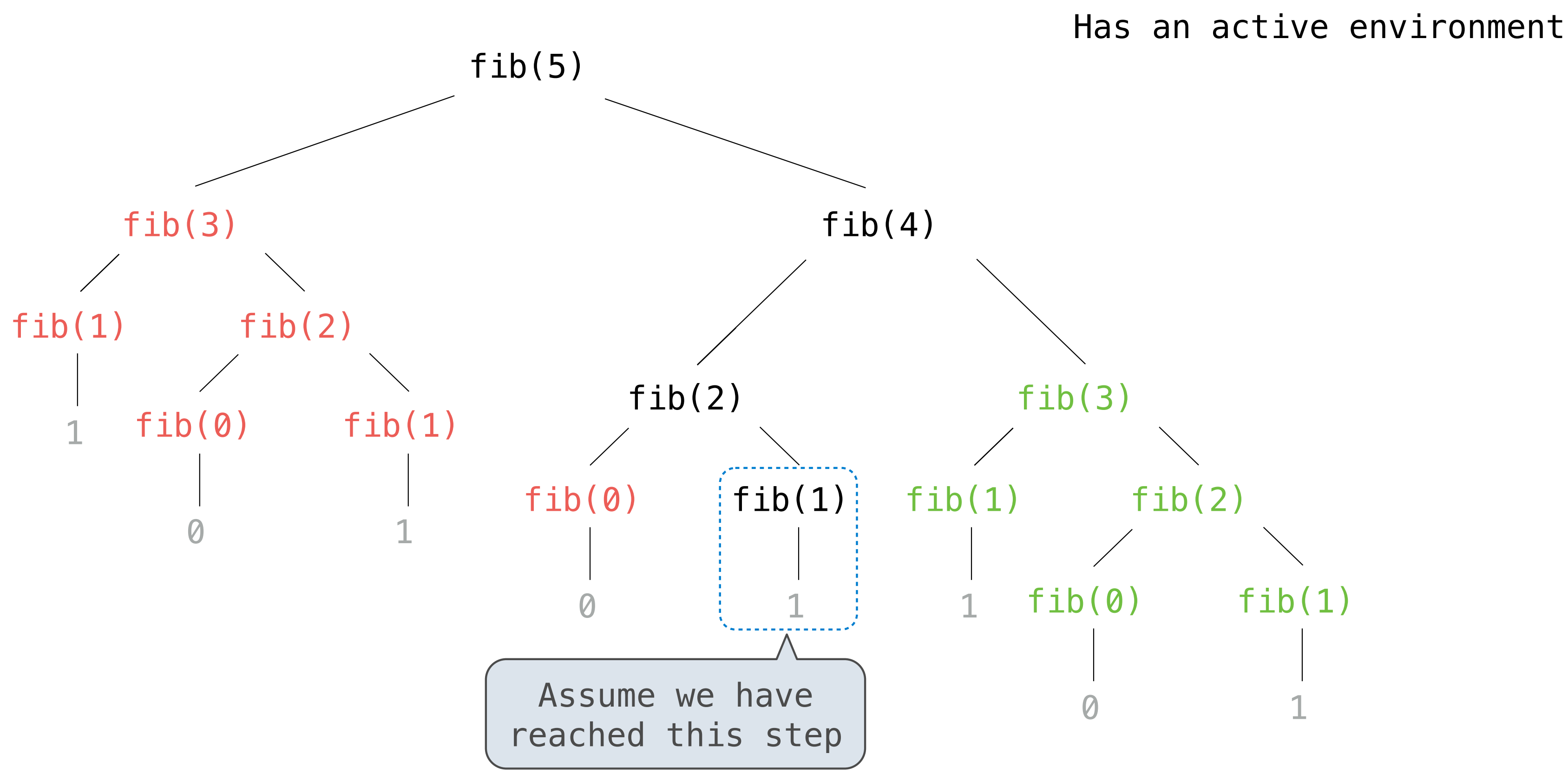

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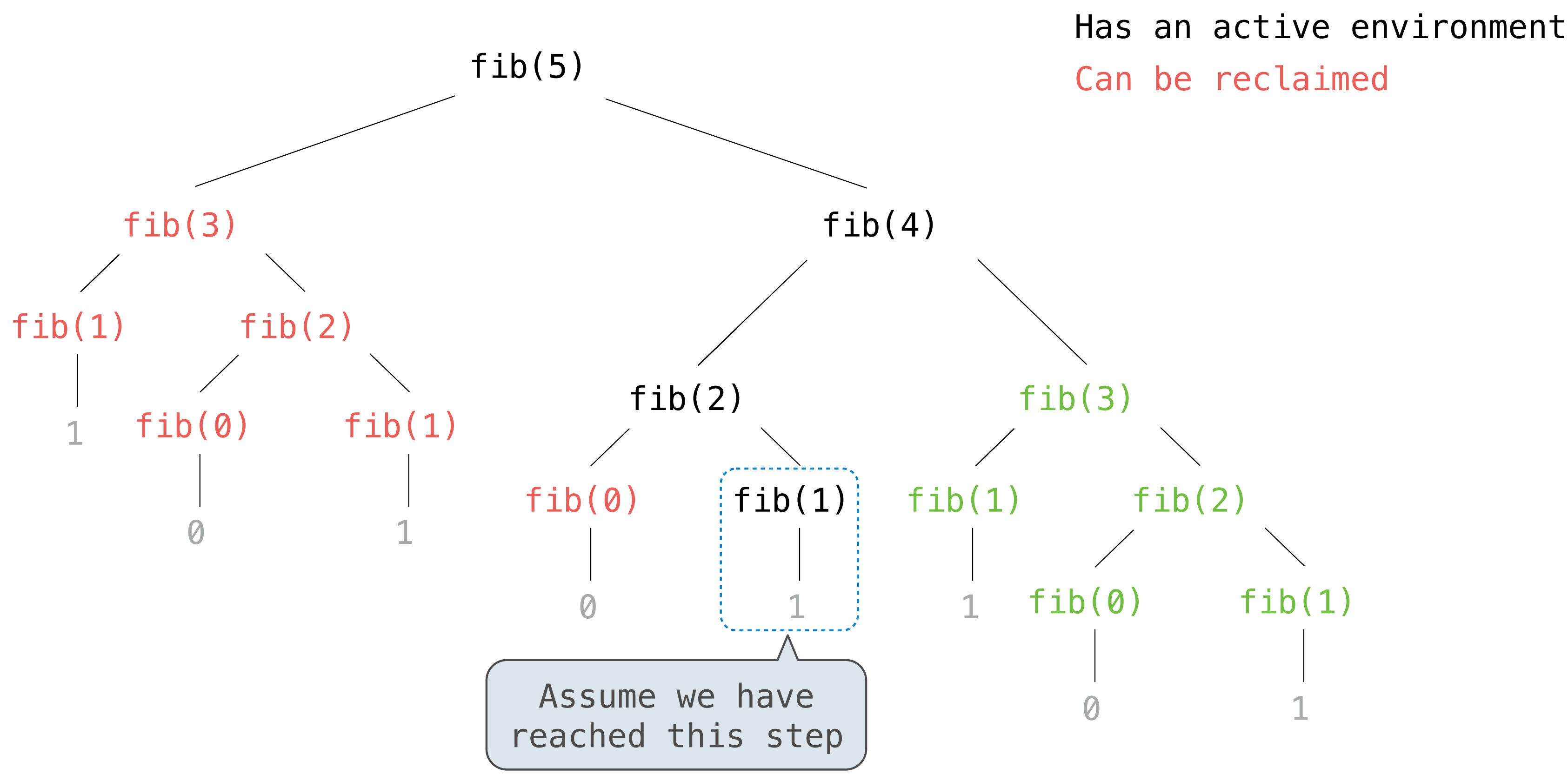

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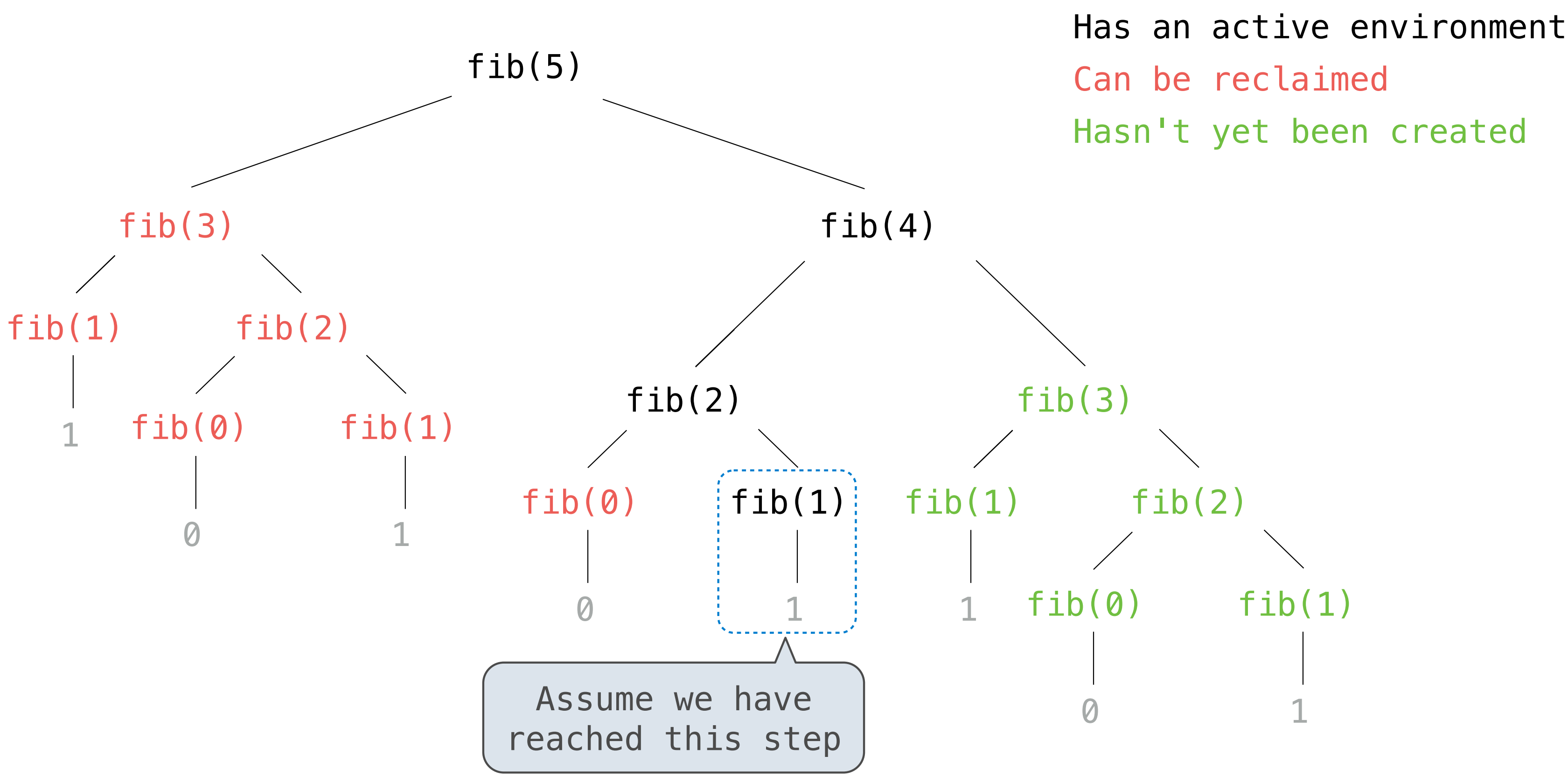

Time

Comparing Implementations

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Implementations of the same functional abstraction can require different resources

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Problem: How many factors does a positive integer n have?

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For every k, n/k is also a factor!

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Fast: Test each k from 1 to square root n For every k, n/k is also a factor!	Greatest integer less than $\sqrt{n}$

Question: How many time does each implementation use division? (Demo)

Orders of Growth

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A method for bounding the resources used by a function by the "size" of a problem

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Order of Growth of Counting Factors

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(Demo)

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Goal: one more multiplication lets us double the problem size

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def exp(b, n):
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```
def square(x):
    return x*x

def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)
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```

	Time	**Space**
`exp`	$\Theta(n)$	$\Theta(n)$
`exp_fast`	$\Theta(\log n)$	$\Theta(\log n)$

Comparing Orders of Growth

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Constants: Constant terms do not affect the order of growth of a process

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def overlap(a, b):
    count = 0
    for item in a:
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            count += 1
    return count
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```
def overlap(a, b):
    count = 0
    for item in a:          Outer: length of a
        if item in b:       Inner: length of b
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Outer: length of a

Inner: length of b

If a and b are both length **n**, then overlap takes $\Theta(n^2)$ steps

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Lower-order terms: The fastest-growing part of the computation dominates the total

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$$\Theta(n^2) \qquad \Theta(n^2 + n)$$

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Lower-order terms: The fastest-growing part of the computation dominates the total

$$\Theta(n^2) \qquad \Theta(n^2 + n) \qquad \Theta(n^2 + 500 \cdot n + \log_2 n + 1000)$$

Comparing orders of growth (n is the problem size)

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$$\Theta(b^n)$$

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$\Theta(b^n)$ `Exponential growth.  Recursive fib takes`

$\Theta(\phi^n)$ `steps, where` $\phi = \dfrac{1+\sqrt{5}}{2} \approx 1.61828$

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Incrementing n increases R(n) by the problem size n

$\Theta(n)$ Linear growth. E.g., slow `factors` or `exp`

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$\Theta(\phi^n)$ steps, where $\phi = \dfrac{1+\sqrt{5}}{2} \approx 1.61828$

Incrementing the problem scales R(n) by a factor

$\Theta(n^2)$ Quadratic growth. E.g., overlap

Incrementing n increases R(n) by the problem size n

$\Theta(n)$ Linear growth. E.g., slow factors or exp

$\Theta(\sqrt{n})$ Square root growth. E.g., factors_fast

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