61A Extra Lecture 1

Announcements

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- If you want 1 unit (pass/no pass) of credit for in CS 98-52, the CCN is 34591.

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- All info and materials will be posted to cs61a.org/extra.html

Newton's Method

Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!

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$f(x) = x^2 - 2$

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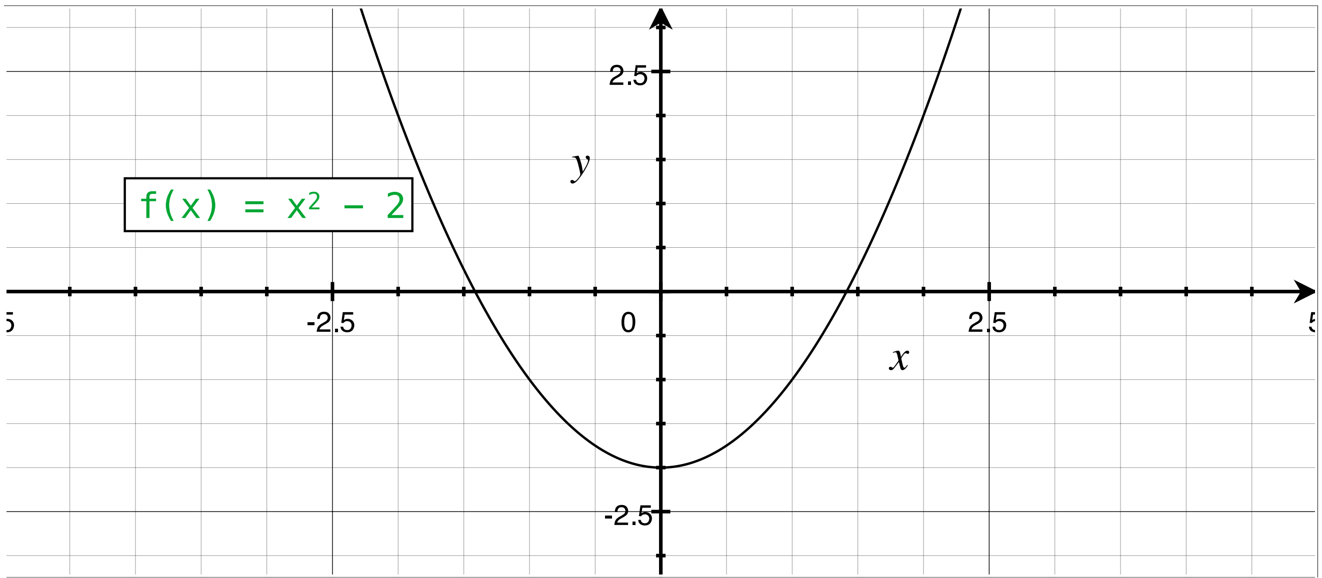

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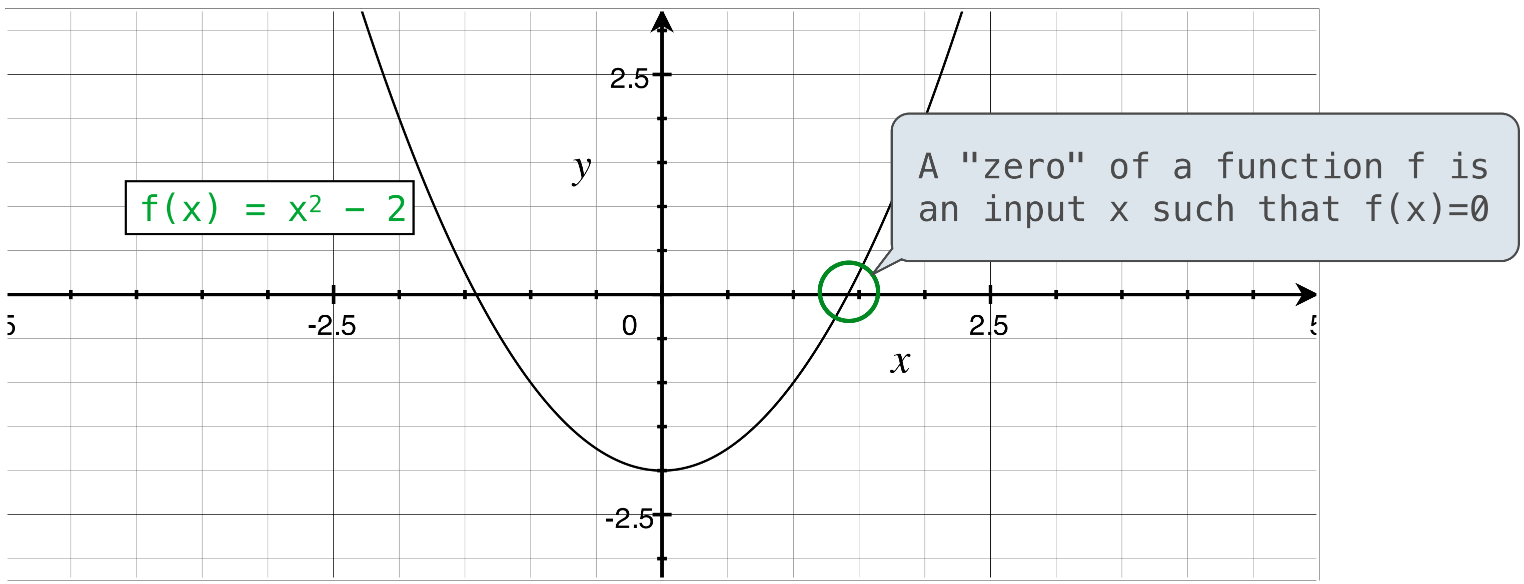

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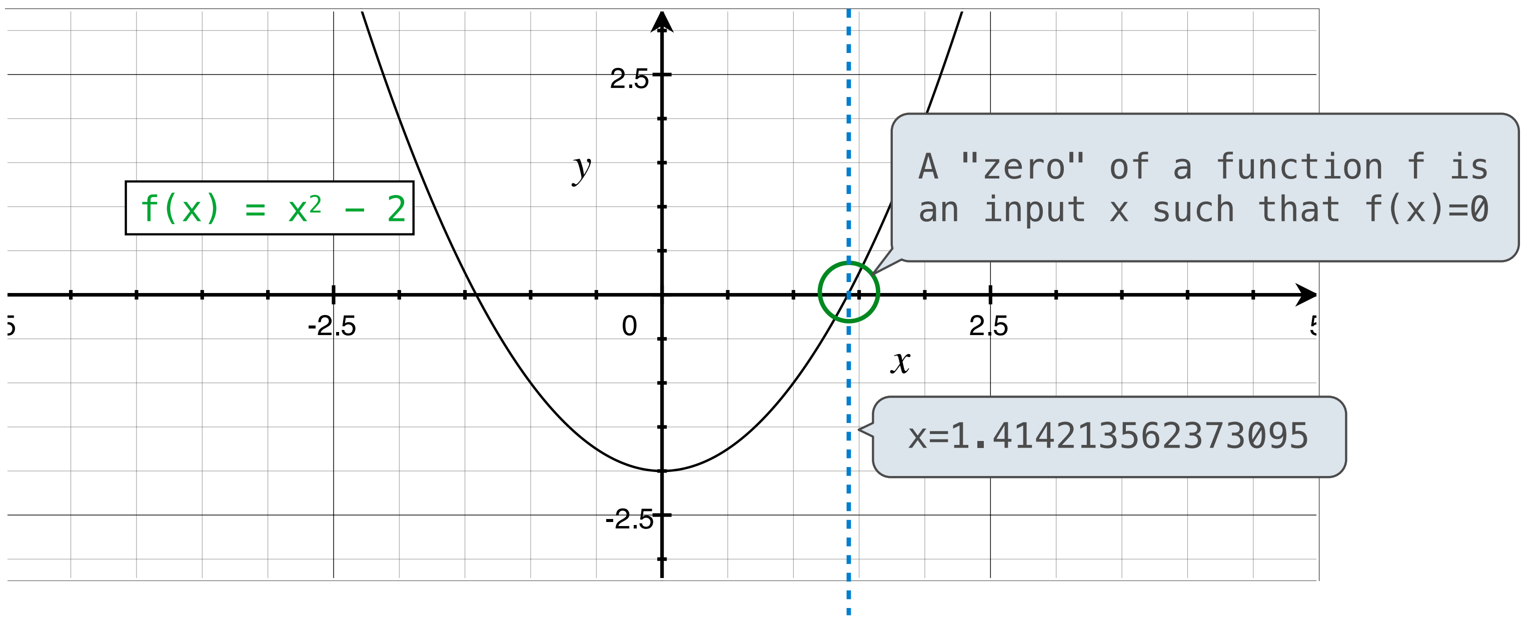

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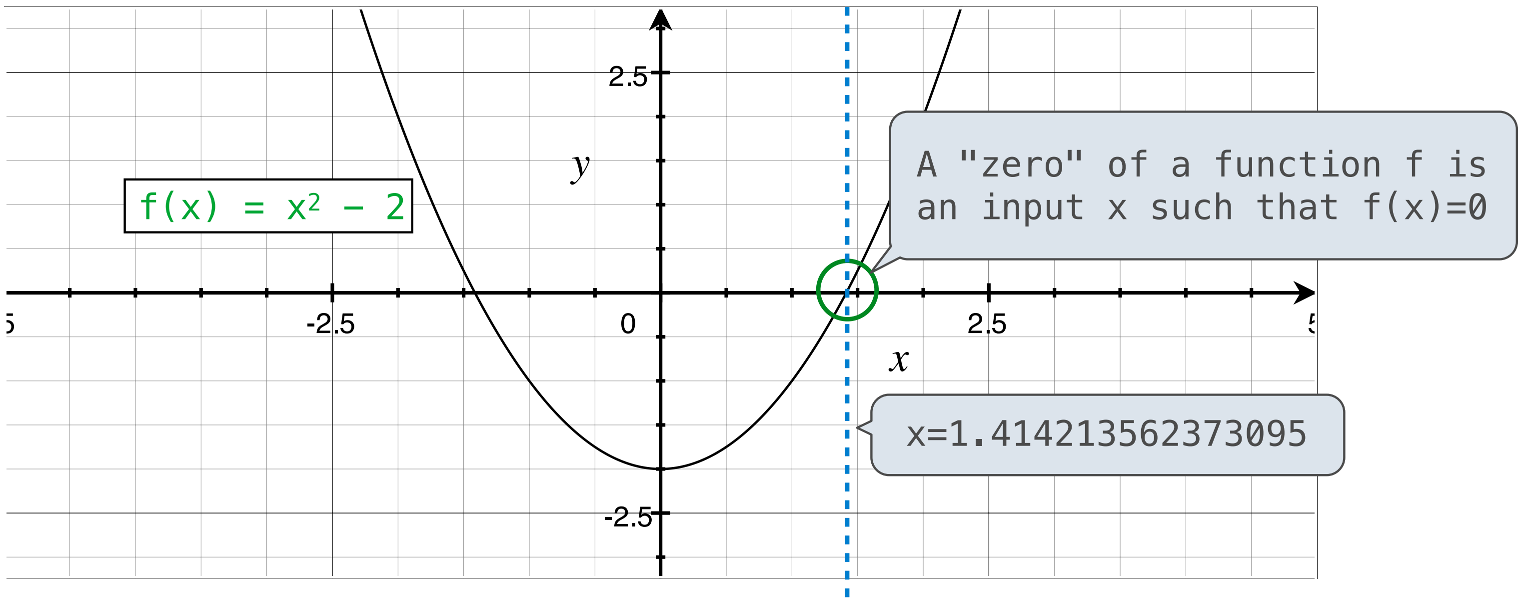

Application: a method for computing square roots, cube roots, etc.

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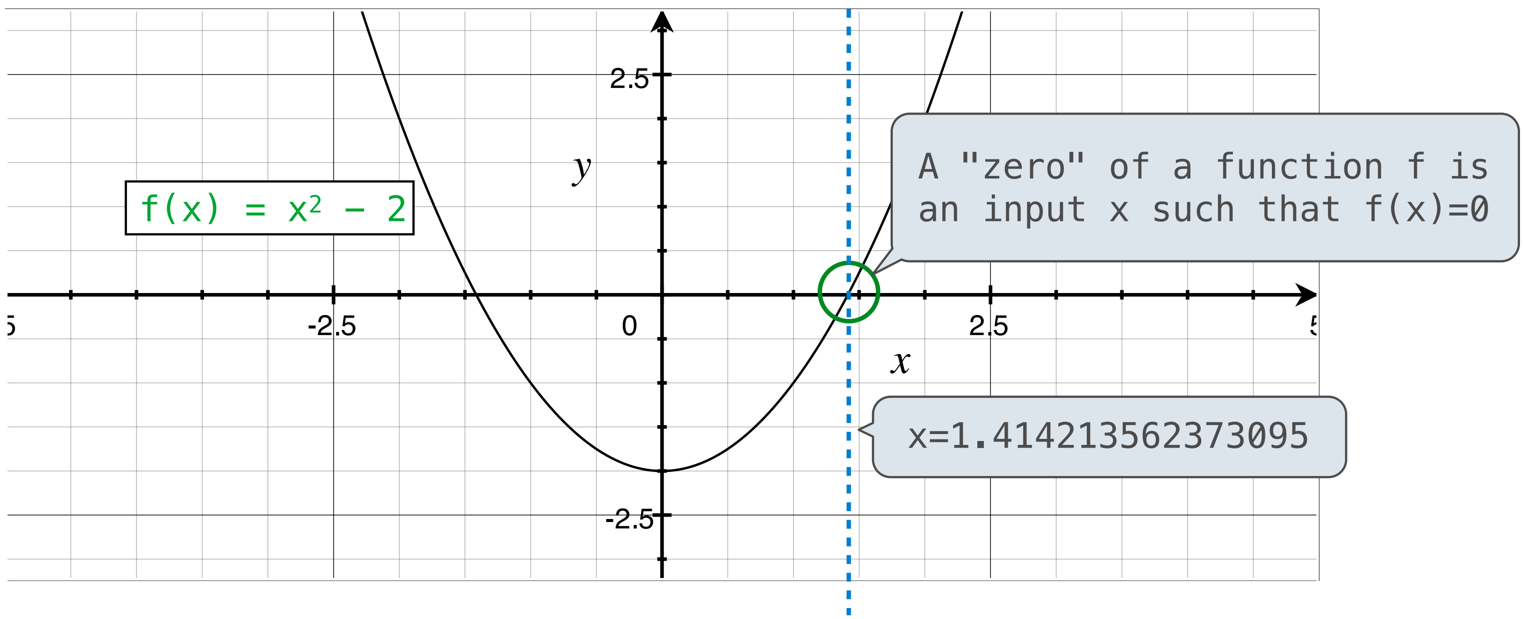

Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)

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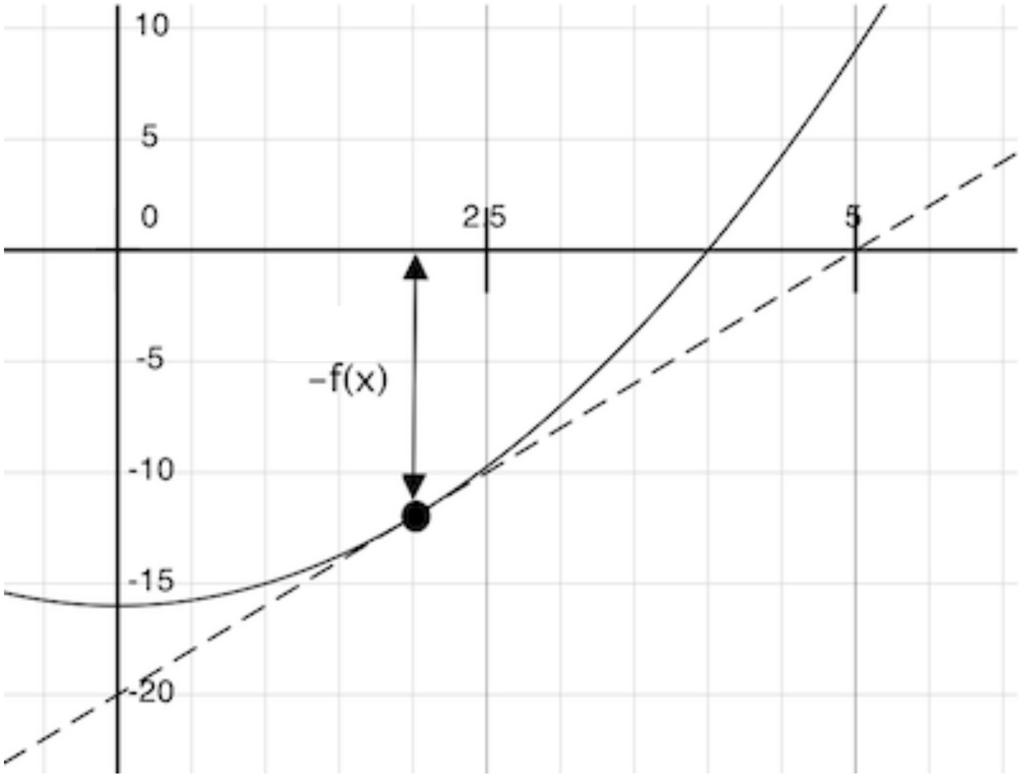

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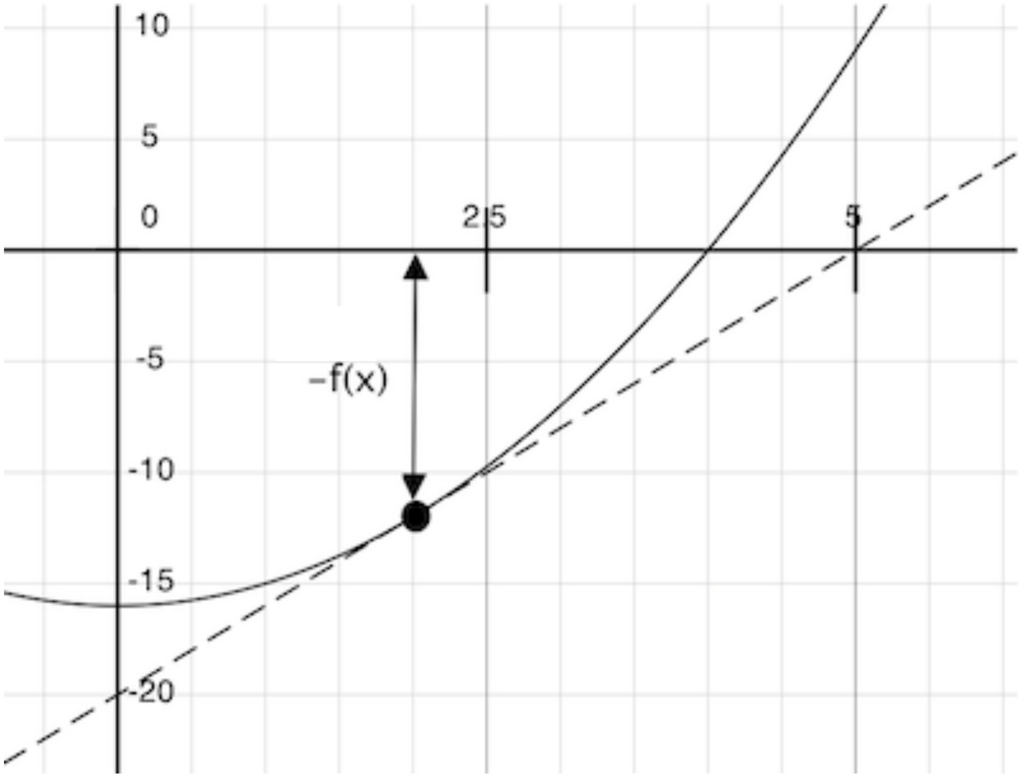

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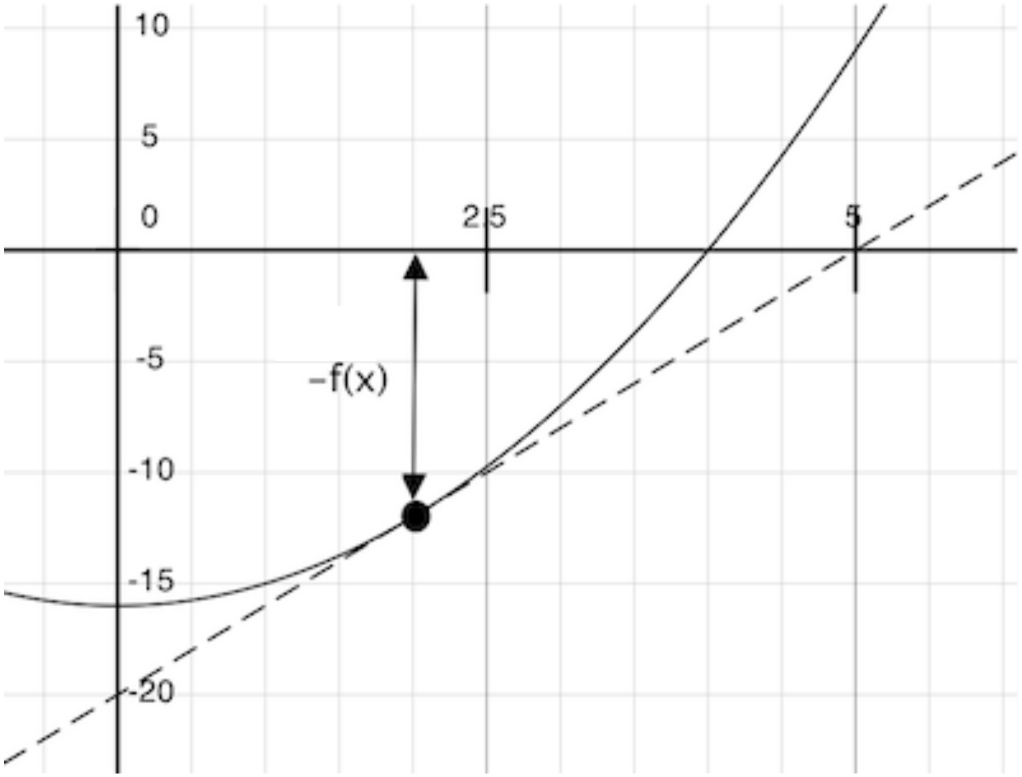

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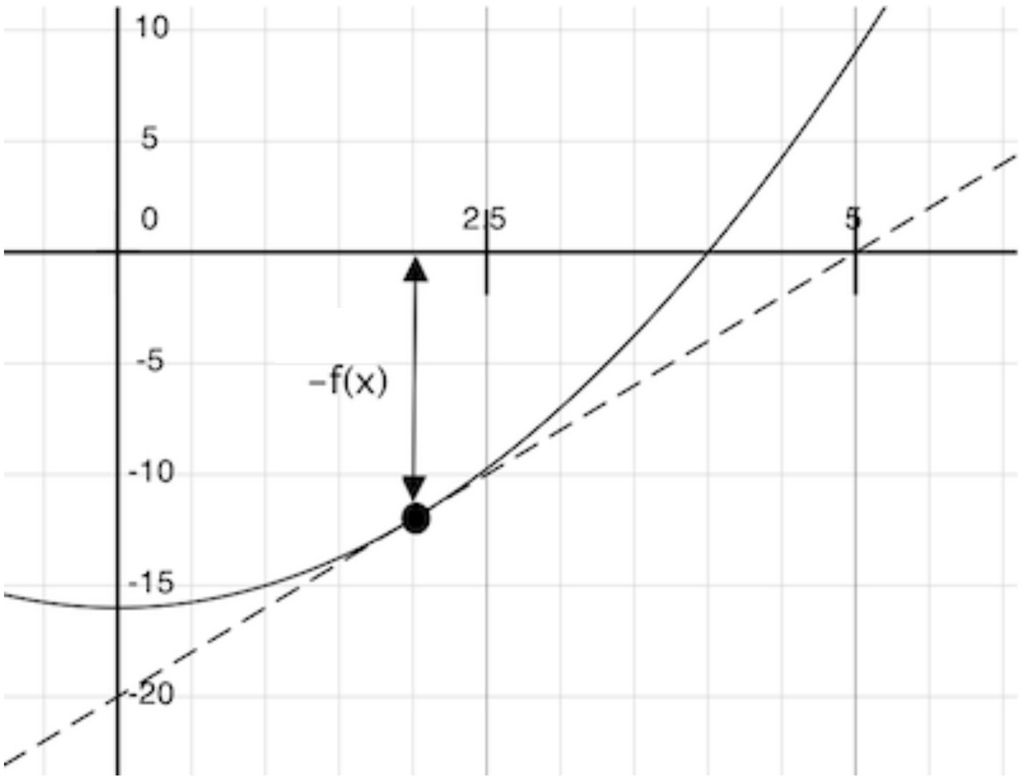
$$x - \frac{f(x)}{f'(x)}$$



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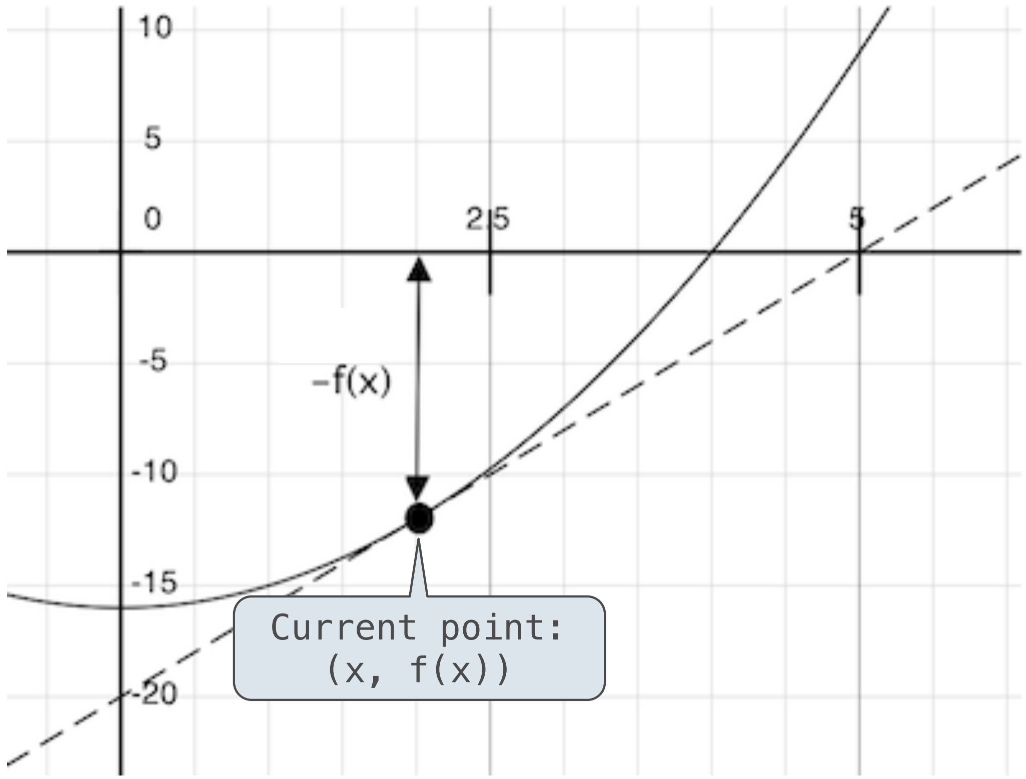

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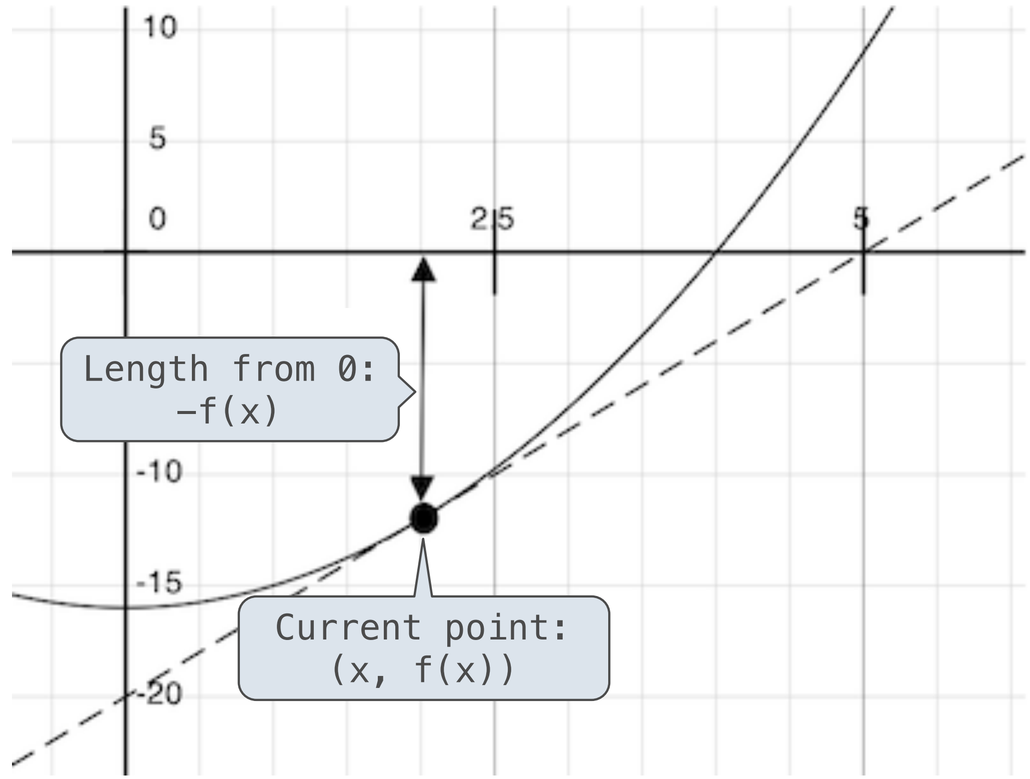

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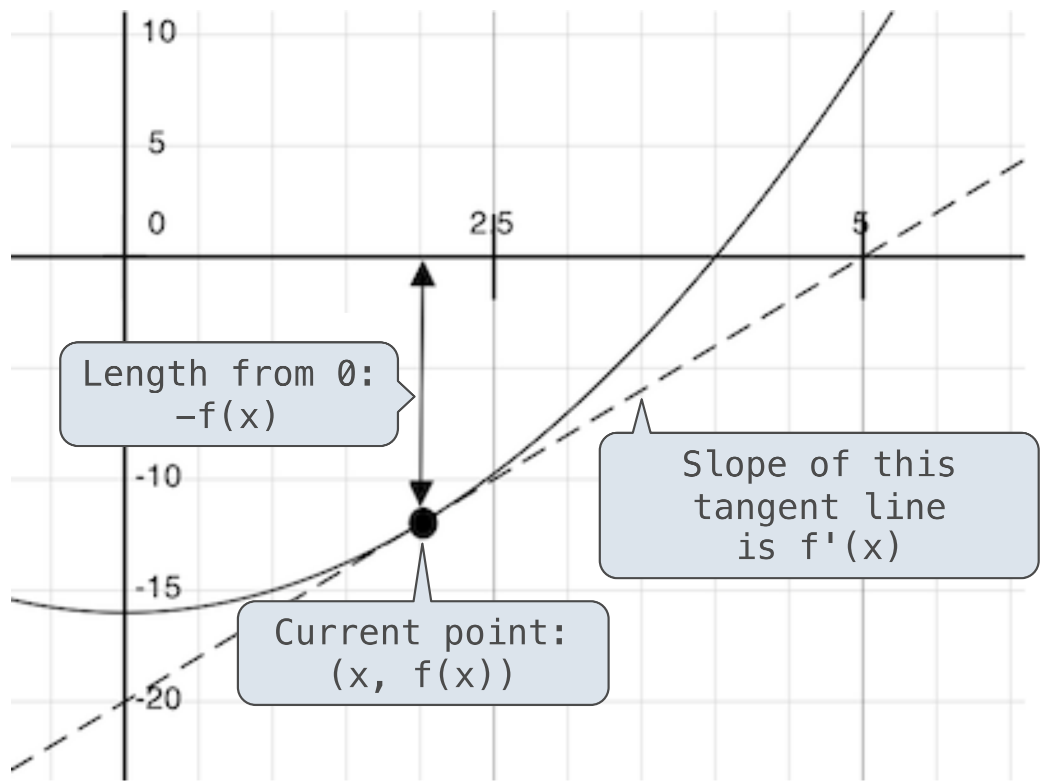

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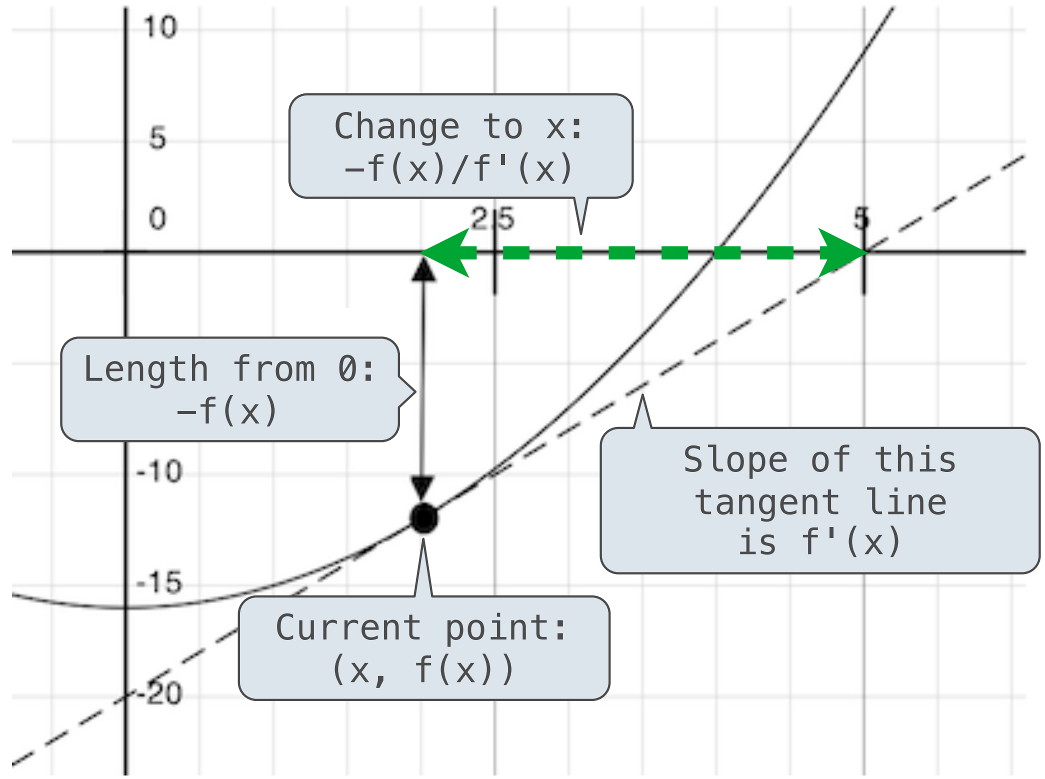

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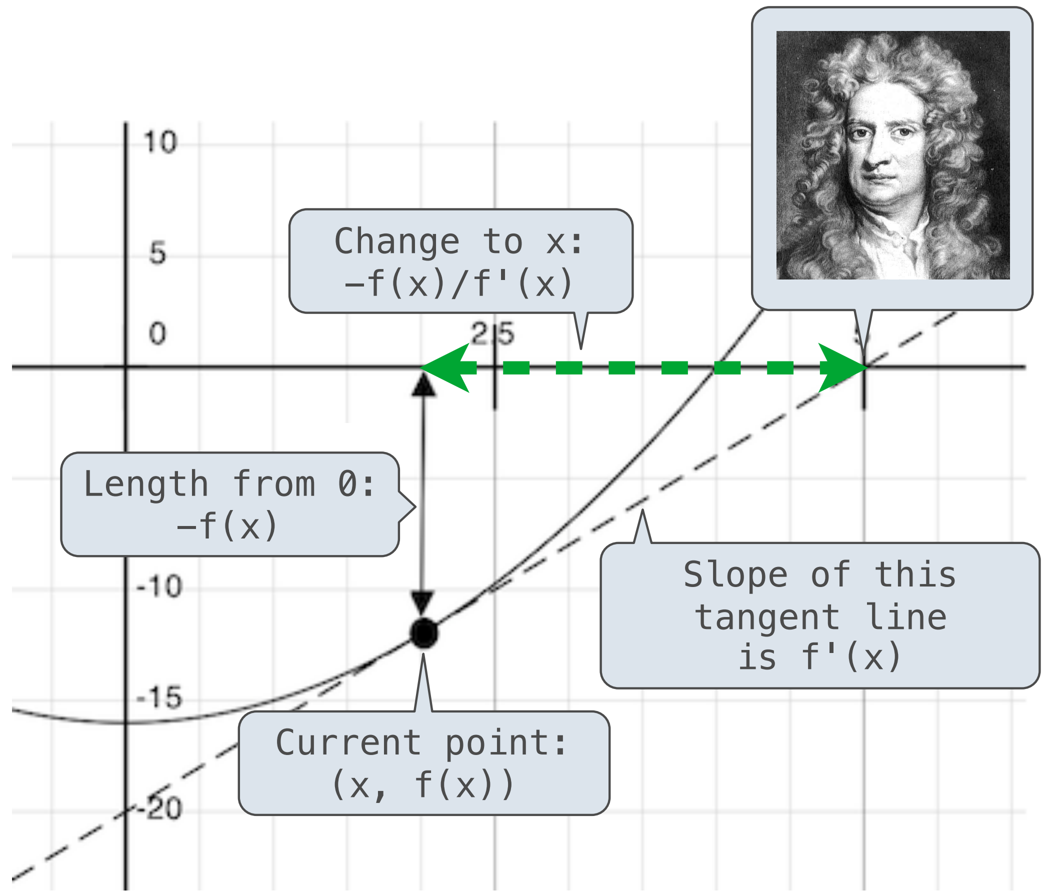

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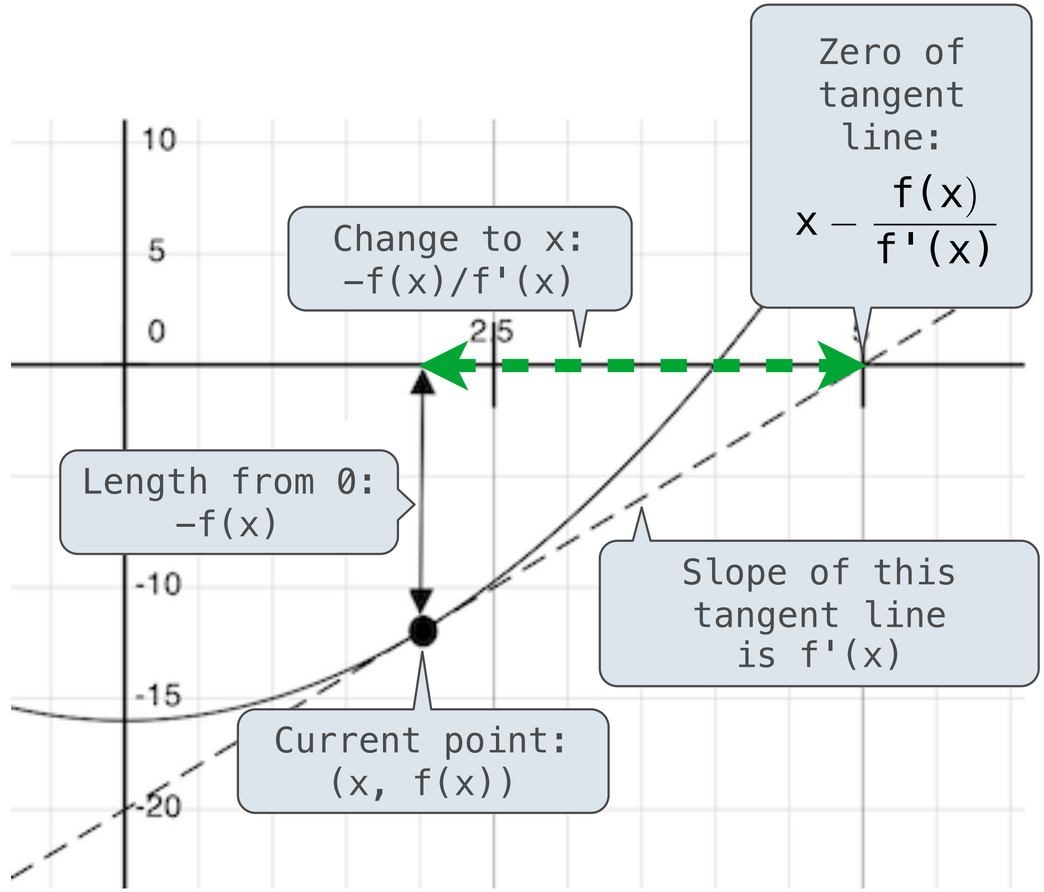

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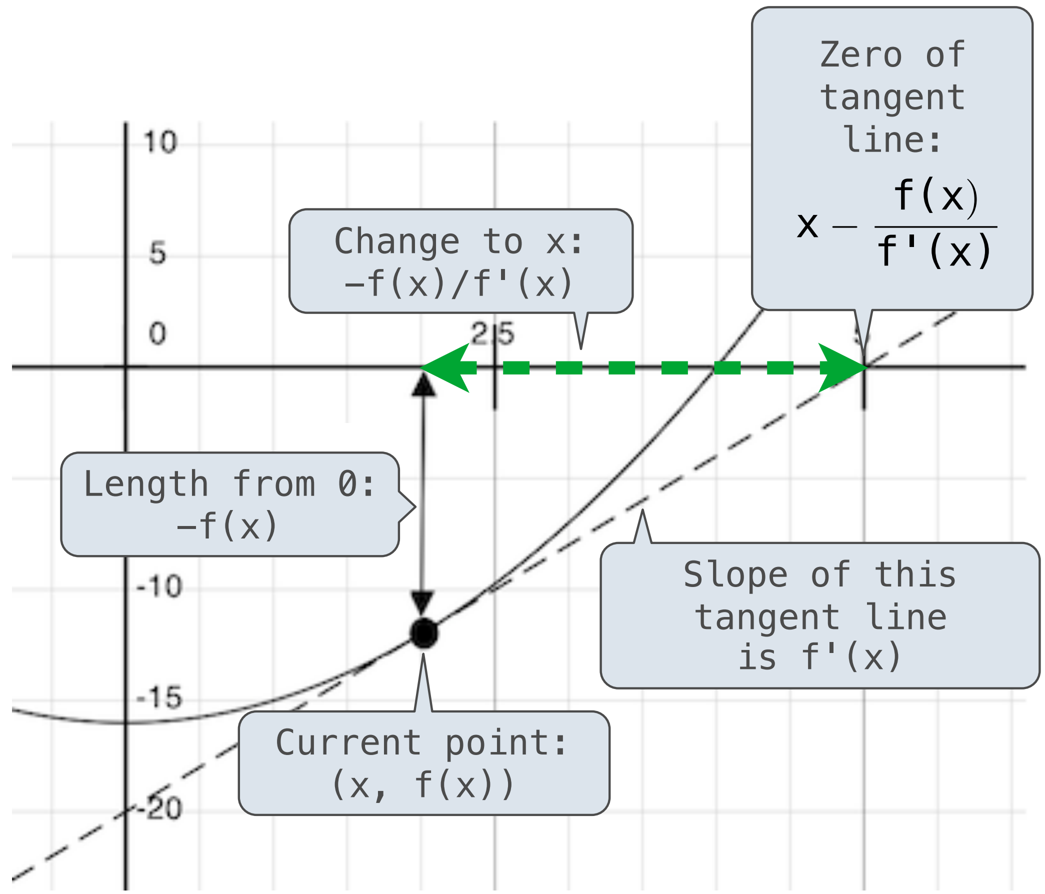

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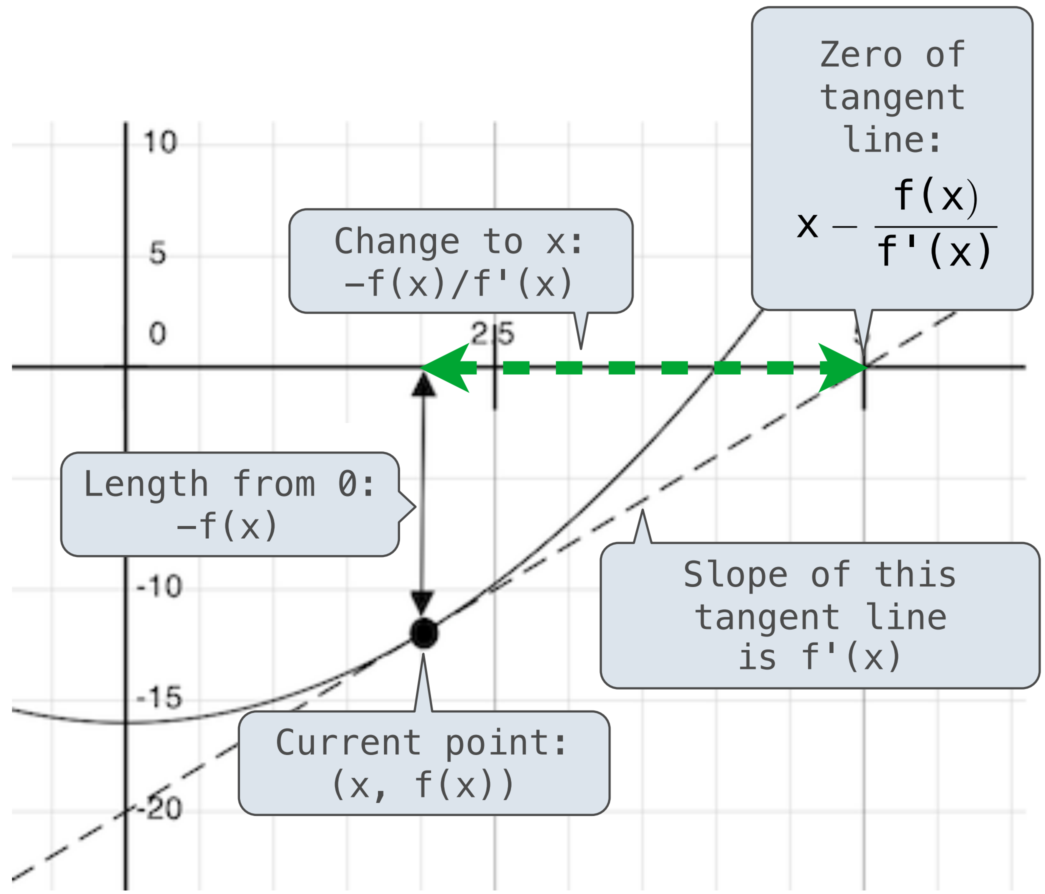

http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

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How to find the square root of 2?
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>>> f  = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
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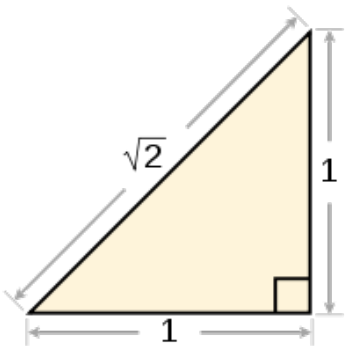
Using Newton's Method

How to find the square root of 2?

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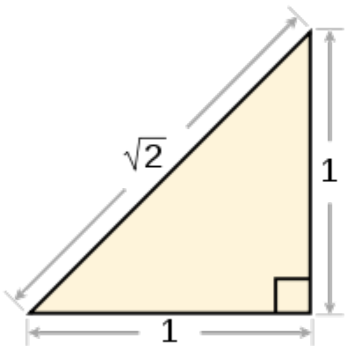


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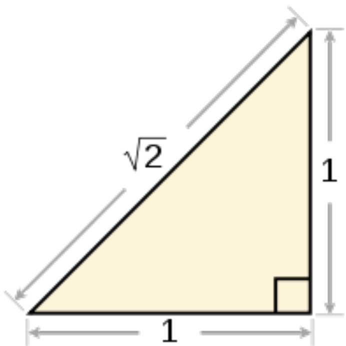


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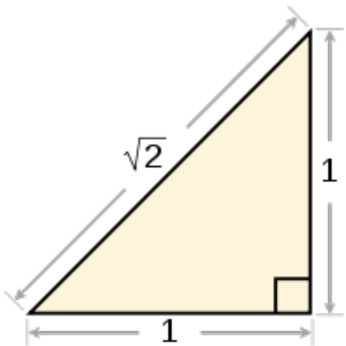


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How to find the cube root of 729?

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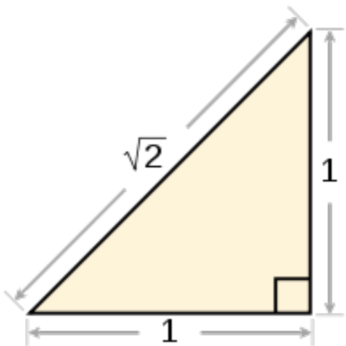


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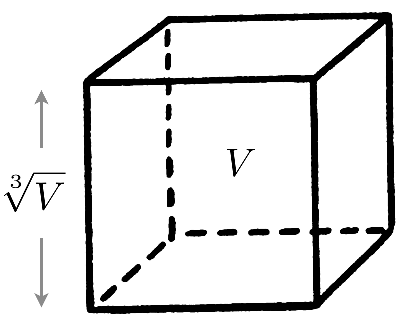

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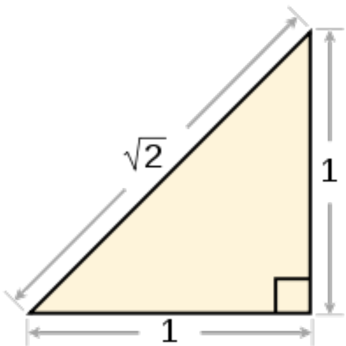


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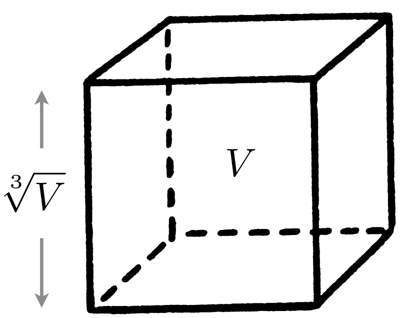


```
>>> g  = lambda x: x*x*x - 729
>>> dg = lambda x: 3*x*x
>>> find_zero(g, dg)
9.0
```

Using Newton's Method

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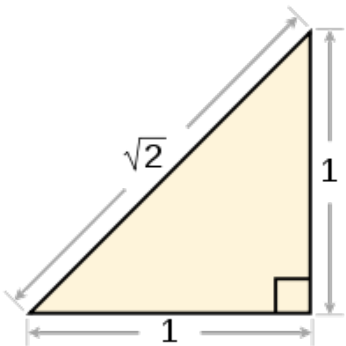


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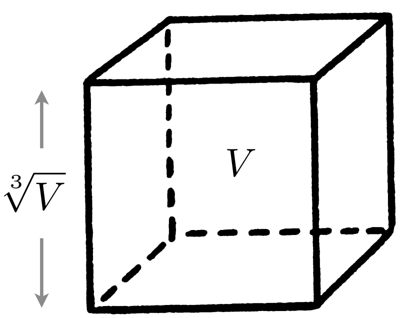


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g(x) = x³ - 729
g'(x) = 3x²

Iterative Improvement

Special Case: Square Roots

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How to compute square_root(a)

Idea: Iteratively refine a guess x about the square root of a

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(Demo)

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Implementation questions:

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How do we know when we are finished?

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(Demo)

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Implementing Newton's Method

(Demo)

Extensions

Approximate Differentiation

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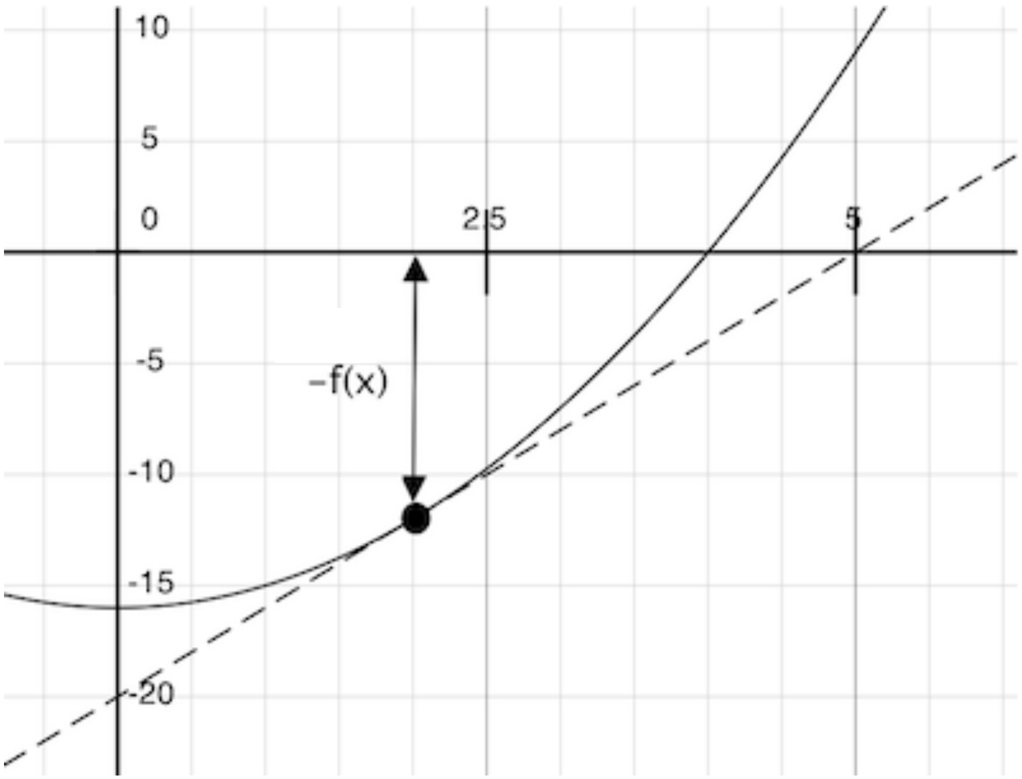

Approximate Differentiation

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Differentiation can be performed
symbolically or numerically
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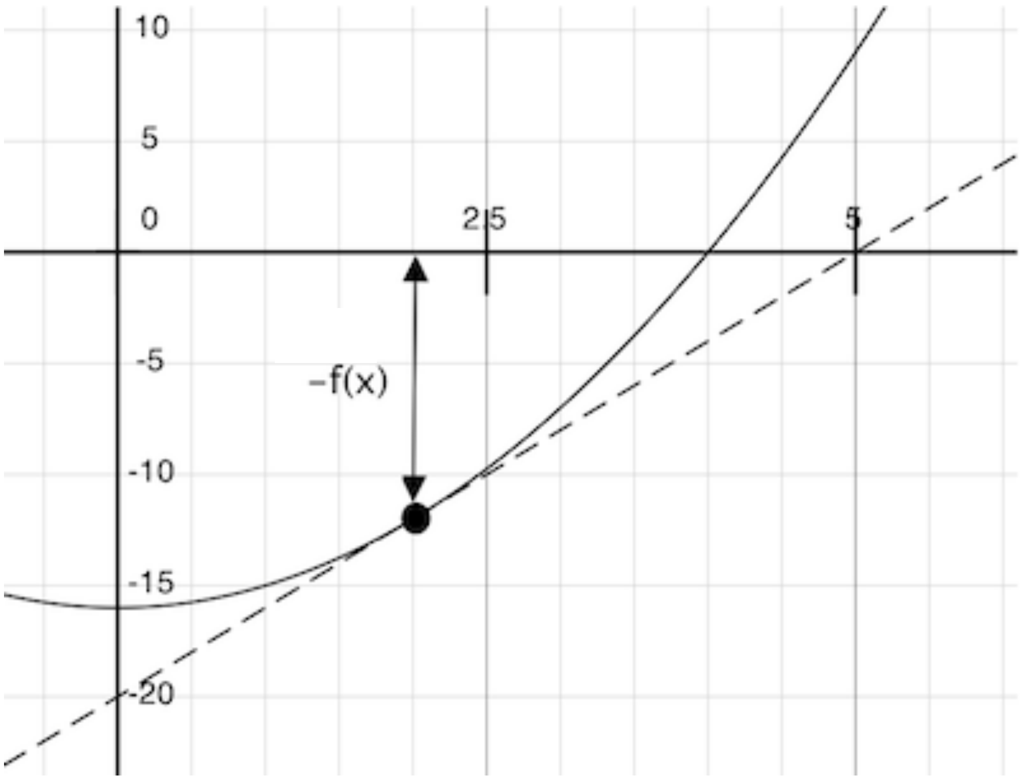
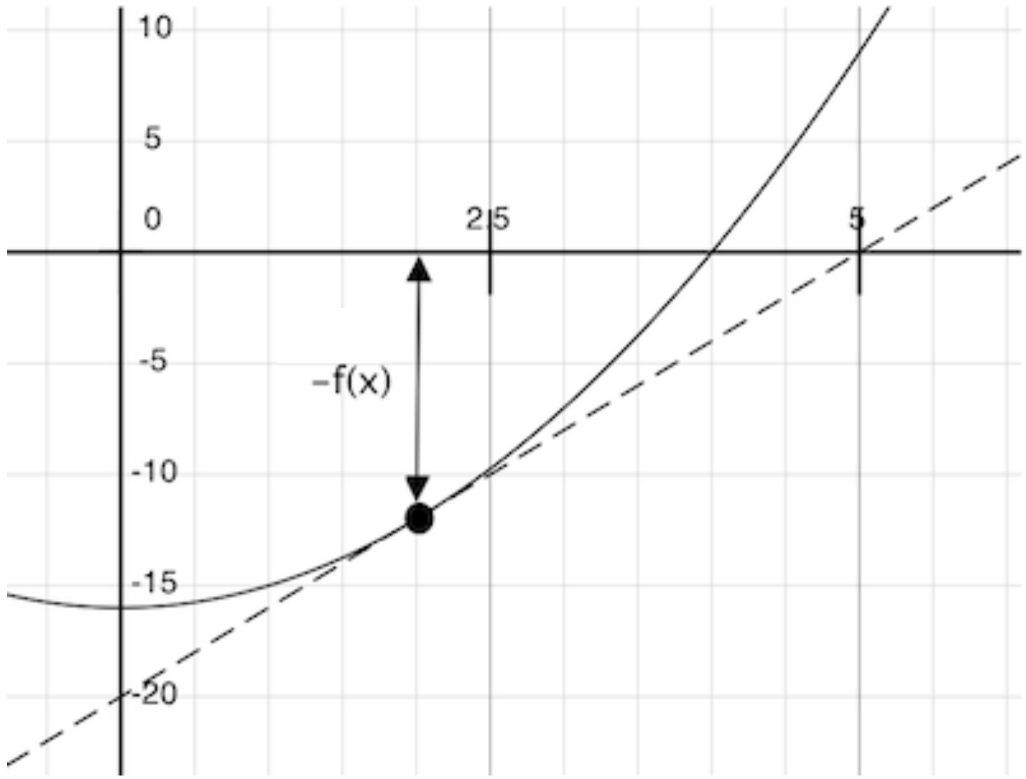


Approximate Differentiation

Differentiation can be performed symbolically or numerically

$f(x) = x^2 - 16$



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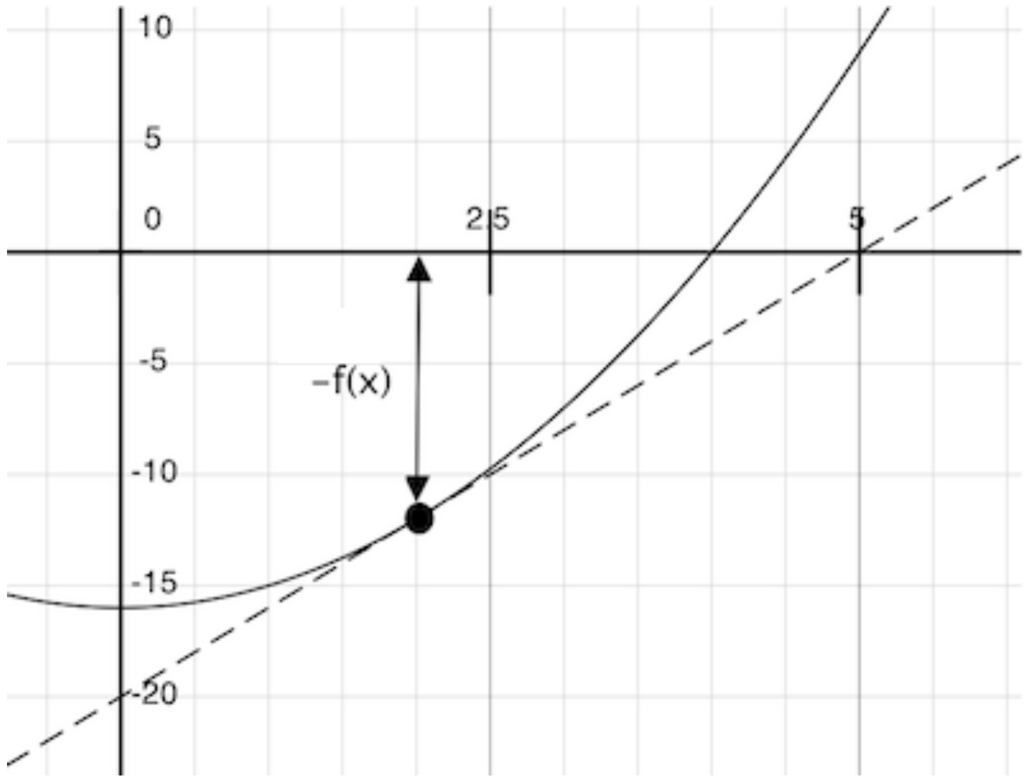

Approximate Differentiation

Differentiation can be performed symbolically or numerically

$f(x) = x^2 - 16$

$f'(x) = 2x$

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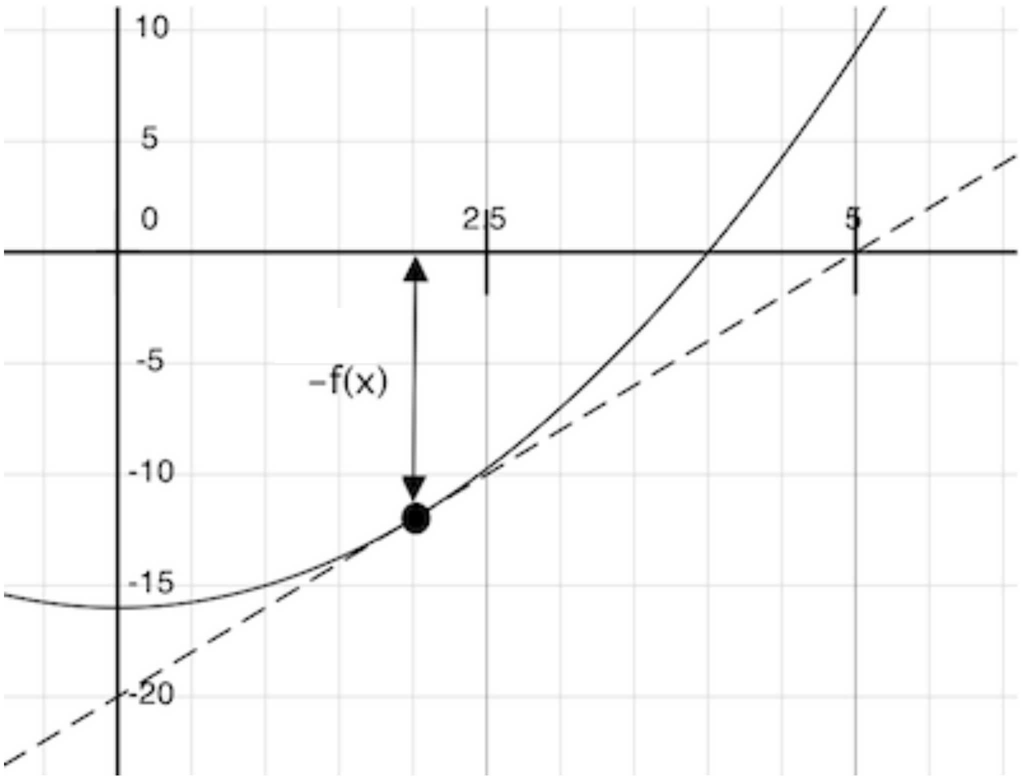

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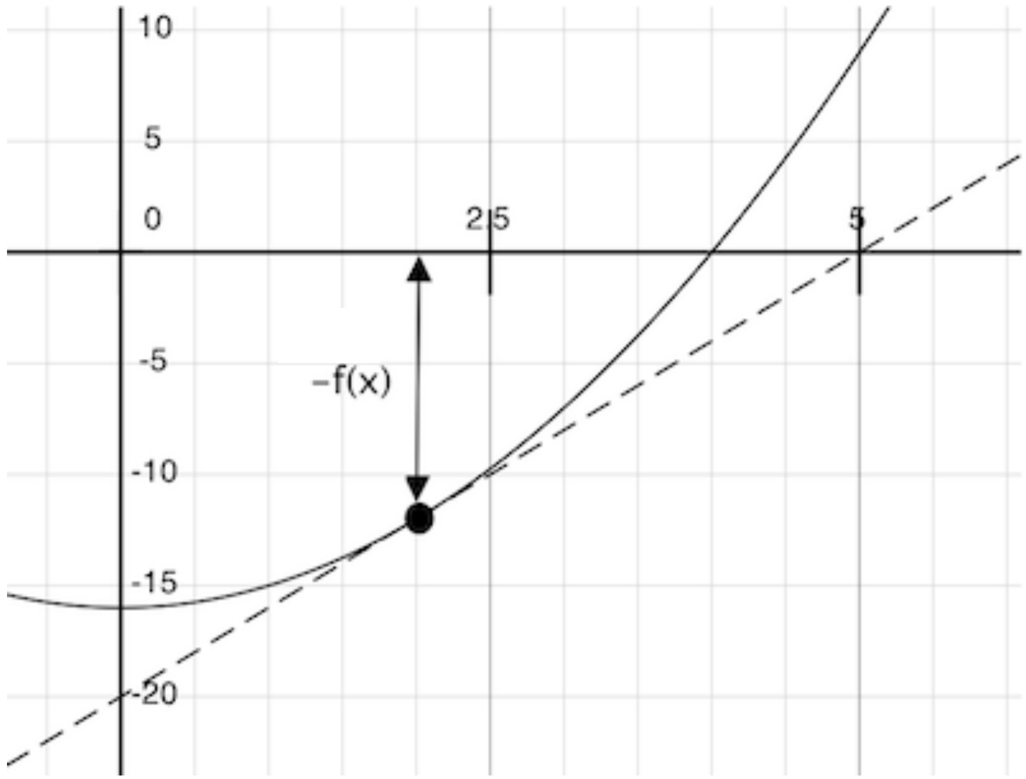

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f(x) = x^2 – 16

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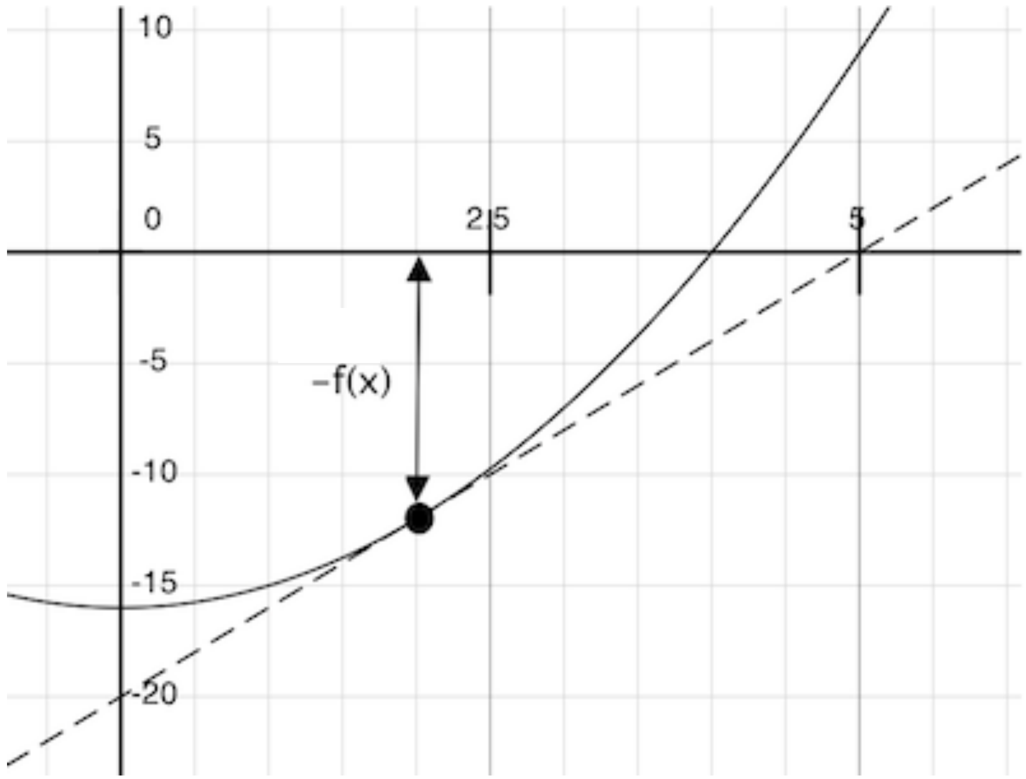

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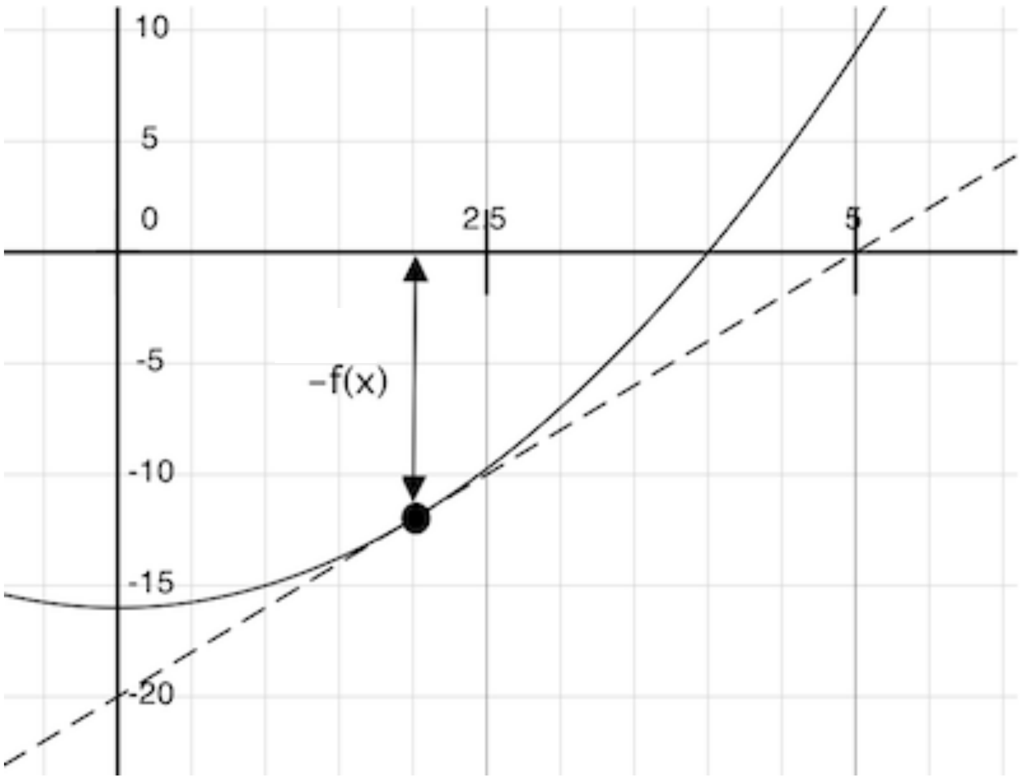

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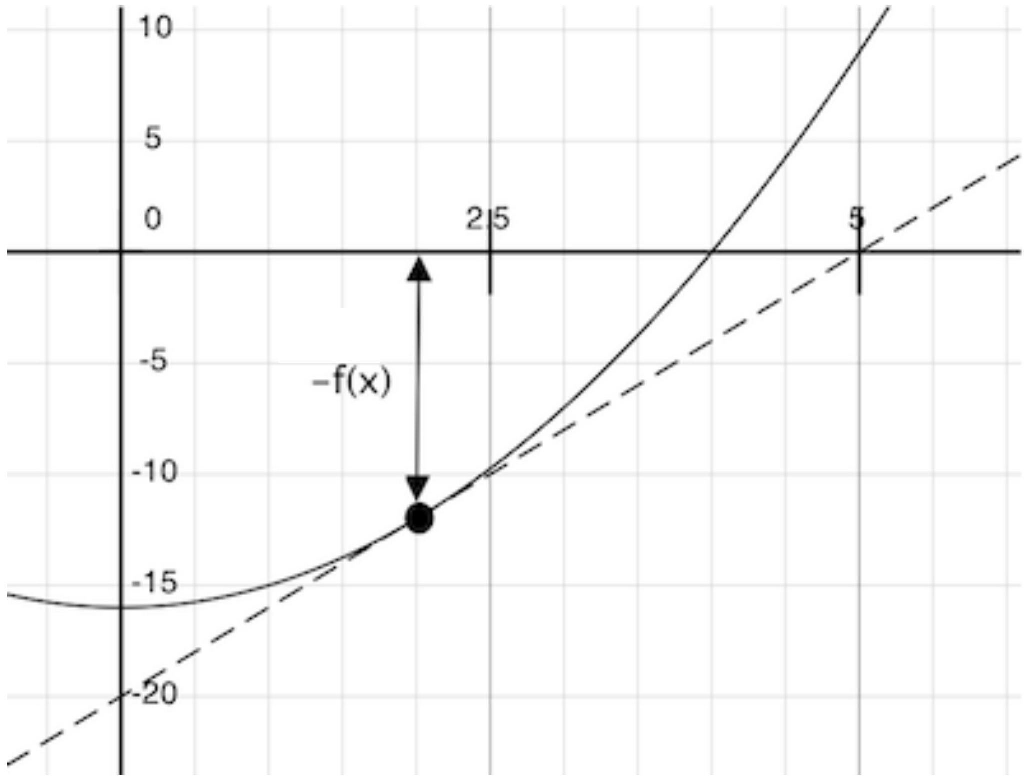

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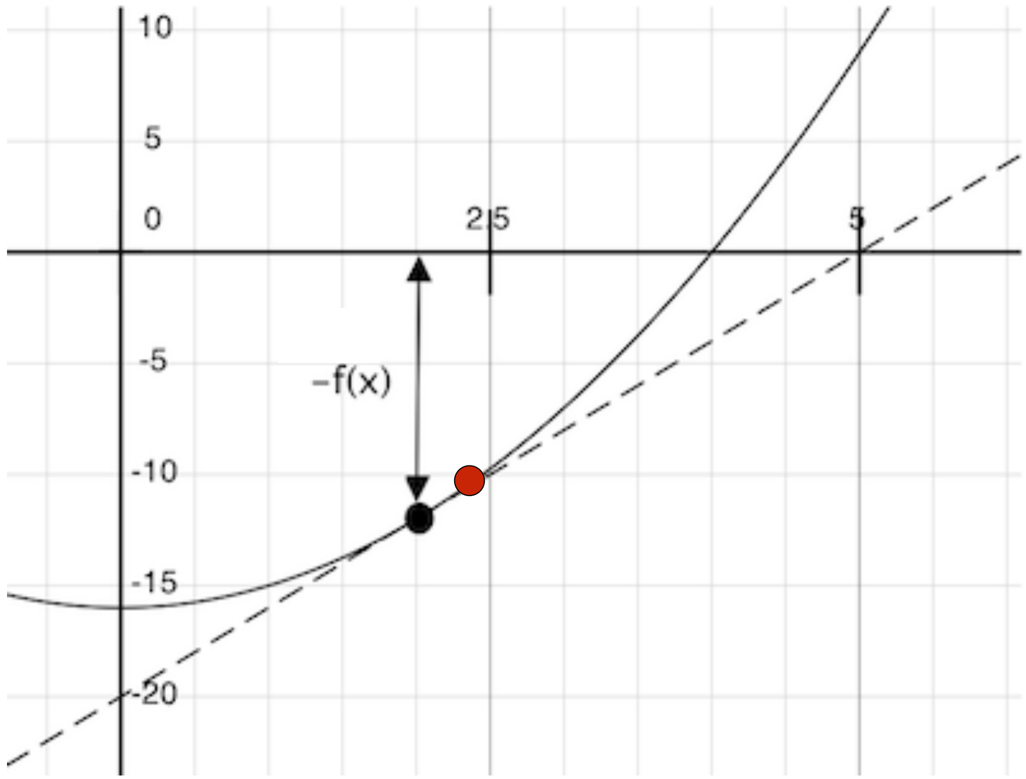

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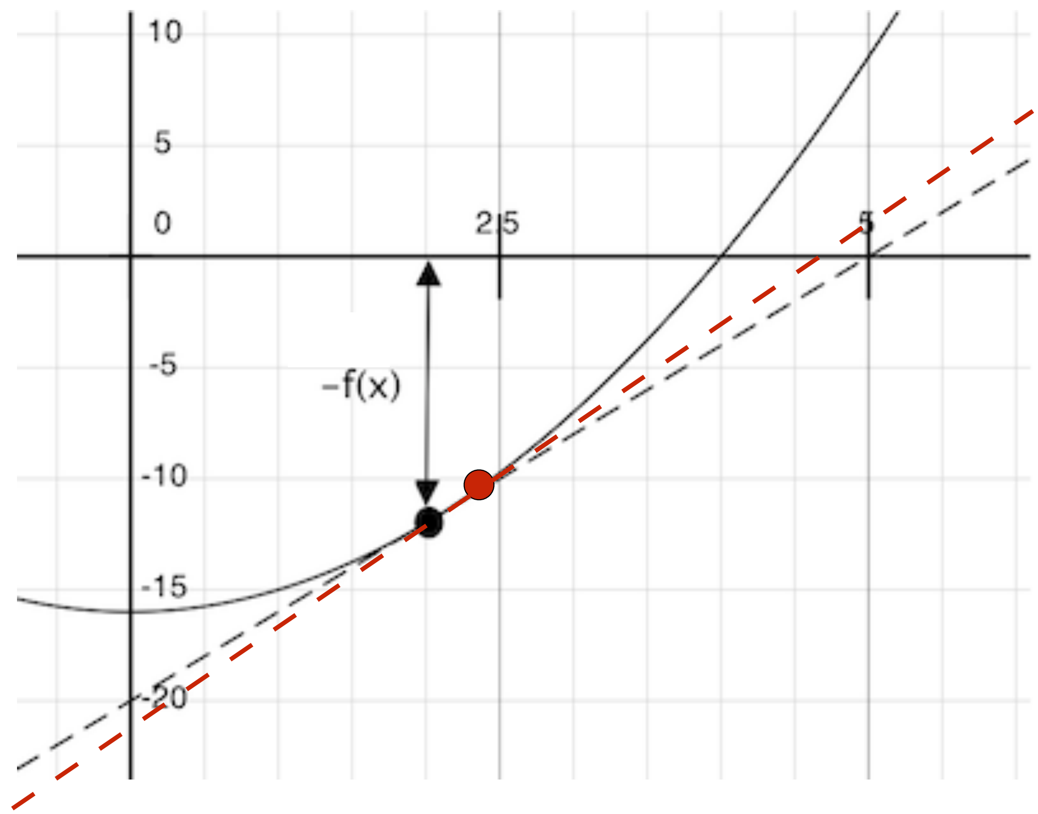

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Differentiation can be performed symbolically or numerically

f(x) = x^2 – 16

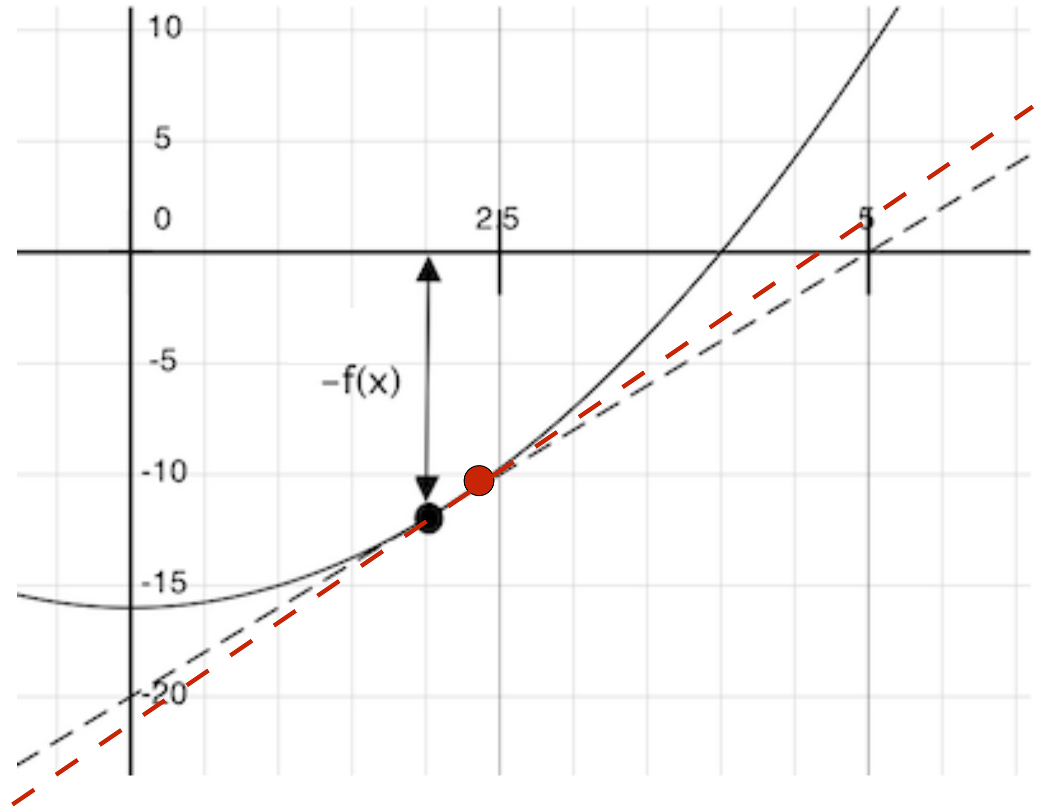
f'(x) = 2x

f'(2) = 4

$$f'(x) = \lim_{a \to 0} \frac{f(x+a) - f(x)}{a}$$

$$f'(x) \approx \frac{f(x+a) - f(x)}{a}$$ (if a is small)

(Demo)



Critical Points and Inverses

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Maxima, minima, and inflection points of a differentiable function occur
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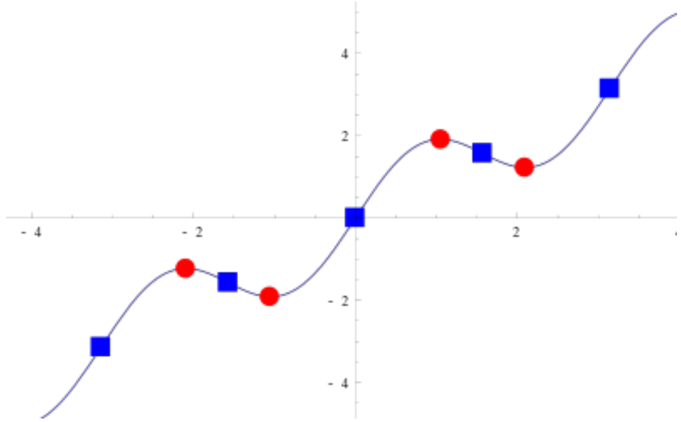

http://upload.wikimedia.org/wikipedia/commons/f/fd/Stationary_vs_inflection_pts.svg

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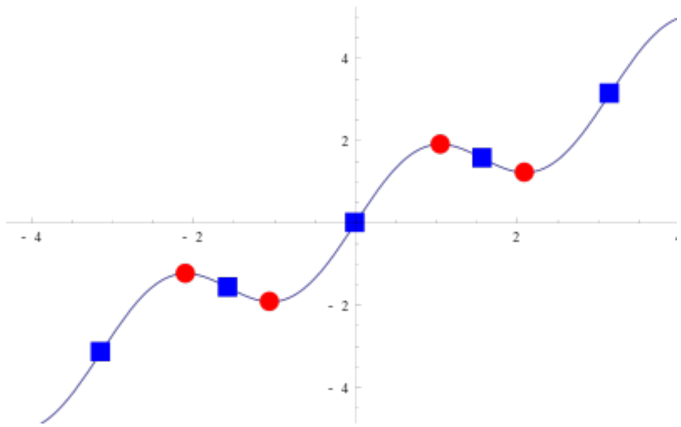

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Critical Points and Inverses

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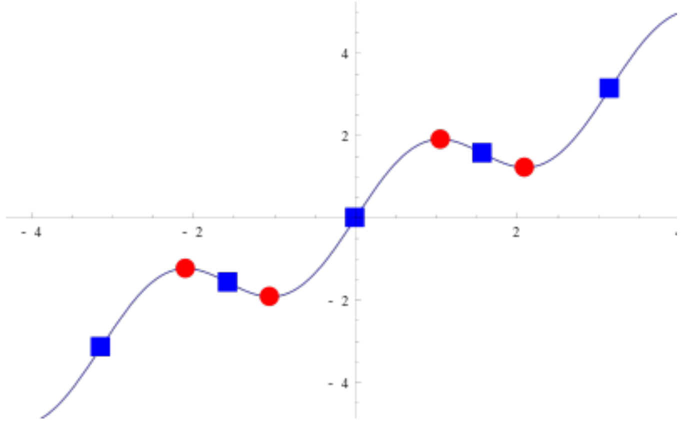
(Demo)

The inverse $f^{-1}(y)$ of a differentiable, one-to-one function computes the value x such that f(x) = y



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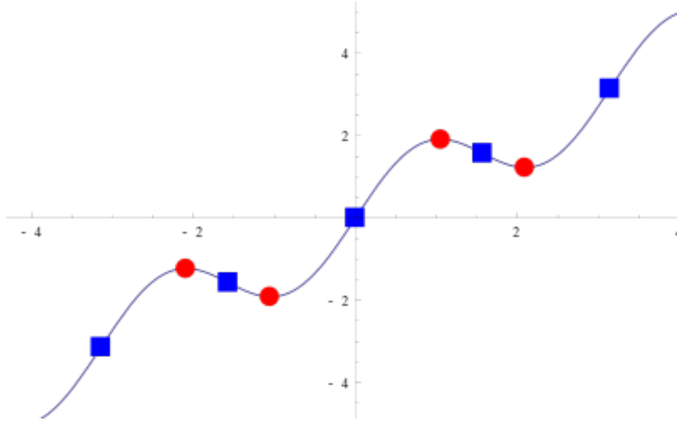

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