Ordered Sets

Announcements

Sets

Sets

One more built-in Python container type

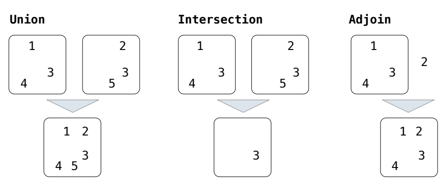
- Set literals are enclosed in braces
- Duplicate elements are removed on construction
- Sets have arbitrary order, just like dictionary entries

```
>>> s = {'one', 'two', 'three', 'four', 'four'}
>>> s
{'three', 'one', 'four', 'two'}
>>> 'three' in s
True
>>> len(s)
4
>>> s.union({'one', 'five'})
{'three', 'five', 'one', 'four', 'two'}
>>> s.intersection({'six', 'five', 'four', 'three'})
{'three', 'four'}
>>> s
{'three', 'one', 'four', 'two'}
```

Implementing Sets

What we should be able to do with a set:

- **Membership testing:** Is a value an element of a set?
- **Union:** Return a set with all elements in set1 or set2
- **Intersection:** Return a set with any elements in set1 and set2
- **Adjoin:** Return a set with all elements in s and a value v



Sets as Linked Lists

Sets as Unordered Sequences

Proposal 1: A set is represented by a linked list that contains no duplicate items.

Time order of growth

```
def empty(s):
    return s is Link.empty
```

$\Theta(1)$

```
def contains(s, v):
    """Return whether set s contains value v.

    >>> s = Link(1, Link(3, Link(2)))
    >>> contains(s, 2)
    True
    """
```

Time depends on whether & where v appears in s.

$\Theta(n)$

In the worst case: v does not appear in s

or

In the average case: appears in a uniformly distributed random location

(Demo)

Sets as Unordered Sequences

Time order of worst-case growth

```
def adjoin(s, v):
    if contains(s, v):
        return s
    else:
        return Link(v, s)
```

$\Theta(n)$

The size of the set

```
def intersect(s, t):
    if s is Link.empty:
        return Link.empty
    rest = intersect(s.rest, t)
    if contains(t, s.first):
        return Link(s.first, rest)
    else:
        return rest
```

$\Theta(n^2)$

If sets are the same size

Sets as Ordered Linked Lists

Sets as Ordered Sequences

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

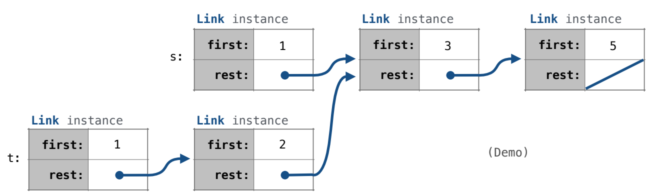
Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	empty, contains, adjoin, intersect, union
Implement set operations	Ordered linked lists	first, rest, <, >, ==

Different parts of a program may make different assumptions about data

Searching an Ordered List

```
>>> s = Link(1, Link(3, Link(5)))
>>> contains(s, 1)
True
>>> contains(s, 2)
False
>>> t = adjoin(s, 2)
```

Operation	Time order of growth
contains	$\Theta(n)$
adjoin	$\Theta(n)$



(Demo)

Set Operations

Intersecting Ordered Linked Lists

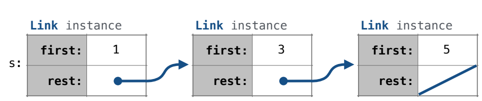
Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(s, t):
    if empty(s) or empty(t):
        return Link.empty
    else:
        e1, e2 = s.first, t.first
        if e1 == e2:
            return Link(e1, intersect(s.rest, t.rest))
        elif e1 < e2:
            return intersect(s.rest, t)
        elif e2 < e1:
            return intersect(s, t.rest)
```

Order of growth? If s and t are sets of size n, then $\Theta(n)$ (Demo)

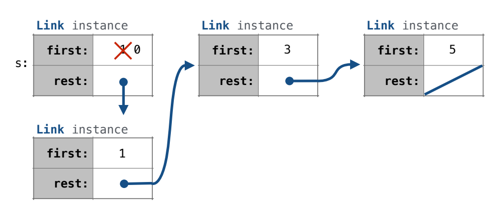
Set Mutation

Adding to an Ordered List



add(s, 0) *Try to return the same object as input*

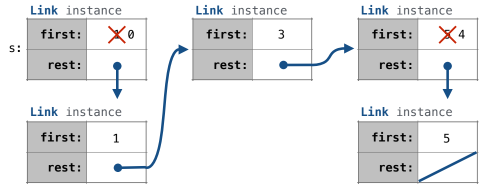
Adding to an Ordered List



add(s, 3)

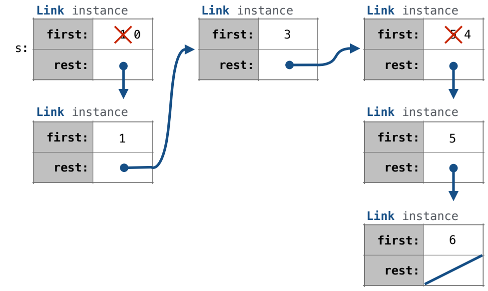
add(s, 4)

Adding to an Ordered List



`add(s, 6)`

Adding to an Ordered List



Adding to a Set Represented as an Ordered List

```
def add(s, v):
    """Add v to a set s, returning modified s."""

    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))
    """
    if empty(s): return Link(v)
    if s.first > v:
        s.first, s.rest = ______v______ , ______Link(s.first, s.rest)______
    elif s.first < v and empty(s.rest):
        s.rest = ______Link(v, s.rest)______
    elif s.first < v:
        ______add(s.rest, v)______
    return s
```