Logic Programming

Announcements

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- Expressions are facts or queries, which contain relations
- Expressions and relations are Scheme lists
- For example, **(likes john dogs)** is a relation

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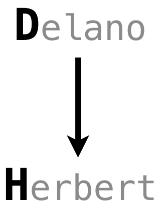

Delano

Herbert

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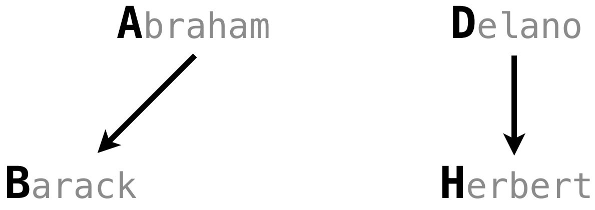

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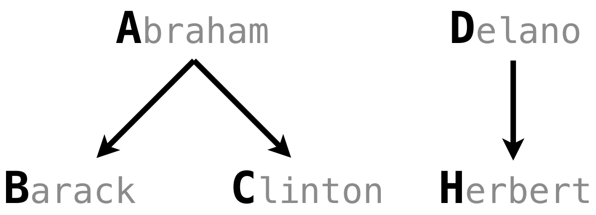

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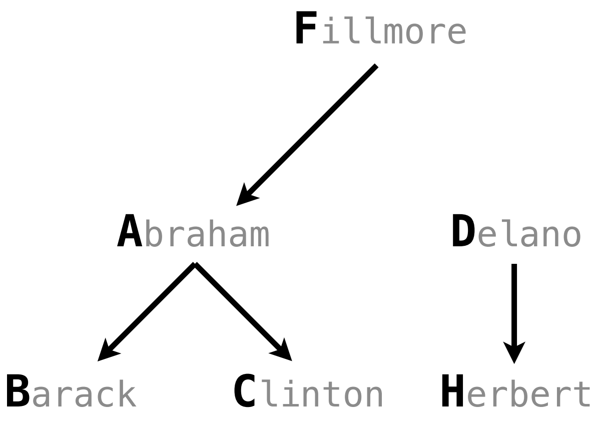

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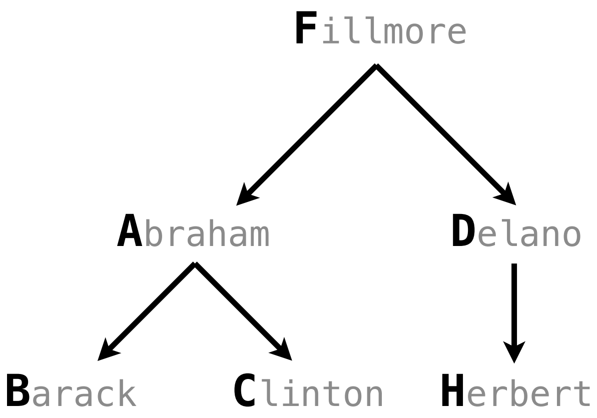

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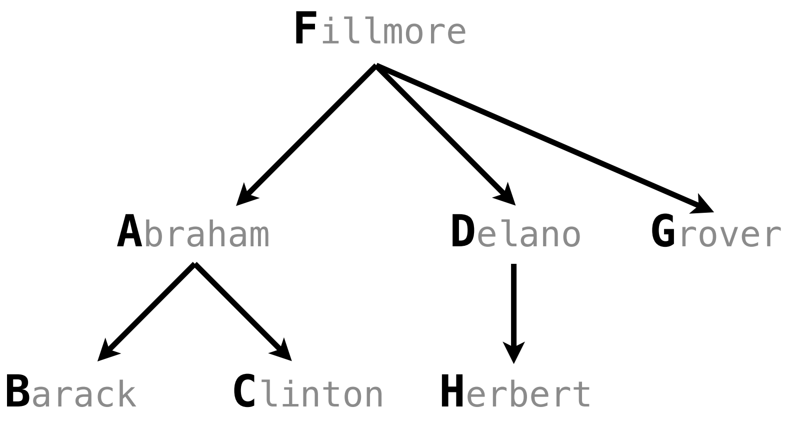

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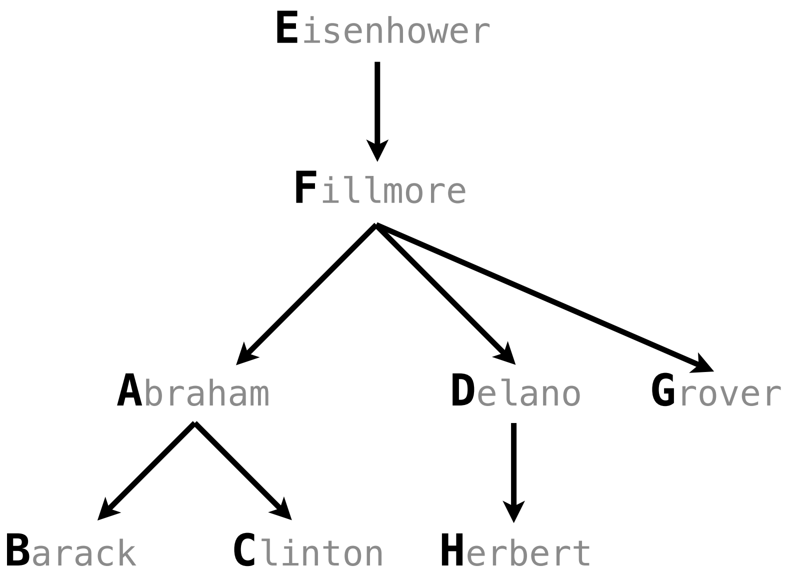

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```
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(add 1 ? 3)     2

(add 1 2 ?)     3

( ?  1 2 3)    add
```

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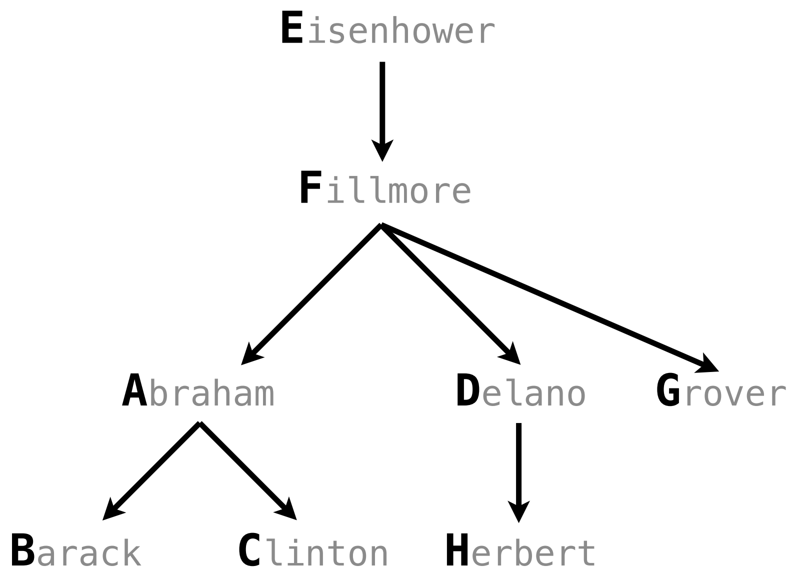

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logic> (query (parent abraham ?puppy))
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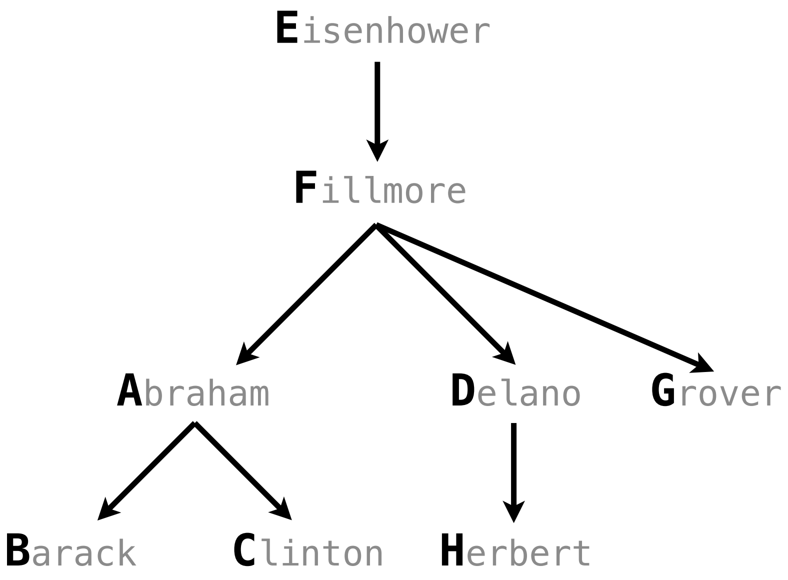

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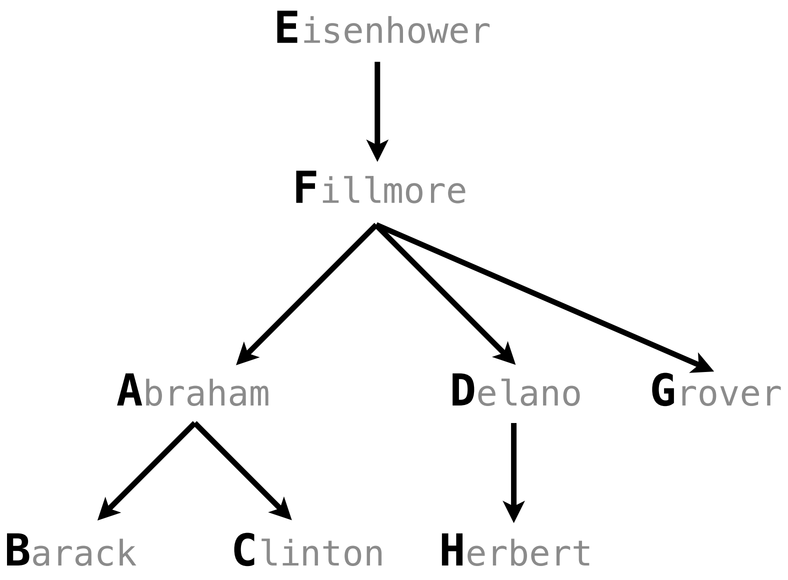
logic> (query (parent abraham ?puppy))
```

A variable can have any name



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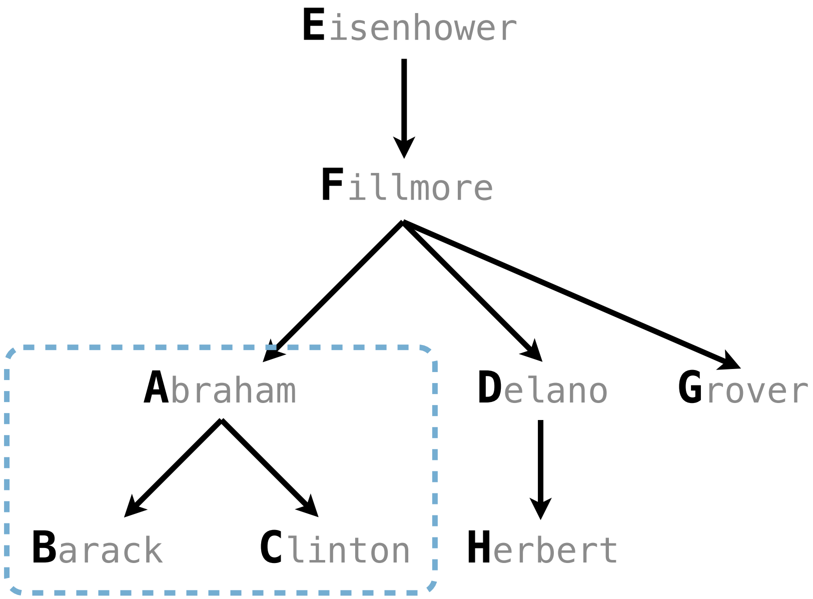

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logic> (query (parent abraham ?puppy))
Success!
```

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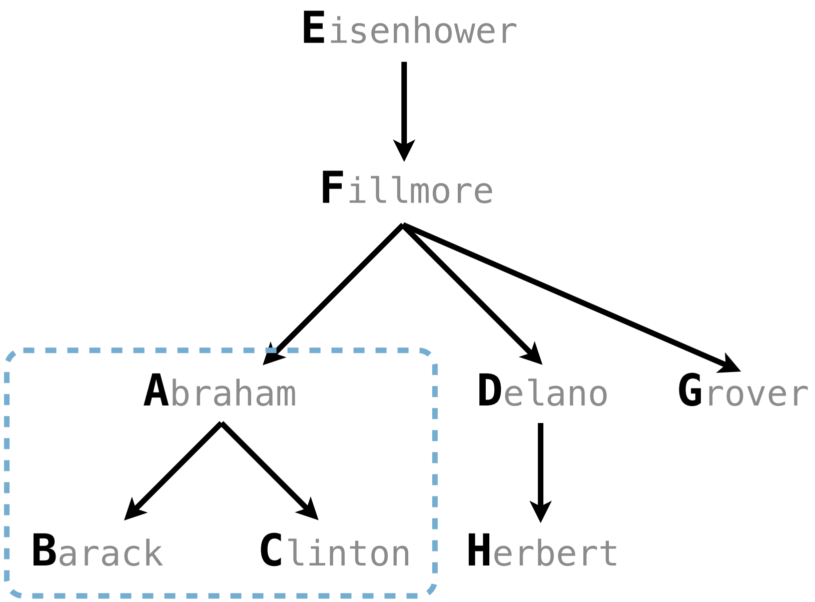

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logic> (query (parent abraham ?puppy))
Success!
puppy: barack
```

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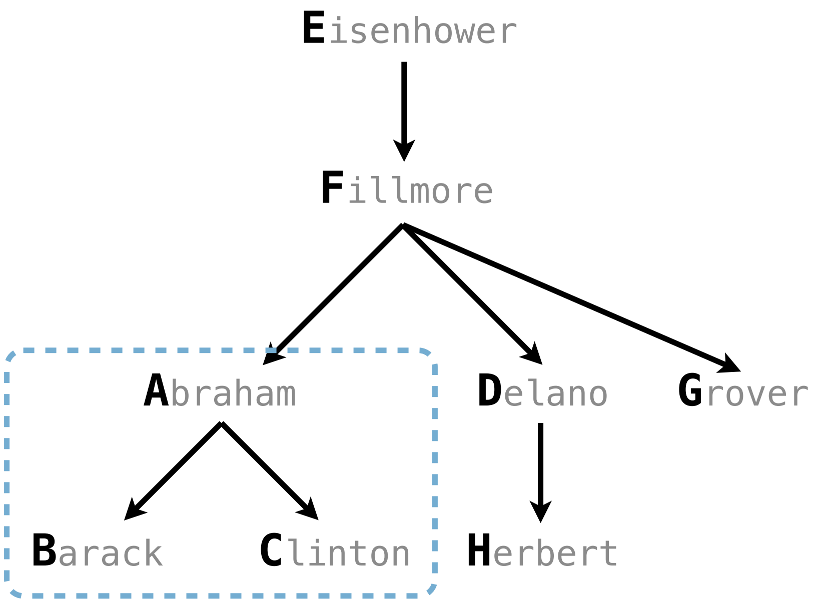

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Success!
puppy: barack
puppy: clinton
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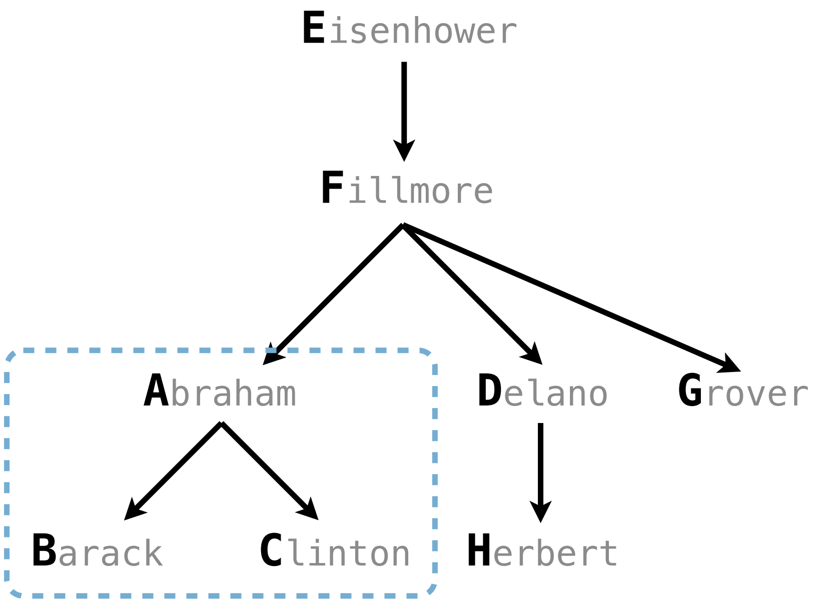

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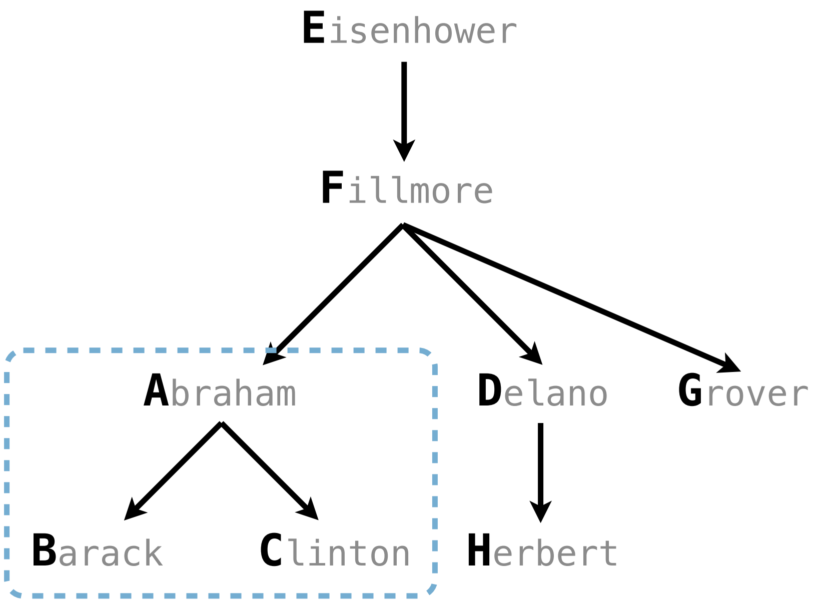
A variable can have any name

Each line is an assignment of variables to values



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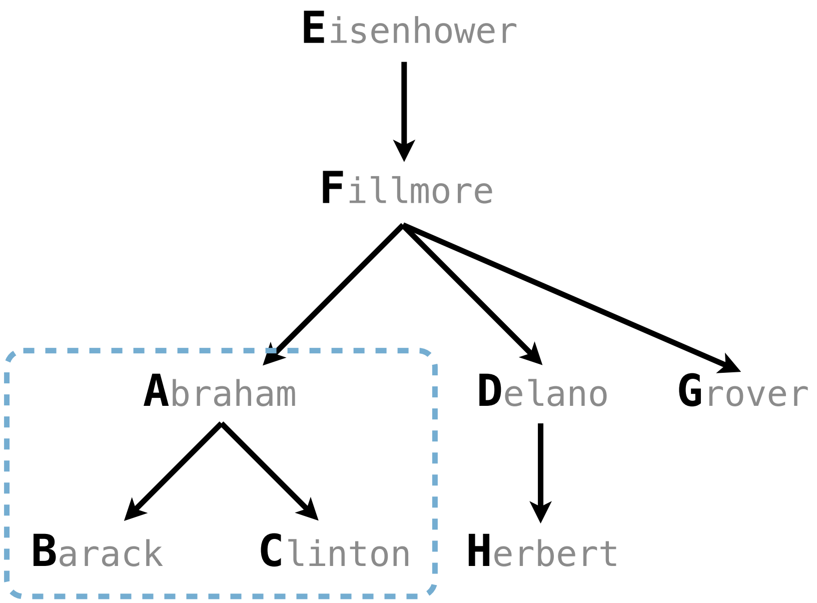
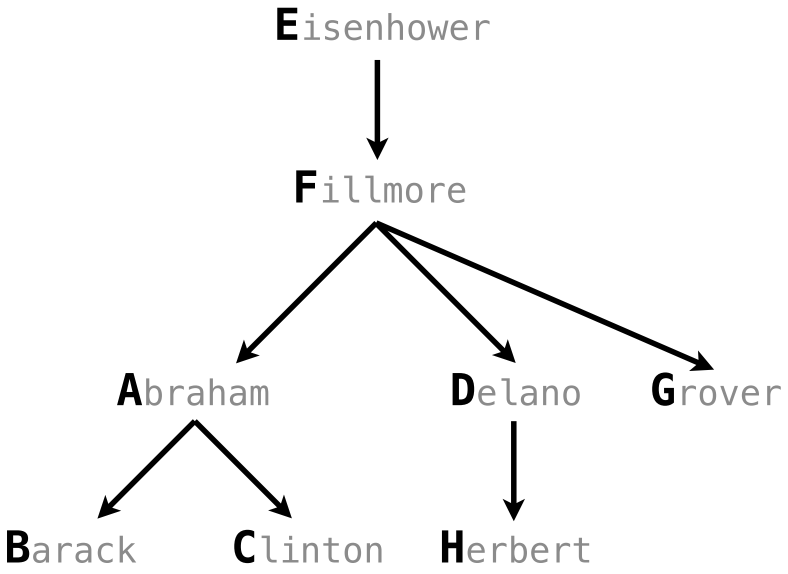


(Demo)

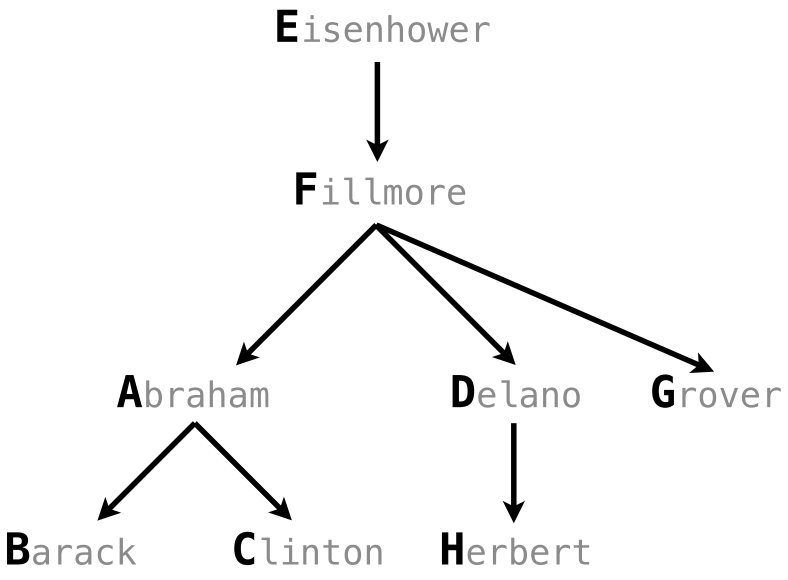
Compound Facts and Queries

Compound Facts



Compound Facts

A fact can include multiple relations and variables as well.



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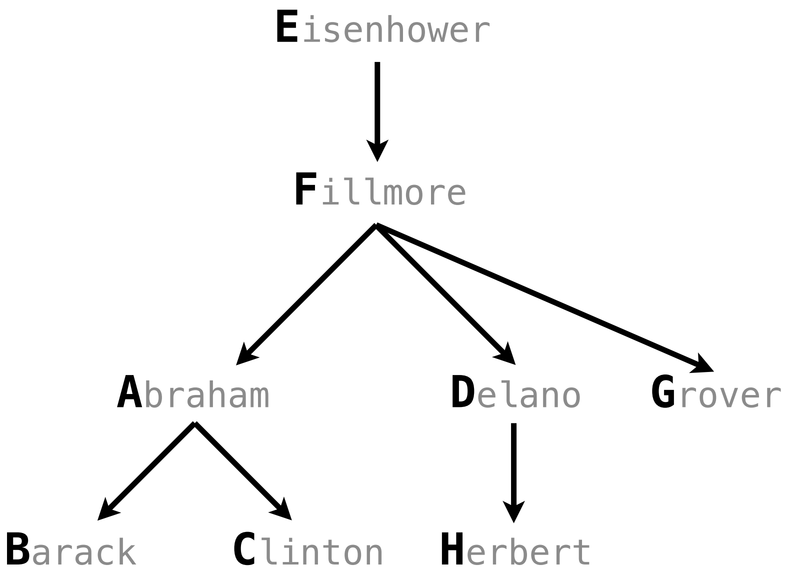
```
(fact <conclusion> <hypothesis₀> <hypothesis₁> ... <hypothesisN>)
```



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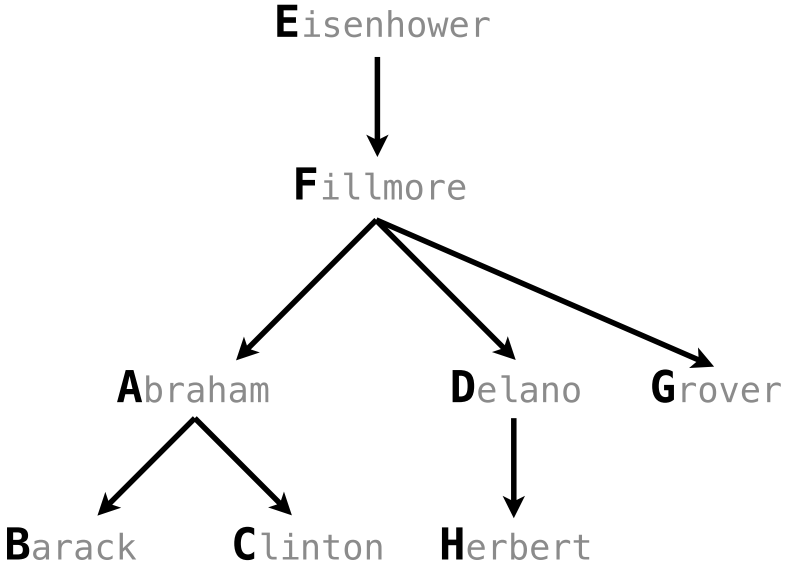
`(fact <conclusion> <hypothesis₀> <hypothesis₁> ... <hypothesisN>)`

Means <conclusion> is true if all the $\text{<hypothesis}_K\text{>}$ are true.



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```
logic> (fact (child ?c ?p) (parent ?p ?c))
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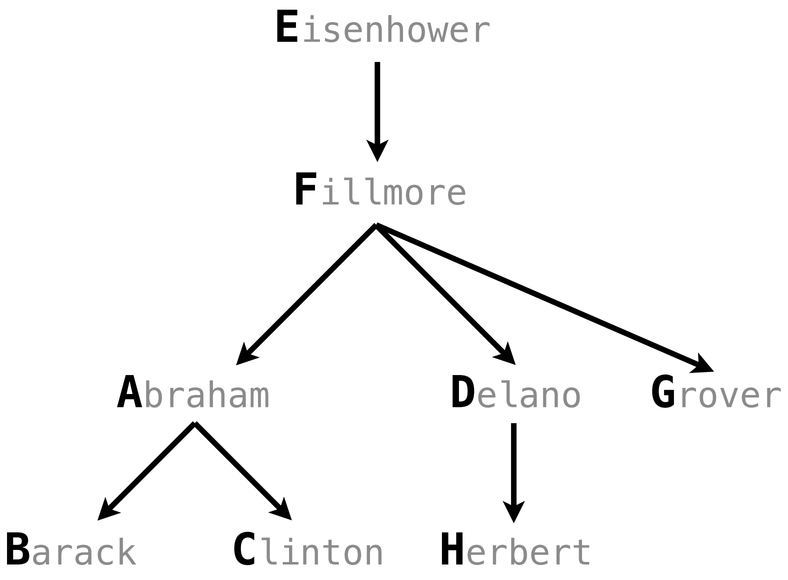

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```
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logic> (query (child herbert delano))
```

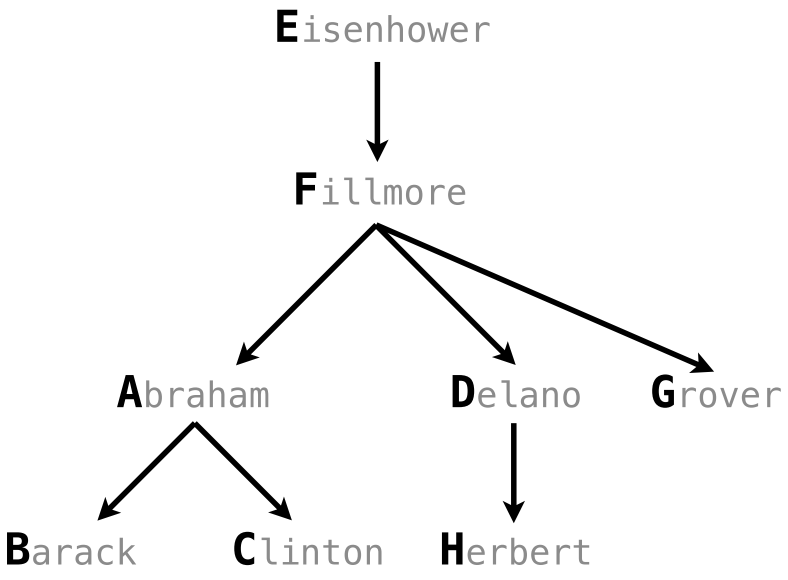

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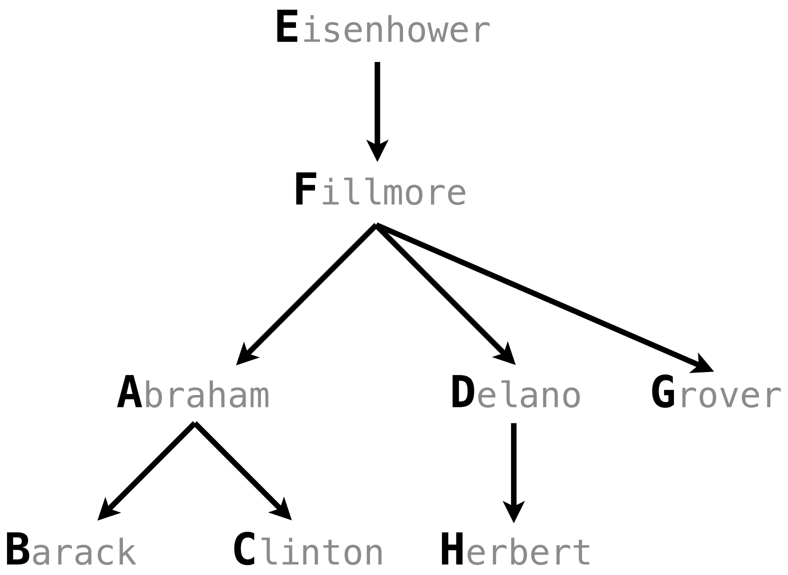

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Success!

logic> (query (child eisenhower clinton))
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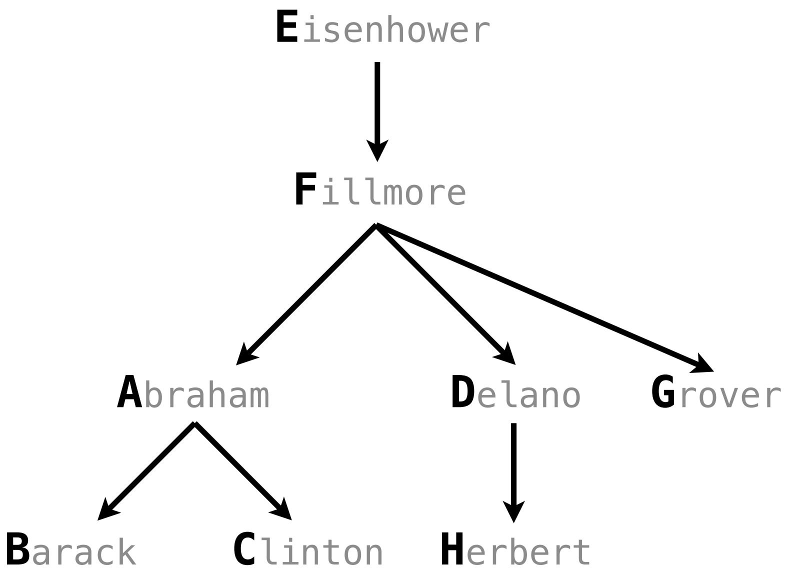

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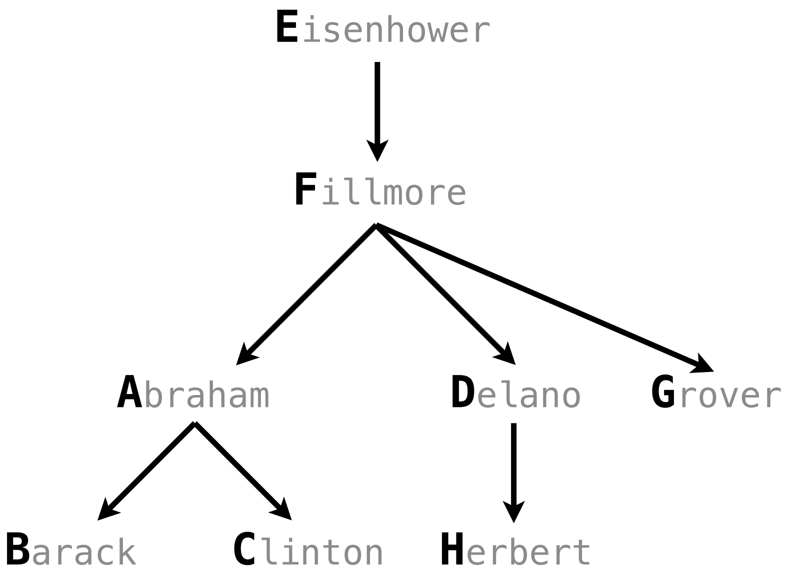
logic> (query (child herbert delano))
Success!

logic> (query (child eisenhower clinton))
Failure.
```



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```

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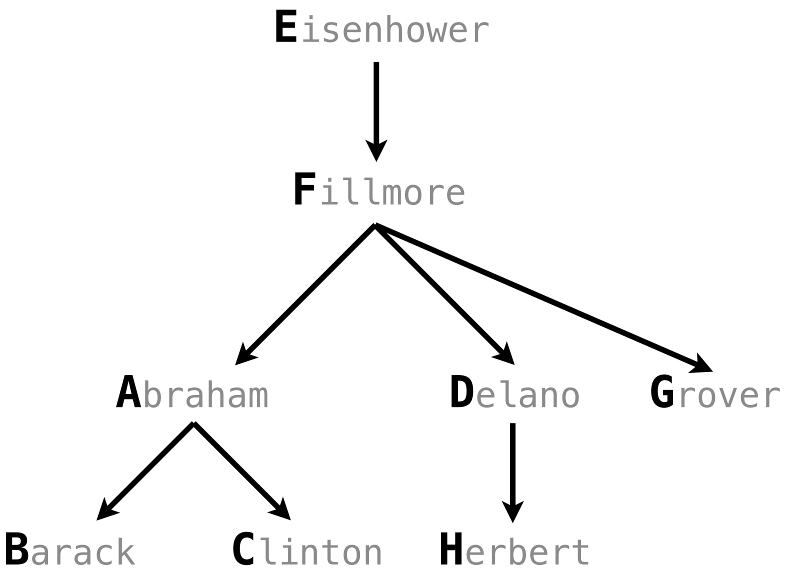
logic> (query (child herbert delano))
Success!

logic> (query (child eisenhower clinton))
Failure.

logic> (query (child ?kid fillmore))
```



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logic> (query (child eisenhower clinton))
Failure.

logic> (query (child ?kid fillmore))
Success!
```

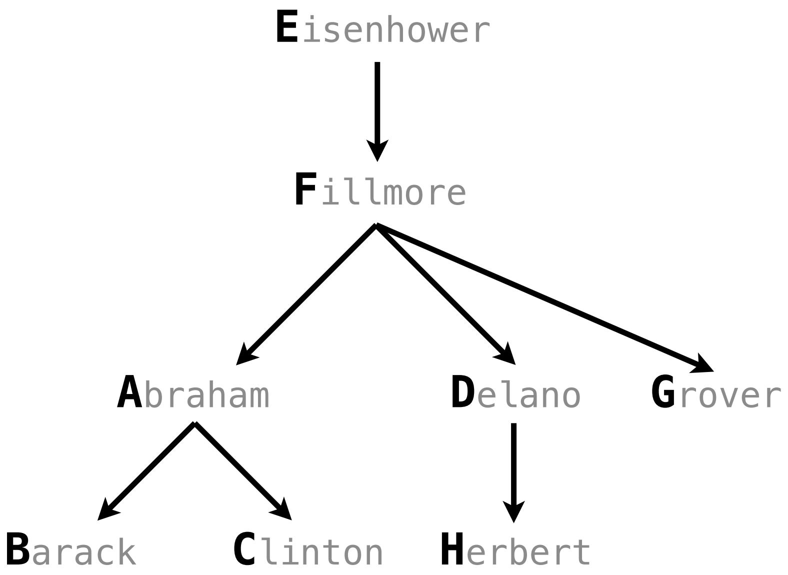

Compound Facts

A fact can include multiple relations and variables as well.

(fact <conclusion> <hypothesis$_0$> <hypothesis$_1$> ... <hypothesis$_N$>)

Means <conclusion> is true if all the <hypothesis$_K$> are true.

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Success!

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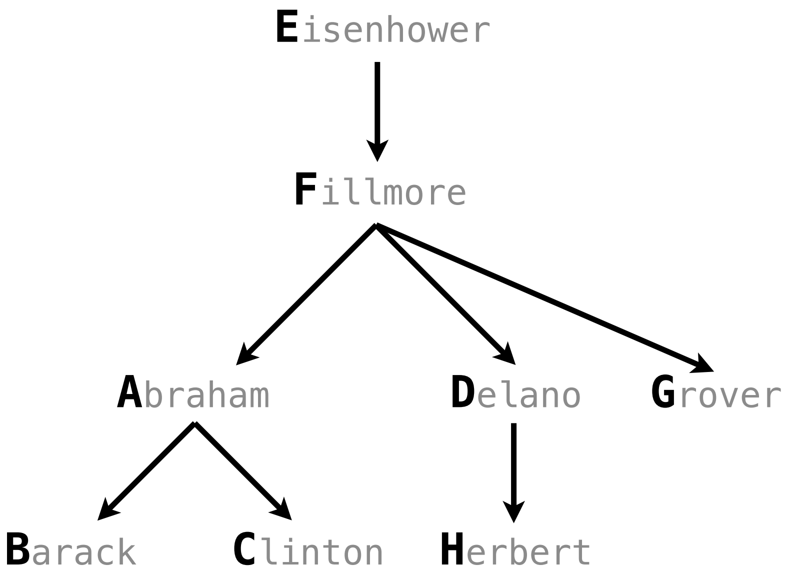
Failure.

logic> (query (child ?kid fillmore))
Success!
kid: abraham
```



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Success!
kid: abraham
kid: delano
```

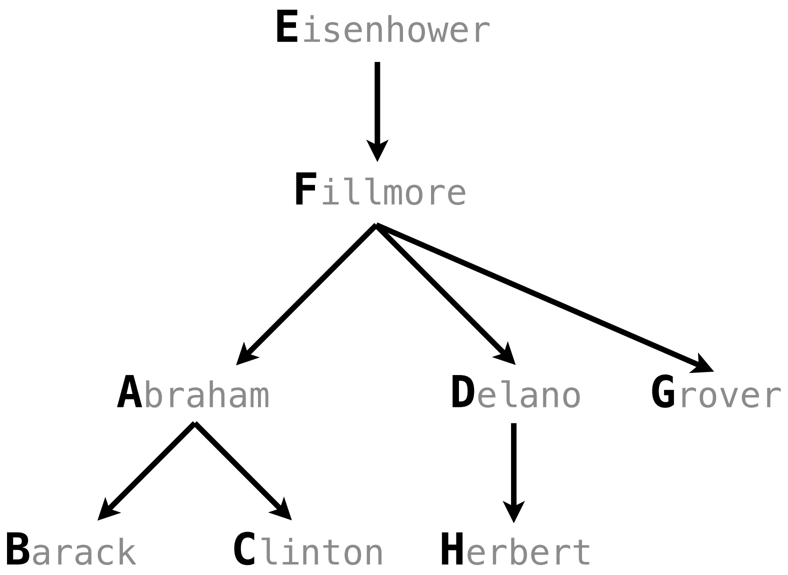

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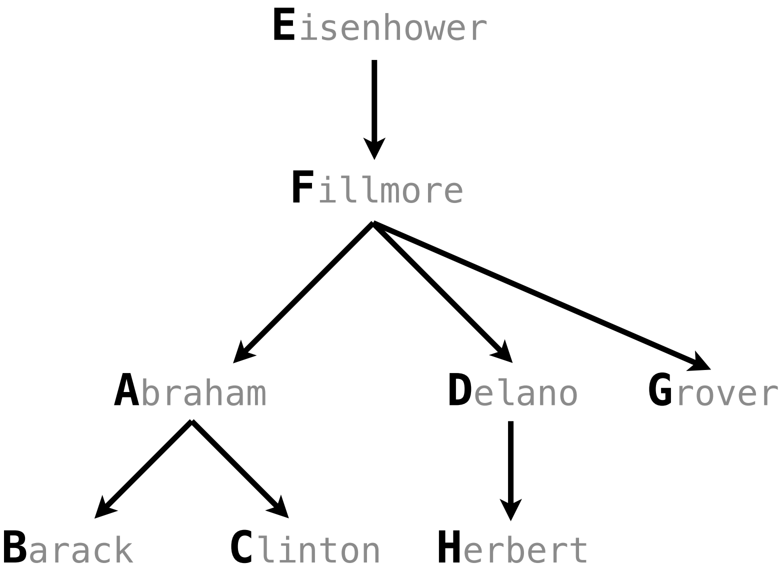
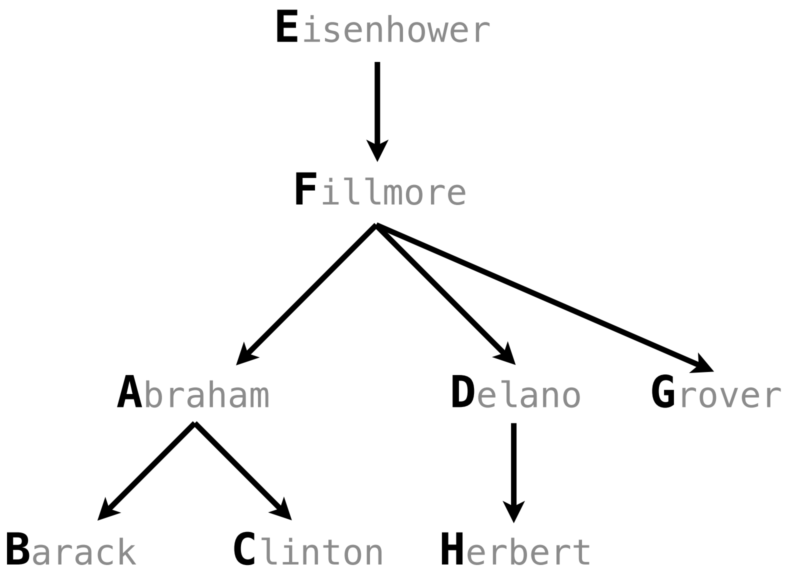
Success!
kid: abraham
kid: delano
kid: grover
```

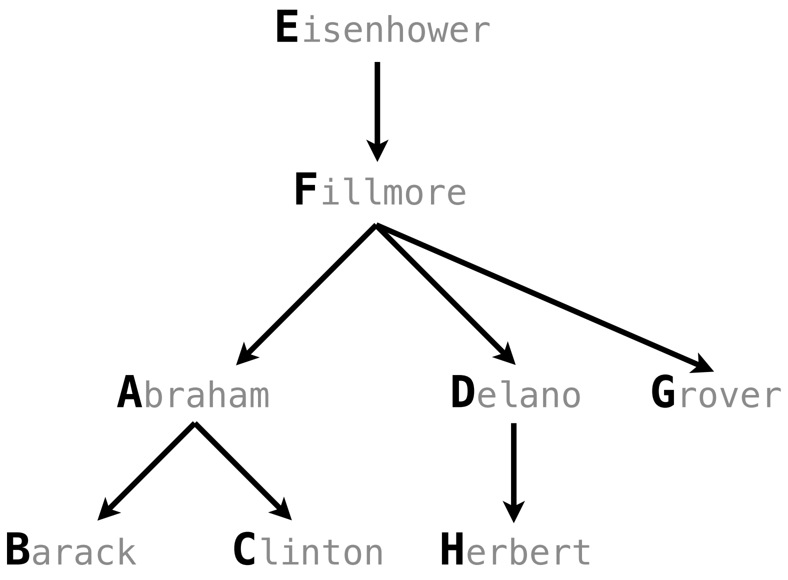


Compound Queries



Compound Queries

An assignment must satisfy all relations in a query.



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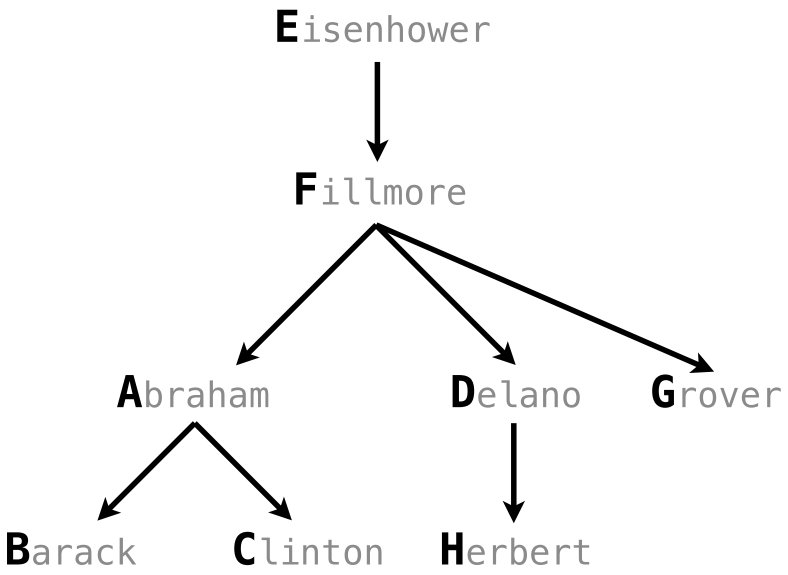

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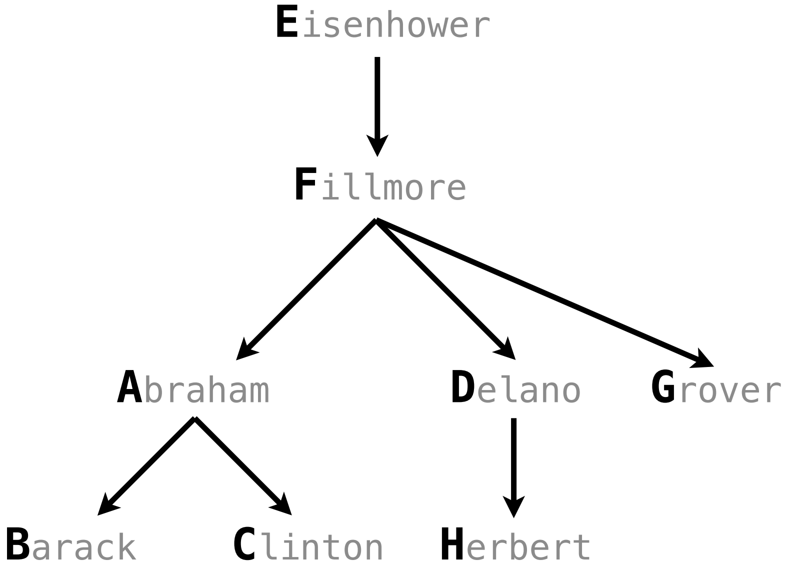

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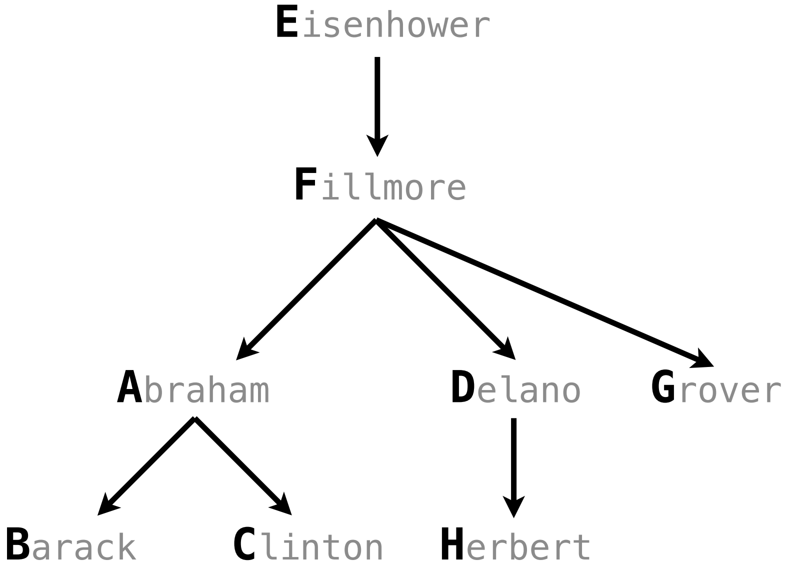

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logic> (query (parent ?grampa ?kid)
```

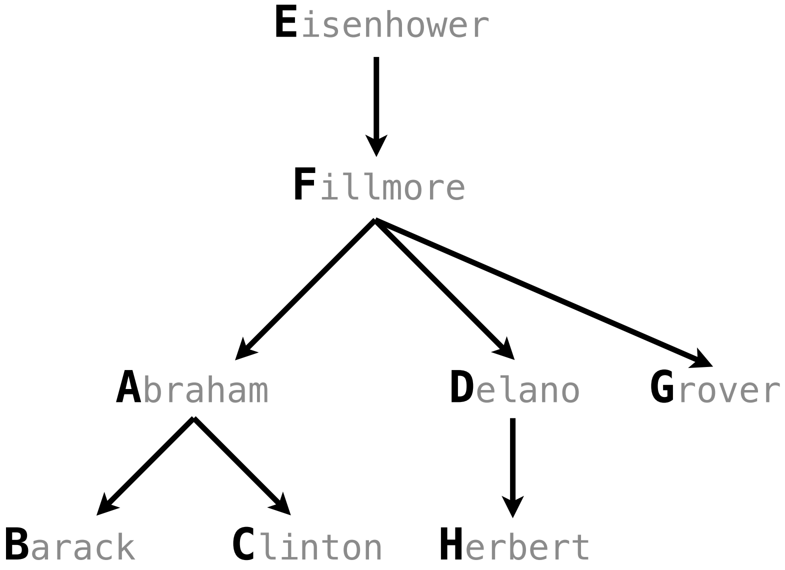

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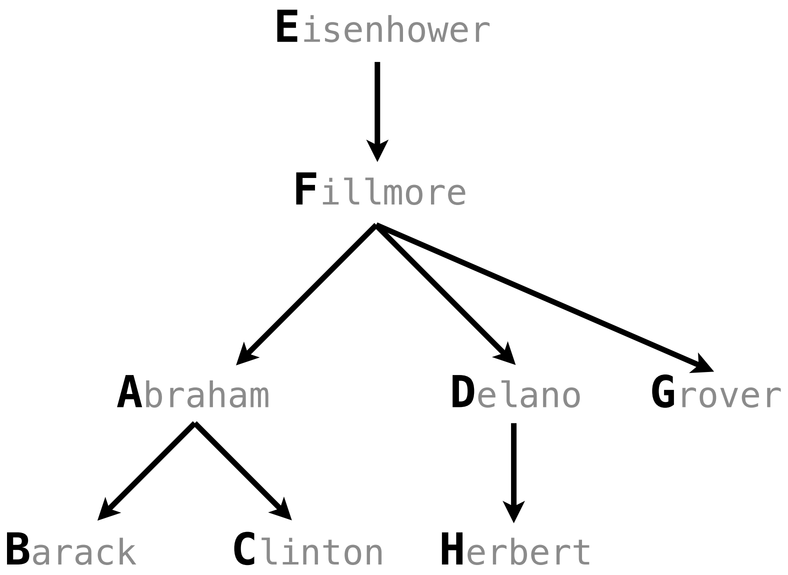
              (child clinton ?kid))
```



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```

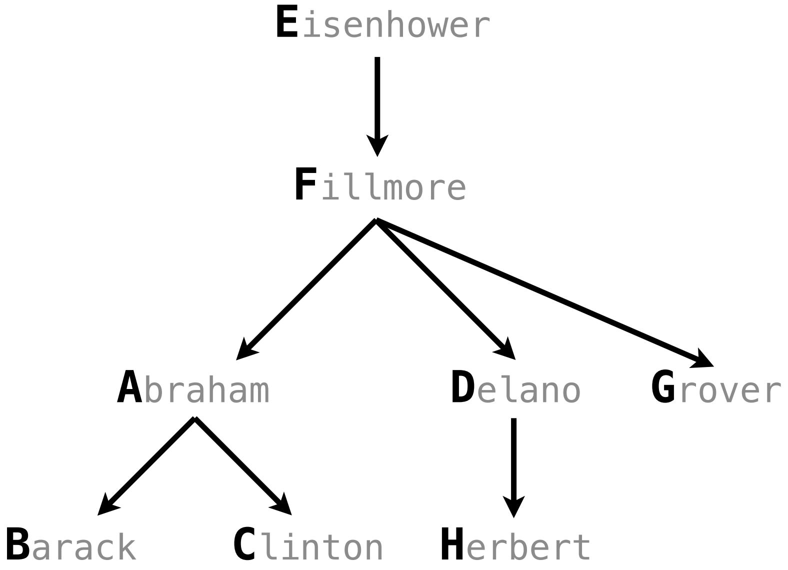

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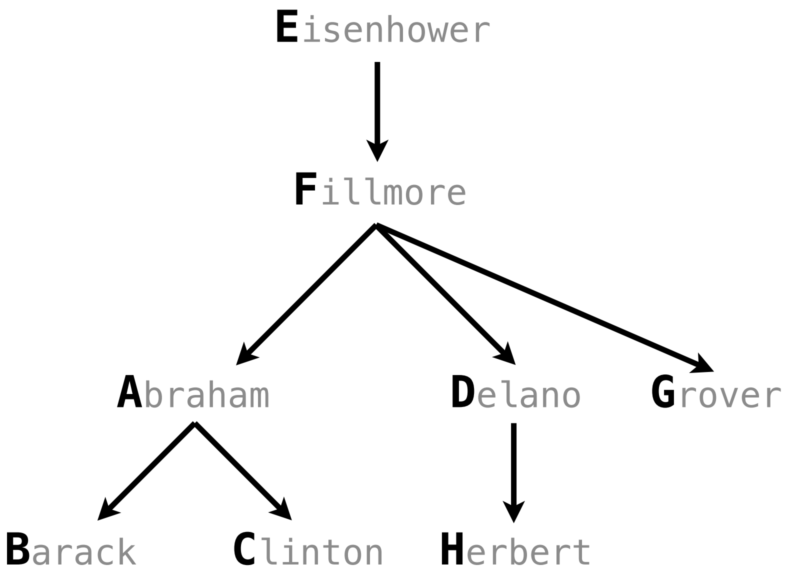
              (child clinton ?kid))
Success!
grampa: fillmore    kid: abraham
```



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grampa: fillmore    kid: abraham

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```

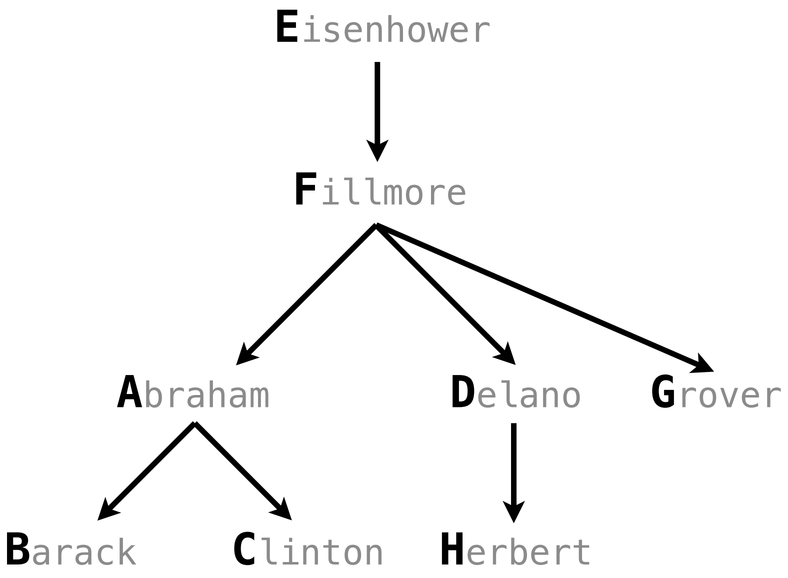

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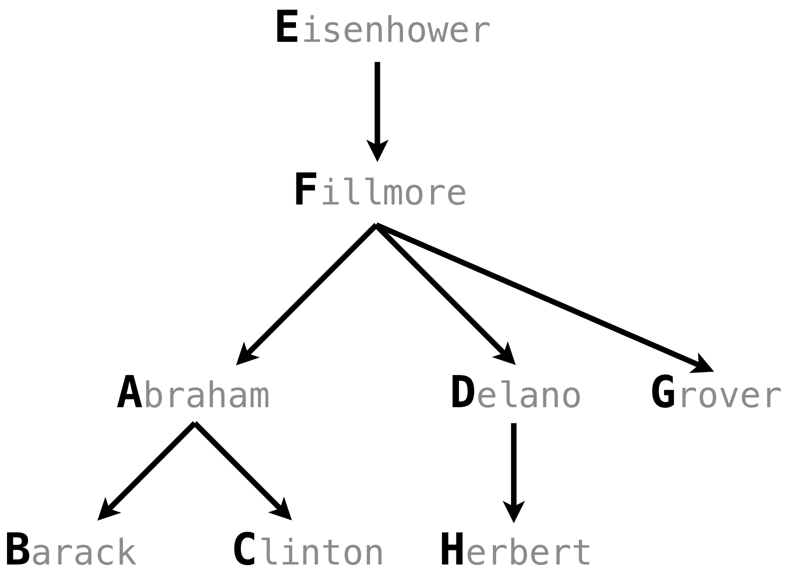
Success!
grampa: fillmore    kid: abraham

logic> (query (child ?y ?x)
              (child ?x eisenhower))
```



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```

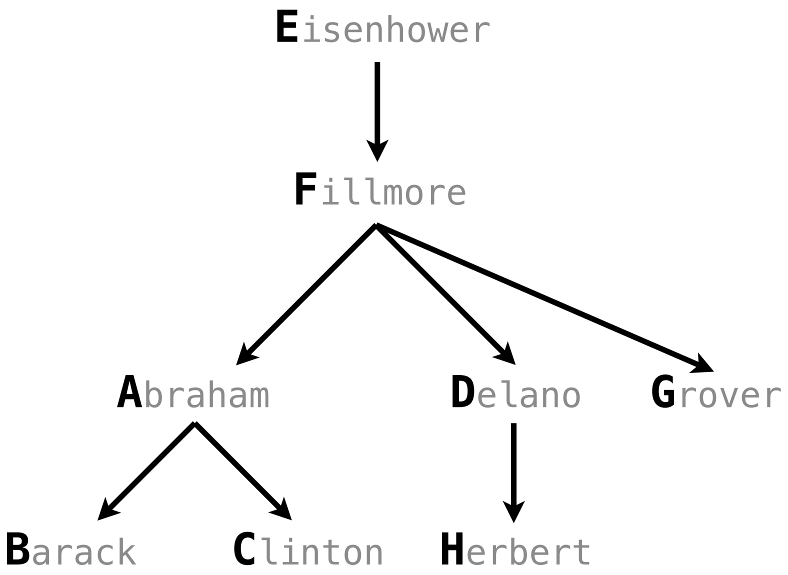

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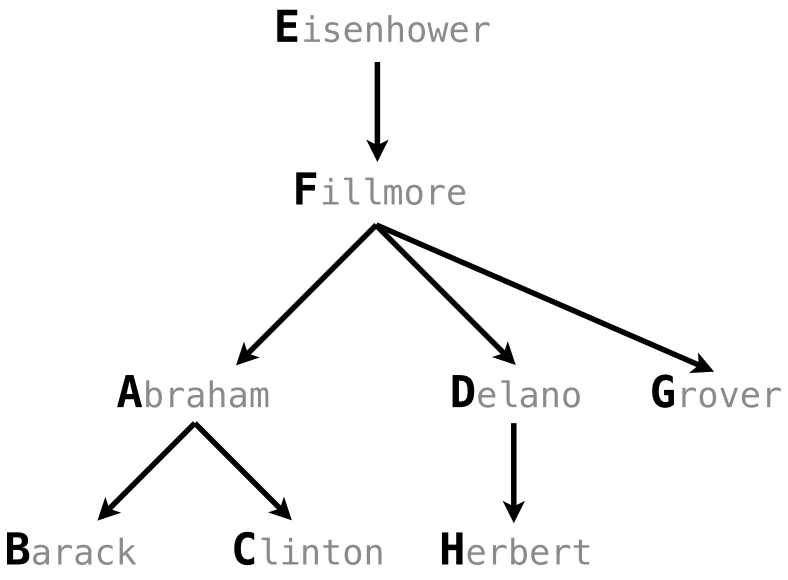

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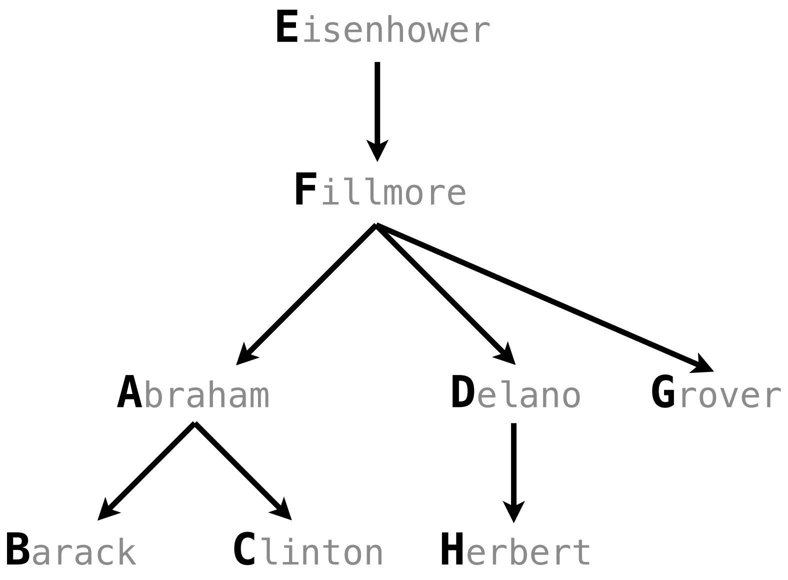
              (child ?x eisenhower))
Success!
y: abraham     x: fillmore
y: delano      x: fillmore
```



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y: abraham    x: fillmore
y: delano     x: fillmore
y: grover     x: fillmore
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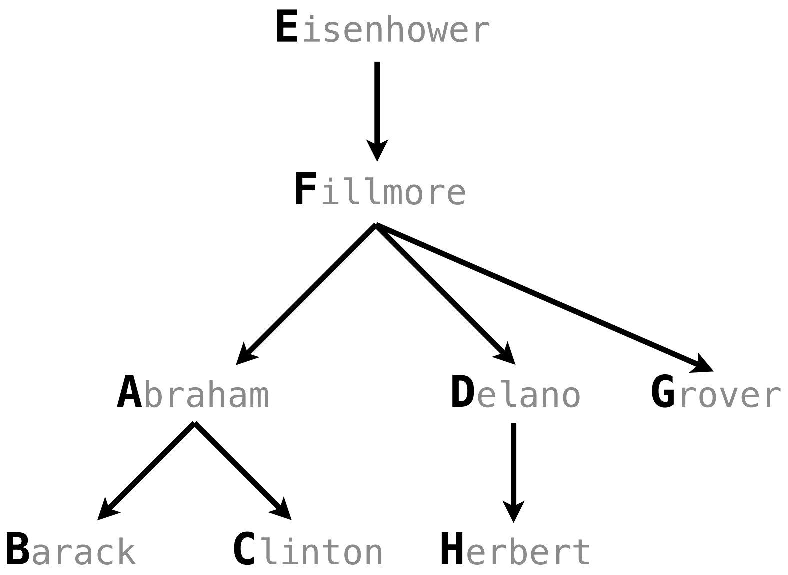

Recursive Facts

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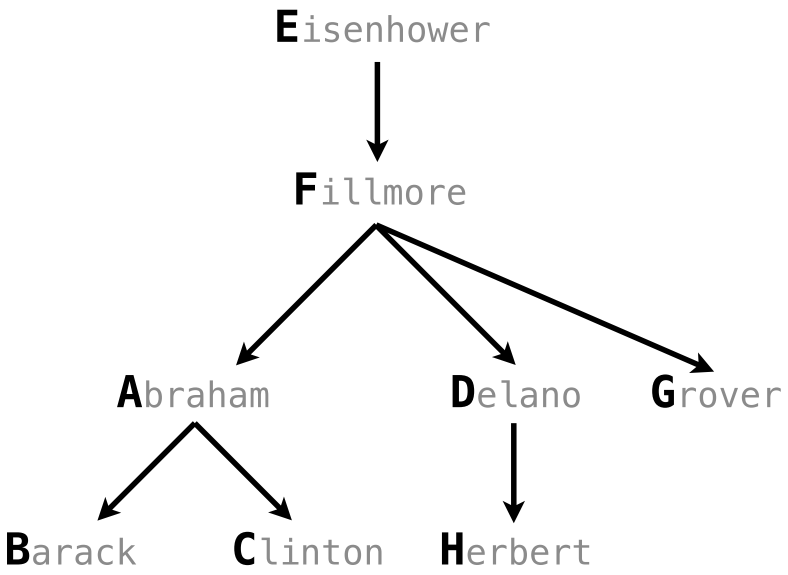
A fact is recursive if the same relation is mentioned in a hypothesis and the conclusion.



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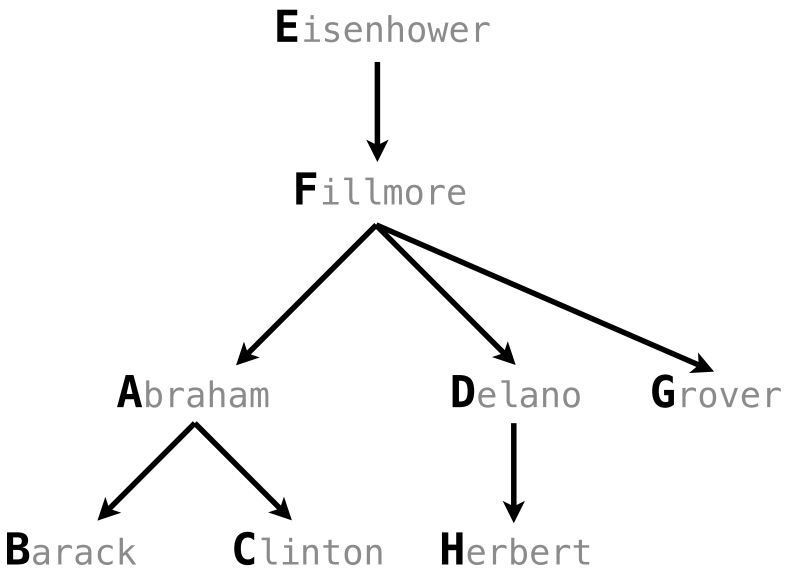

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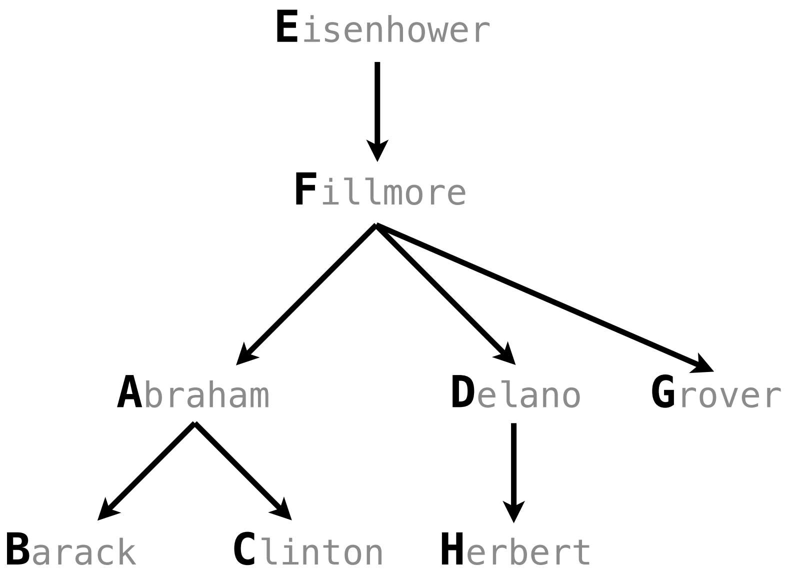

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logic> (query (ancestor ?a herbert))
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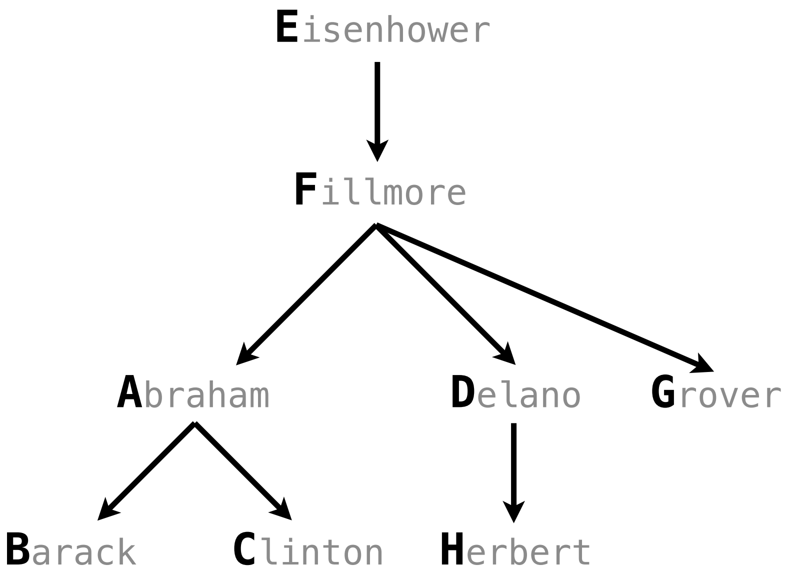

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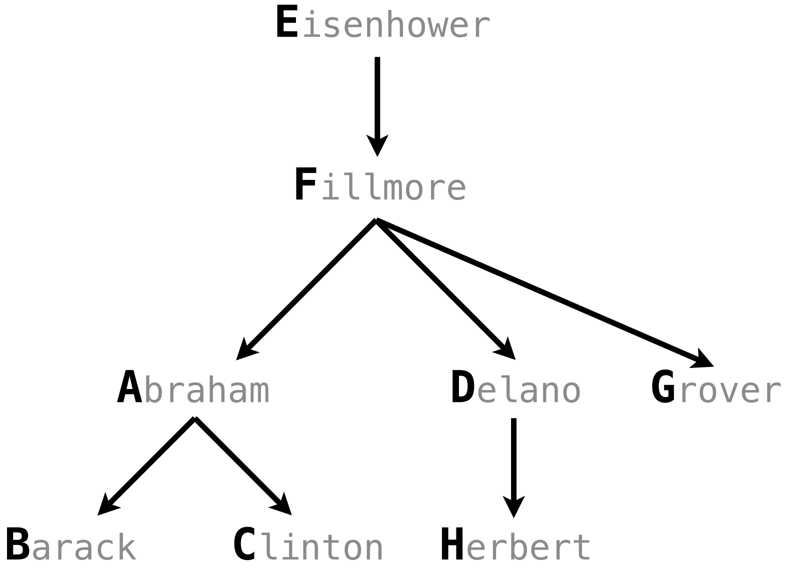

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logic> (query (ancestor ?a herbert))
Success!
a: delano
```

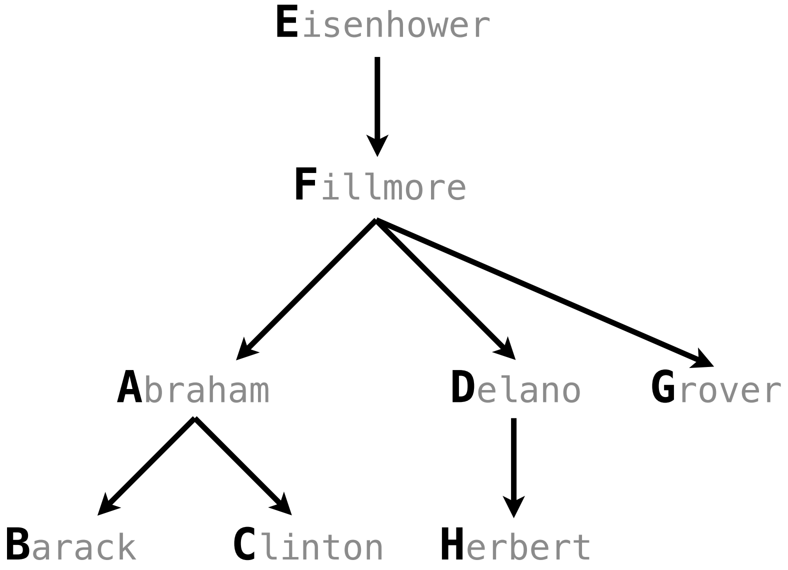

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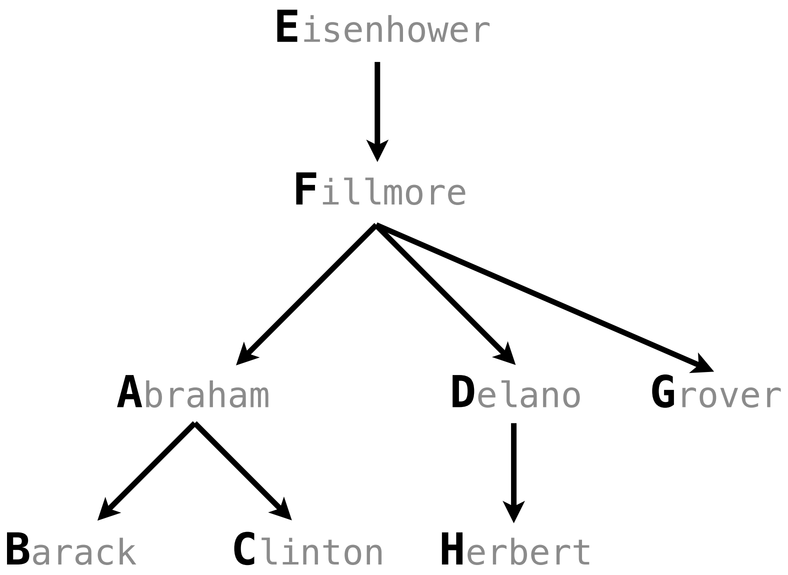
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
```



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a: eisenhower
```

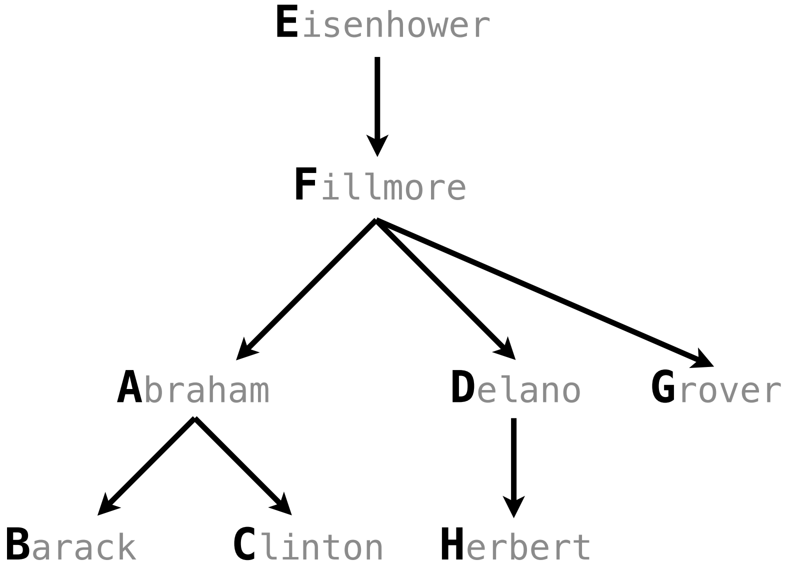

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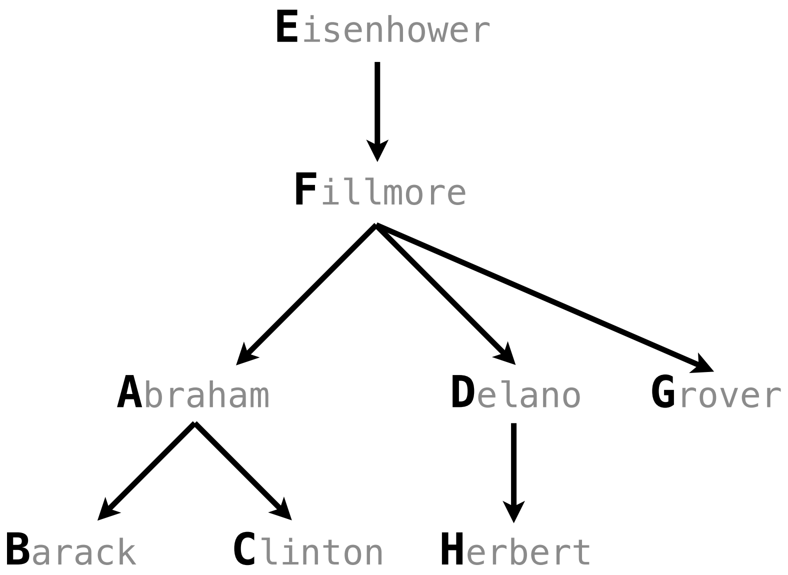

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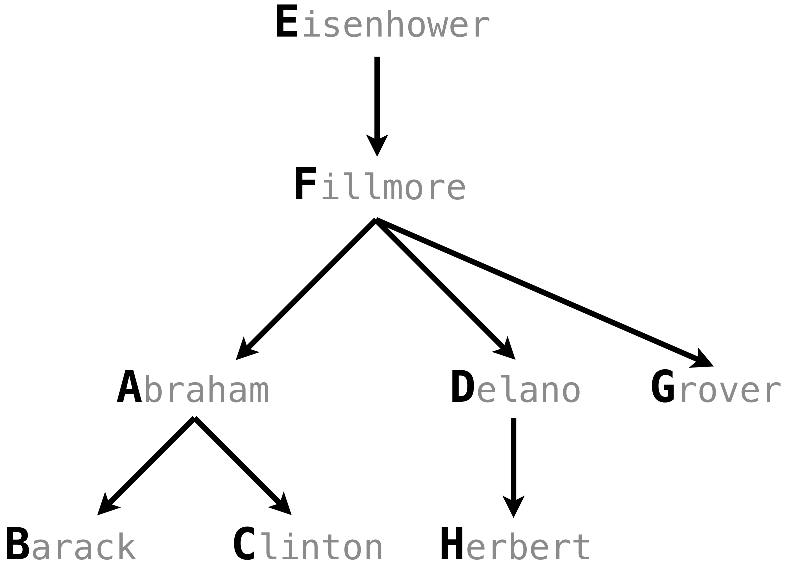
```



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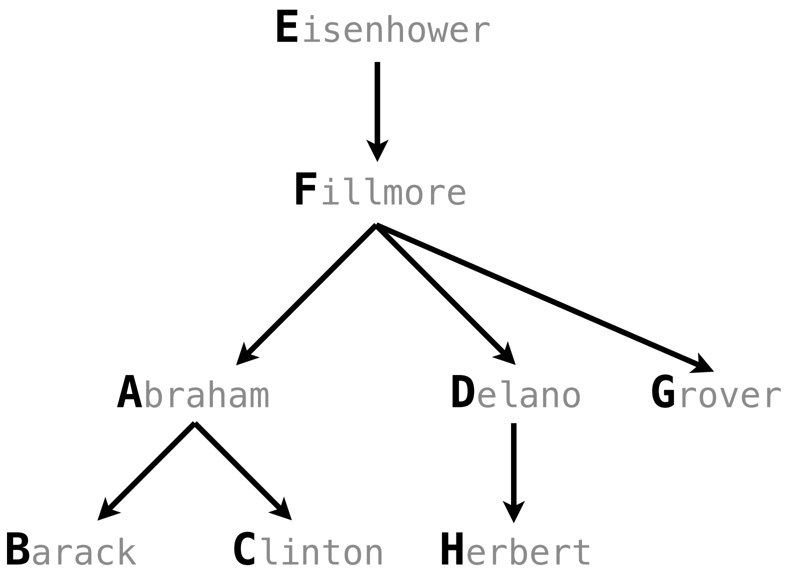

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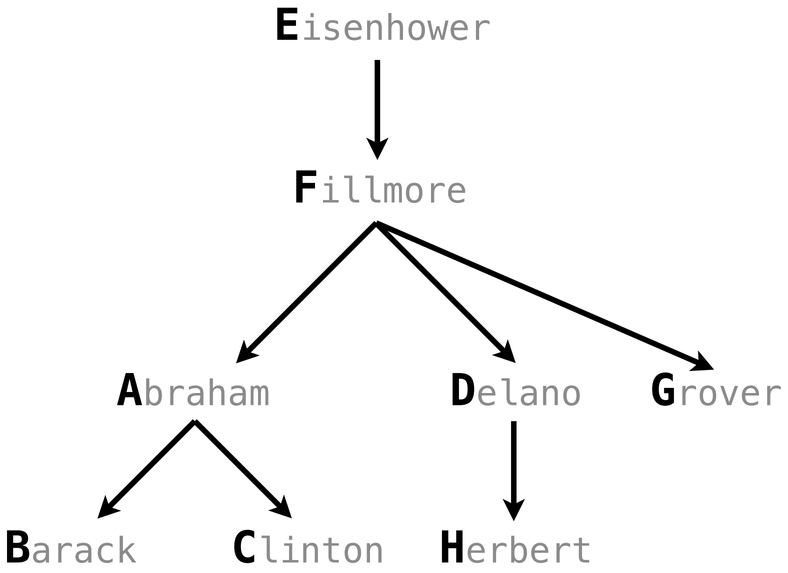

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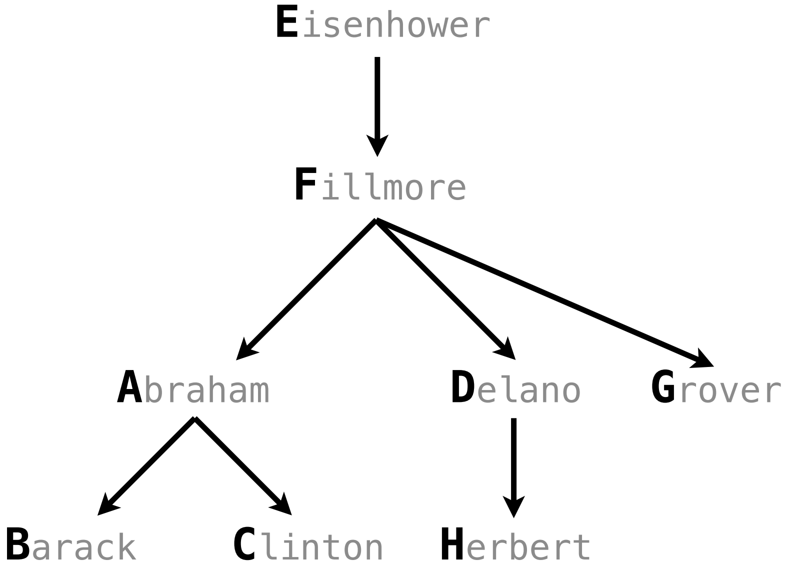

Searching to Satisfy Queries

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logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
a: eisenhower
logic> (fact (parent delano herbert))
```

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logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
```

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```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
```

```
logic> (fact (parent delano herbert))
```

```
logic> (fact (parent fillmore delano))
```

```
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
```

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The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

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logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

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logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
```

```
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

```
(parent delano herbert)     ; (1), a simple fact
```

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logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
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logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

```
(parent delano herbert)      ; (1), a simple fact
(ancestor delano herbert)    ; (2), from (1) and the 1st ancestor fact
```

Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
```

```
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

```
(parent delano herbert)      ; (1), a simple fact
(ancestor delano herbert)    ; (2), from (1) and the 1st ancestor fact
(parent fillmore delano)     ; (3), a simple fact
```

Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
```

```
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

```
(parent delano herbert)     ; (1), a simple fact
(ancestor delano herbert)   ; (2), from (1) and the 1st ancestor fact
(parent fillmore delano)    ; (3), a simple fact
(ancestor fillmore herbert) ; (4), from (2), (3), & the 2nd ancestor fact
```

Hierarchical Facts

Hierarchical Facts

Hierarchical Facts

```
Relations can contain relations in addition to symbols.
```

Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
```

Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
```

Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
```

Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
```

Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
```

Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
```

Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
```

Hierarchical Facts

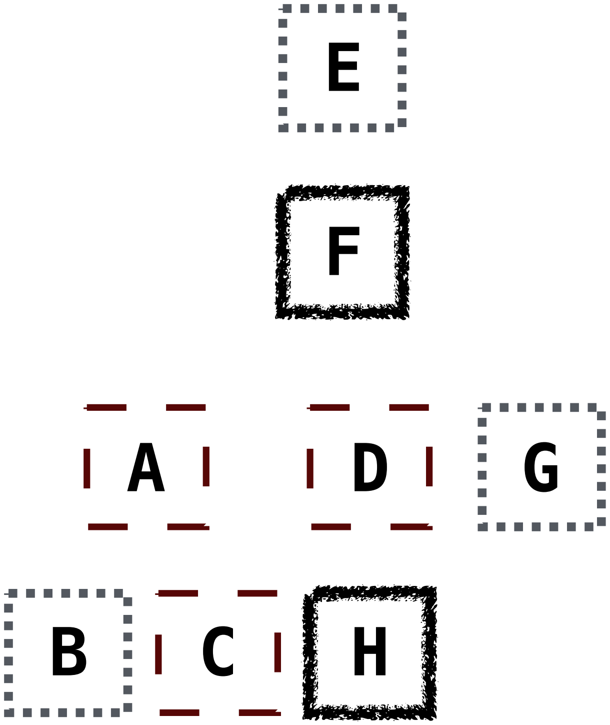
Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Hierarchical Facts

Relations can contain relations in addition to symbols.

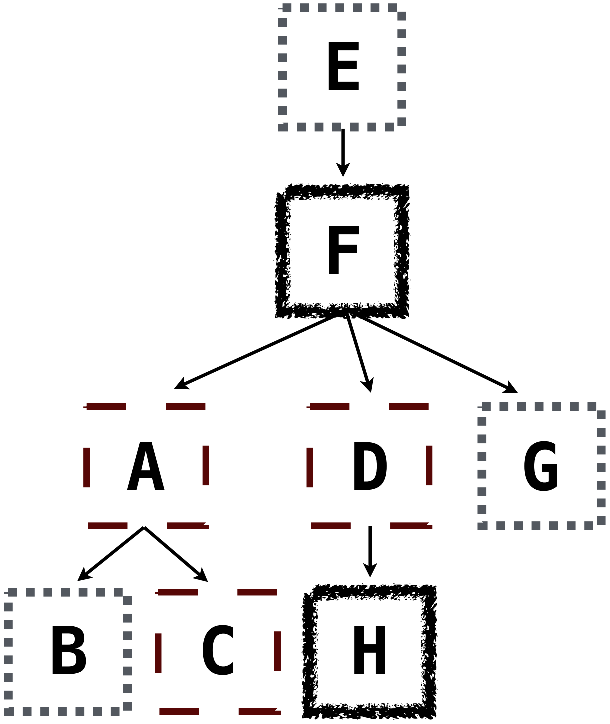
```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```



Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

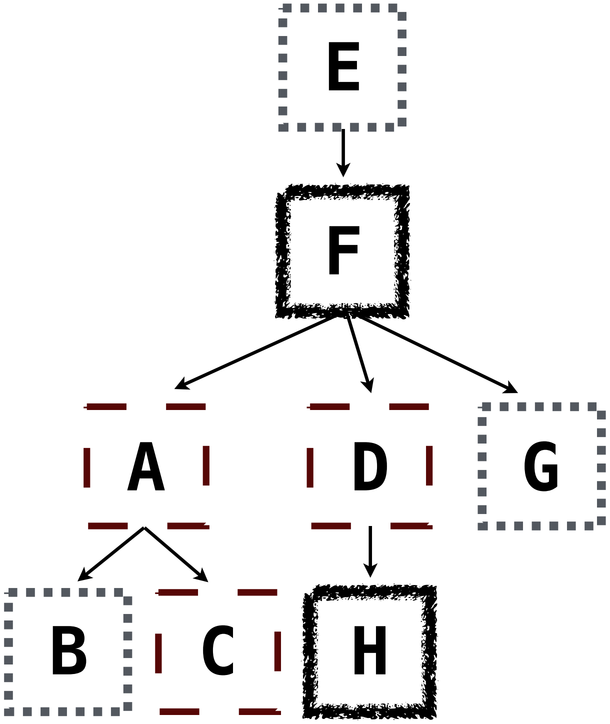


Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.



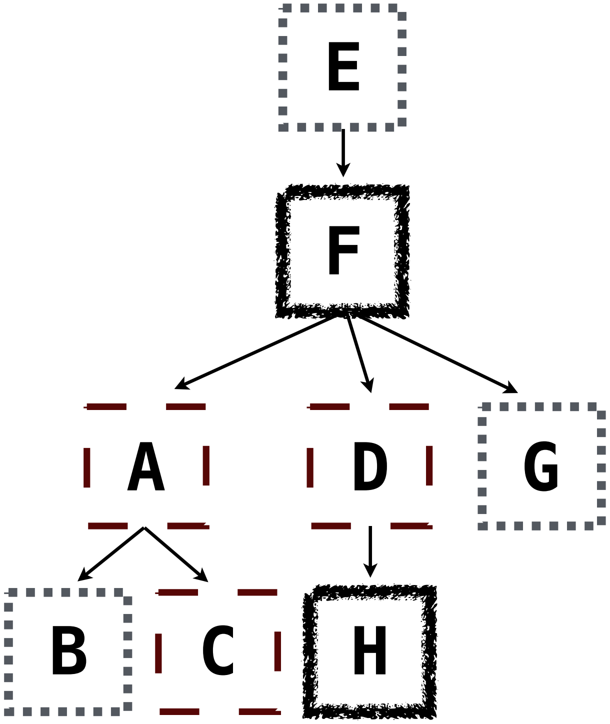
Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
```



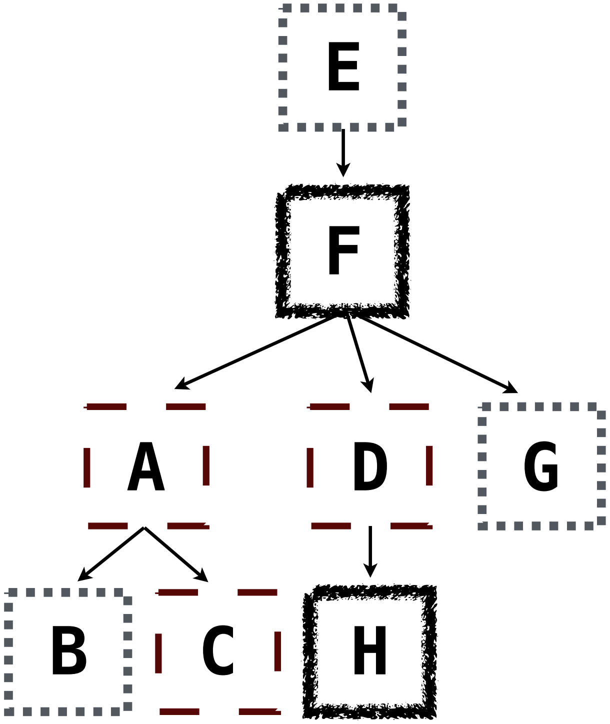
Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
Success!
```



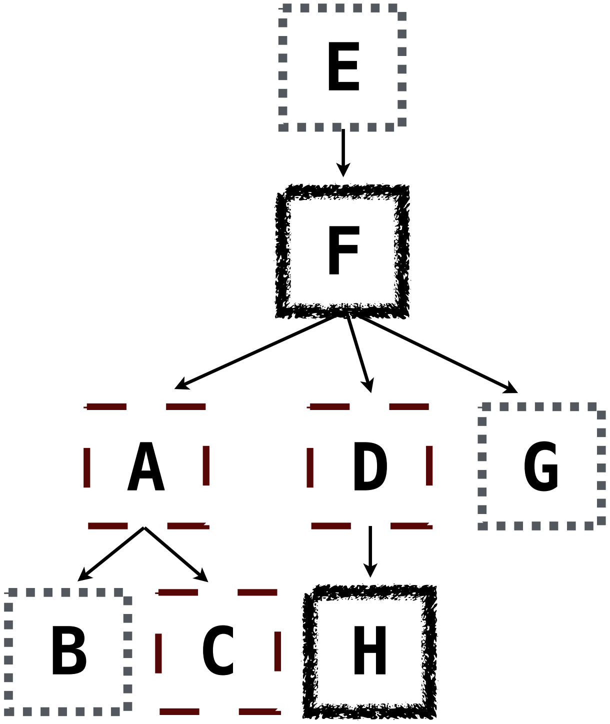
Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.

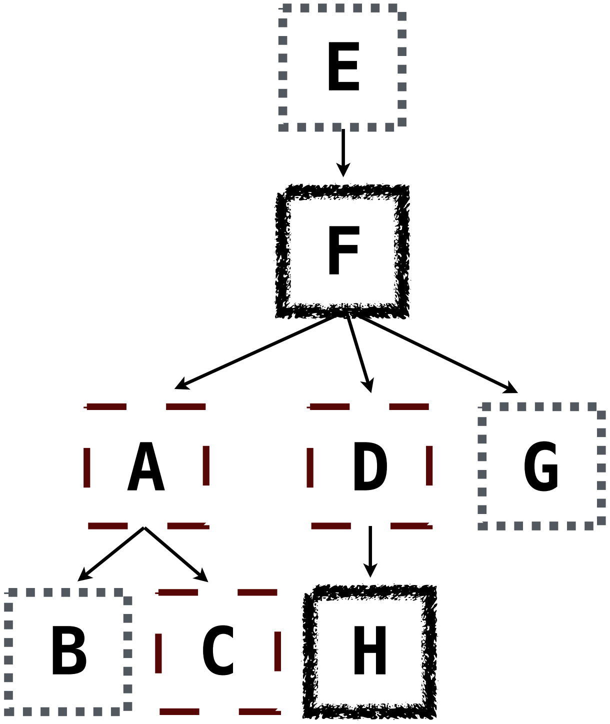
```
logic> (query (dog (name clinton) (fur ?type)))
Success!
type: long
```



Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```



Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
Success!
type: long
```

```
logic> (query (dog (name clinton) ?stats))
```

Hierarchical Facts

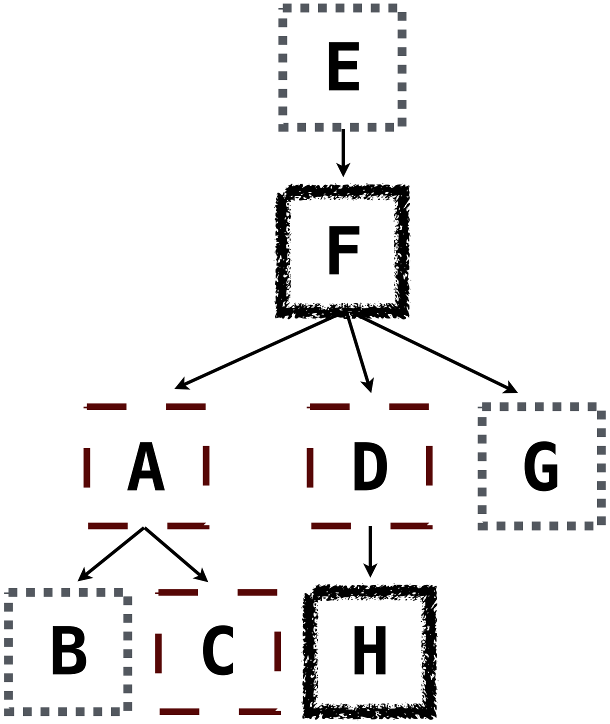
Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

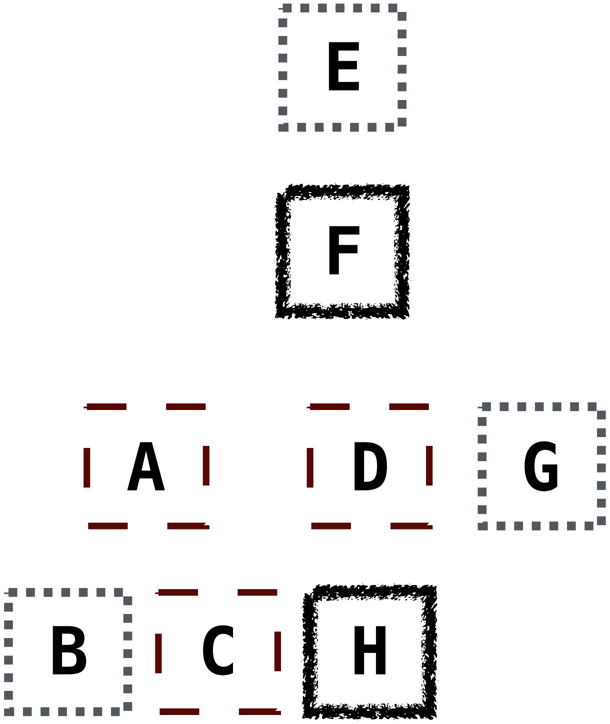
Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
Success!
type: long
```

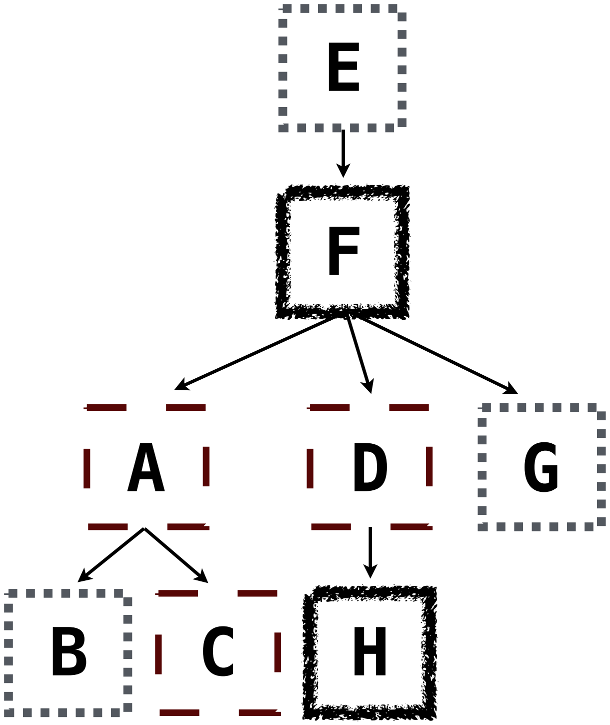
```
logic> (query (dog (name clinton) ?stats))
Success!
stats: (fur long)
```



Combining Multiple Data Sources

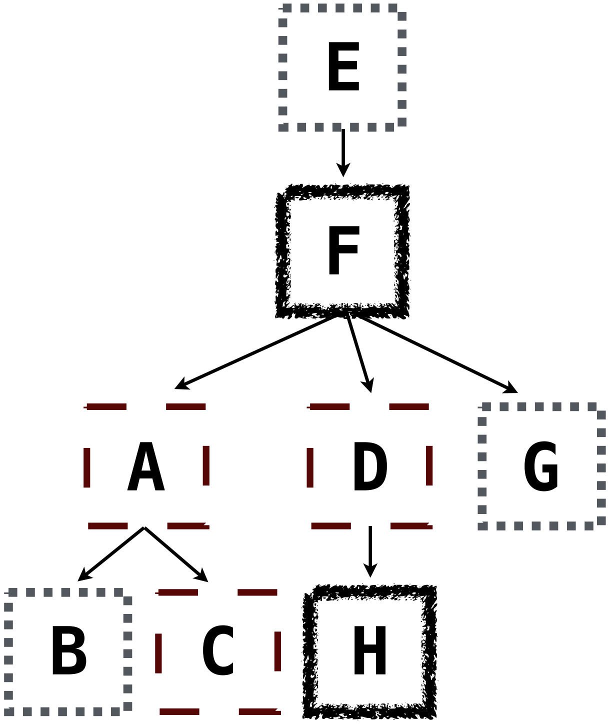


Combining Multiple Data Sources



Combining Multiple Data Sources

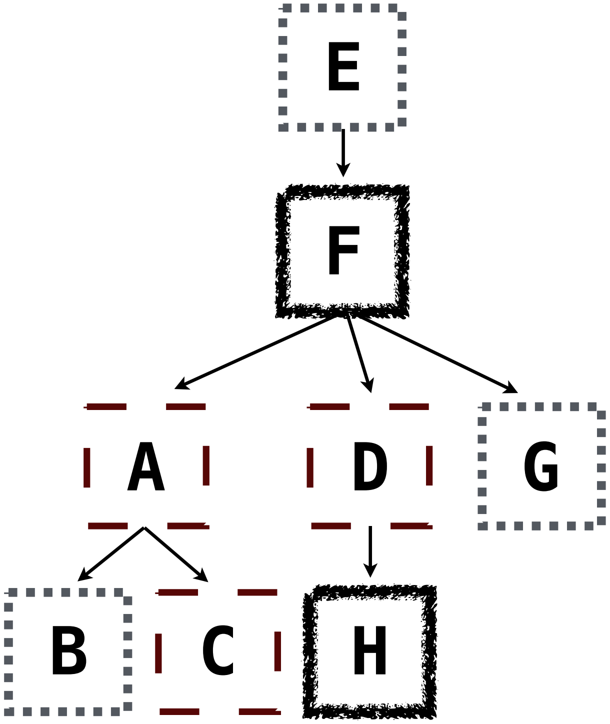
```
Which dogs have an ancestor of the same fur?
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

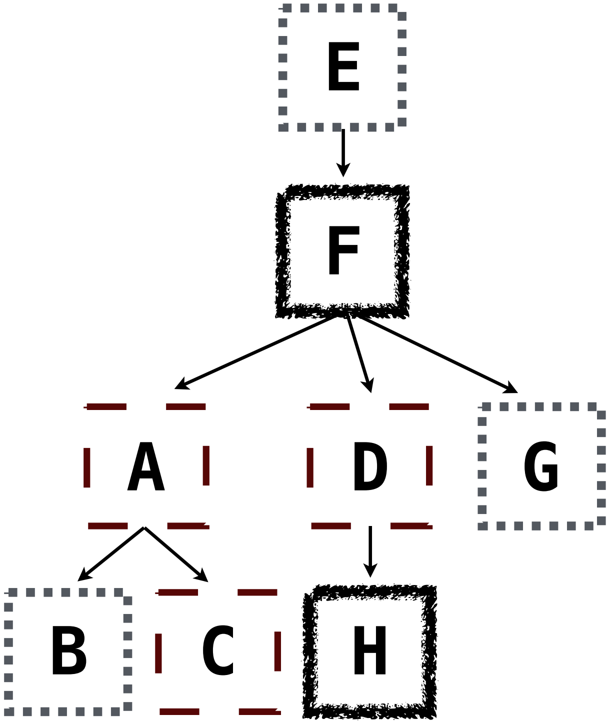
```
logic> (query (dog (name ?x) (fur ?fur))
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

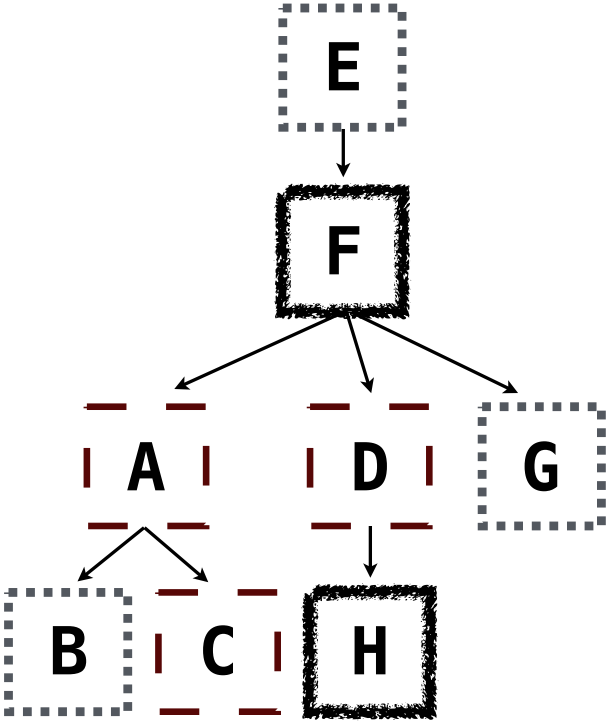
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x))
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

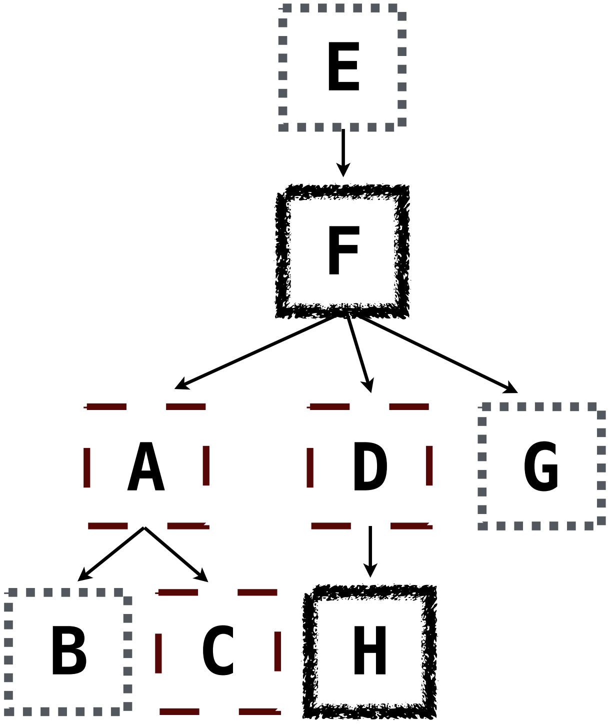
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

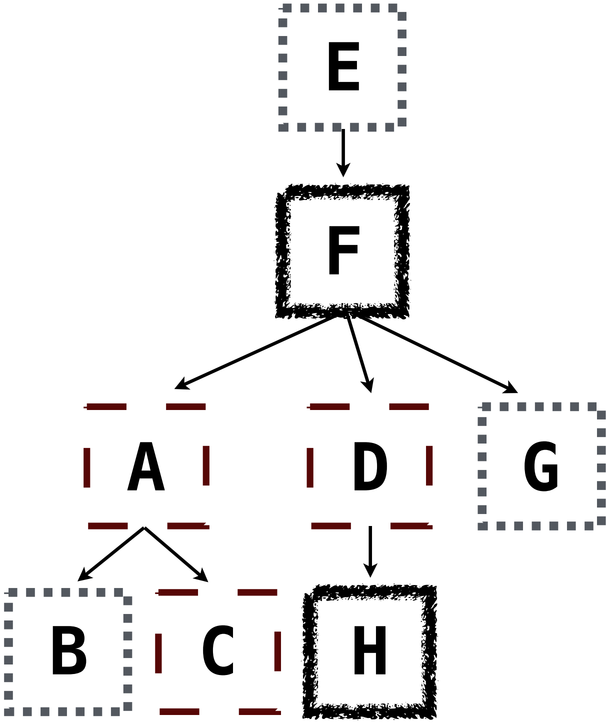
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

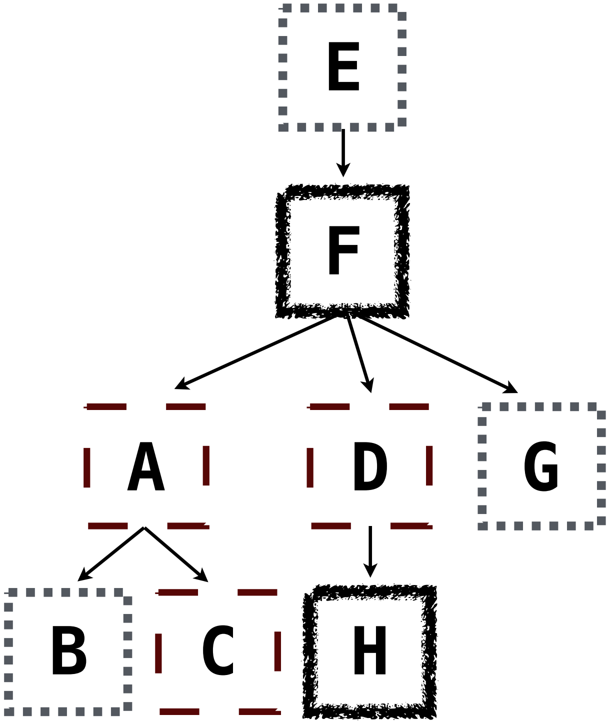
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack     fur: short    y: eisenhower
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

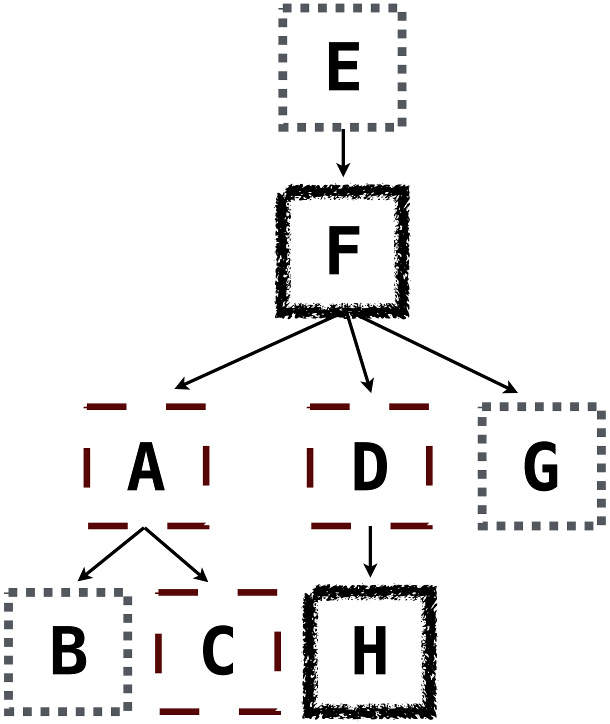
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack    fur: short   y: eisenhower
x: clinton   fur: long    y: abraham
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

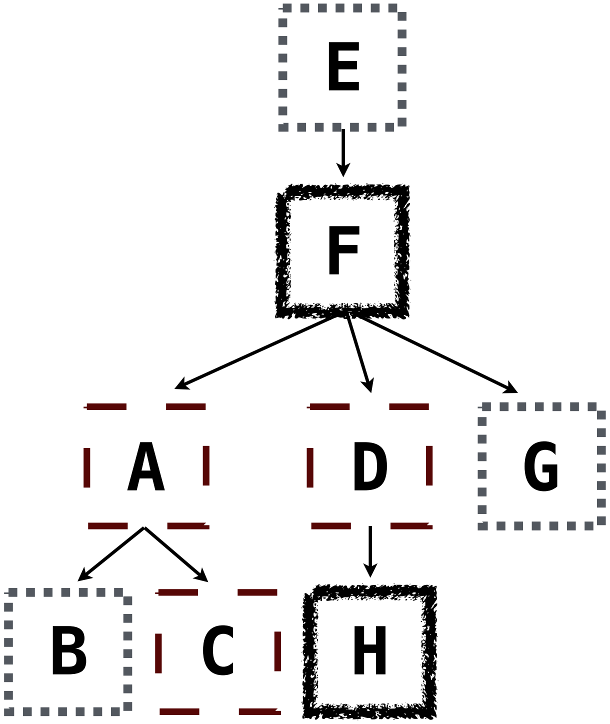
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack     fur: short   y: eisenhower
x: clinton    fur: long    y: abraham
x: grover     fur: short   y: eisenhower
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack     fur: short    y: eisenhower
x: clinton    fur: long     y: abraham
x: grover     fur: short    y: eisenhower
x: herbert    fur: curly    y: fillmore
```



Appending Lists

(Demo)

Lists in Logic

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))


(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

() (c d) => (c d)

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

```
    ?x
() (c d) => (c d)
```

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

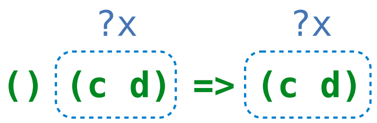
```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)


?x ?x

() (c d) => (c d)

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y        ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

```
      ?x        ?x
() (c d) => (c d)

(b) (c d) => (b c d)
```

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

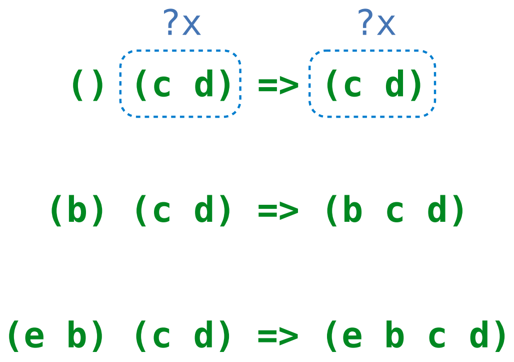
```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y        ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



```
   ?x          ?x
() (c d) => (c d)

(b) (c d) => (b c d)

(e b) (c d) => (e b c d)
```

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

```
    ?x        ?x
() (c d) => (c d)

(b) (c d) => (b c d)

(e b) (c d) => (e b c d)

(e . (b)) (c d) => (e . (b c d))
```

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

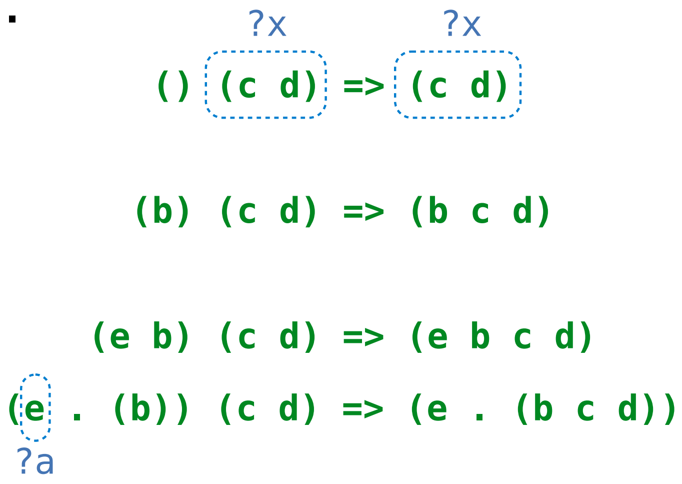
```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



?x ?x

```
() (c d) => (c d)

(b) (c d) => (b c d)

(e b) (c d) => (e b c d)

(e . (b)) (c d) => (e . (b c d))
```

?a

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

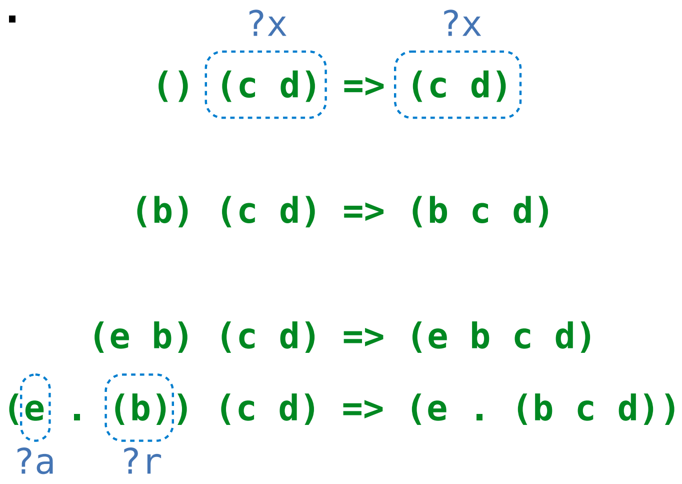
```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y        ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



```
        ?x         ?x
   () (c d) => (c d)

  (b) (c d) => (b c d)

(e b) (c d) => (e b c d)
(e . (b)) (c d) => (e . (b c d))
 ?a   ?r
```

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Simple fact: Conclusion

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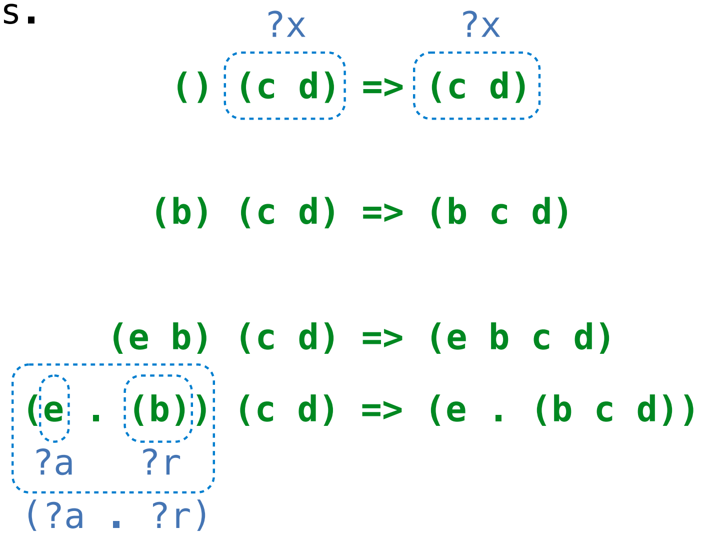
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



```
       ?x         ?x
    () (c d) => (c d)

   (b) (c d) => (b c d)

 (e b) (c d) => (e b c d)
(e . (b)) (c d) => (e . (b c d))
 ?a   ?r
(?a . ?r)
```

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Expressions begin with *query* or *fact* followed by relations.

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(fact (app () ?x ?x))
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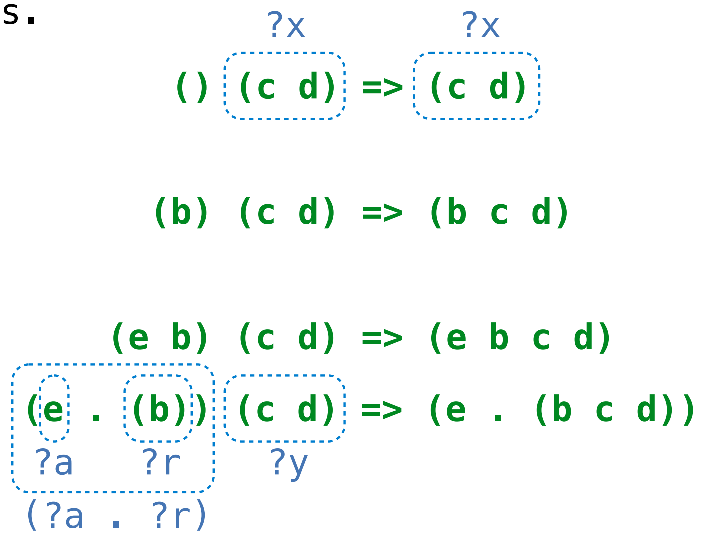
```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



```
    ?x       ?x
() (c d) => (c d)

(b) (c d) => (b c d)

(e b) (c d) => (e b c d)
(e . (b)) (c d) => (e . (b c d))
 ?a   ?r   ?y
(?a . ?r)
```

Lists in Logic

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      (app       ?r  ?y       ?z ))
```
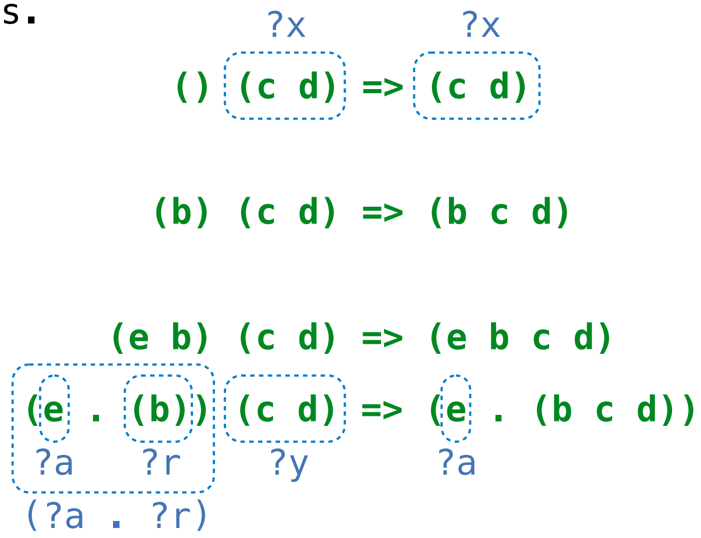

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



```
     ?x      ?x
()  (c d) => (c d)

(b) (c d) => (b c d)

(e b) (c d) => (e b c d)
(e . (b)) (c d) => (e . (b c d))
 ?a  ?r    ?y      ?a
(?a . ?r)
```

Lists in Logic

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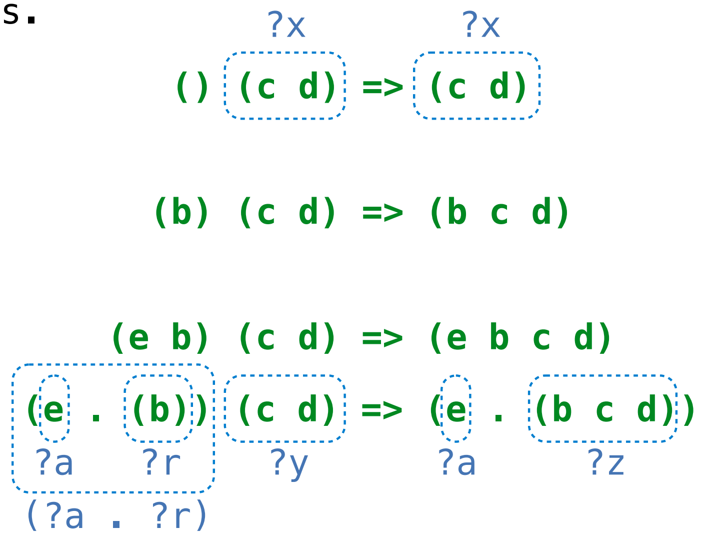
Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



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```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

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(query (app ?left (c d) (e b c d)))
Success!
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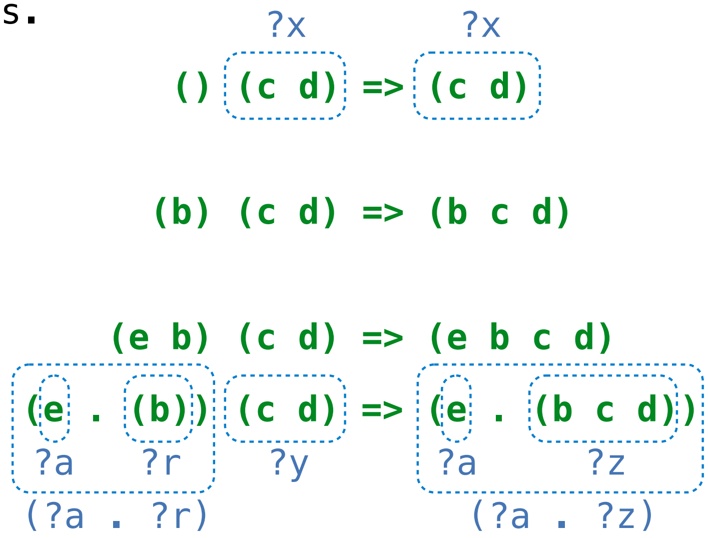
```

What ?left can append with (c d) to create (e b c d)



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```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

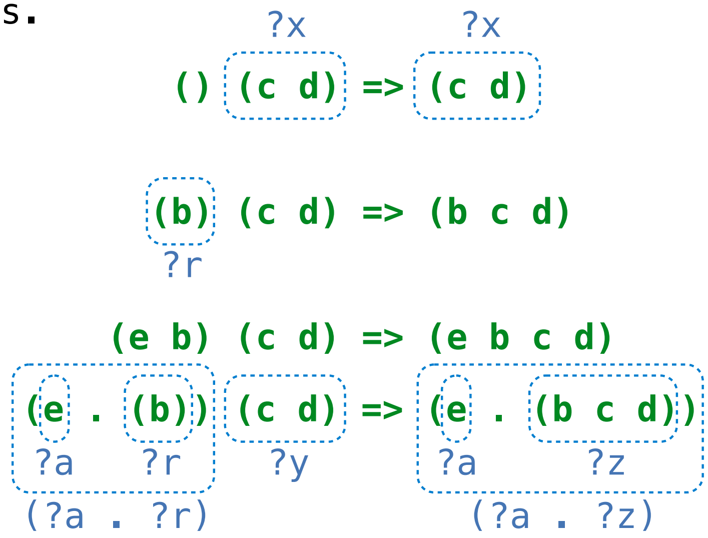
```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y        ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

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```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

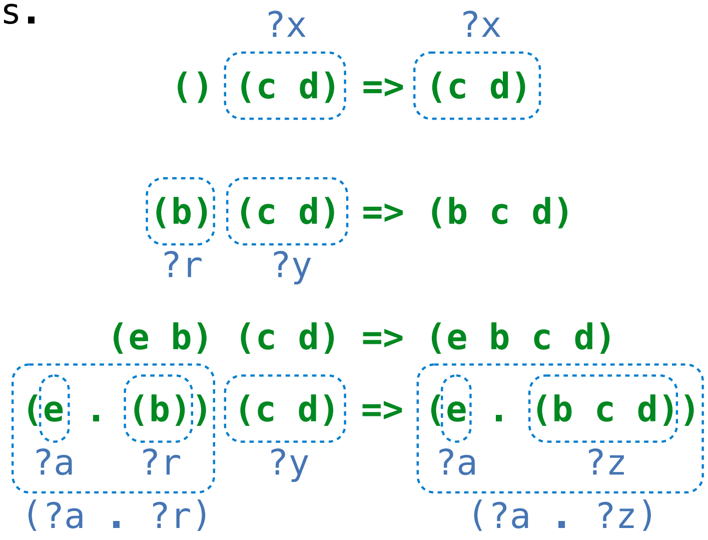
```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



Lists in Logic

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```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

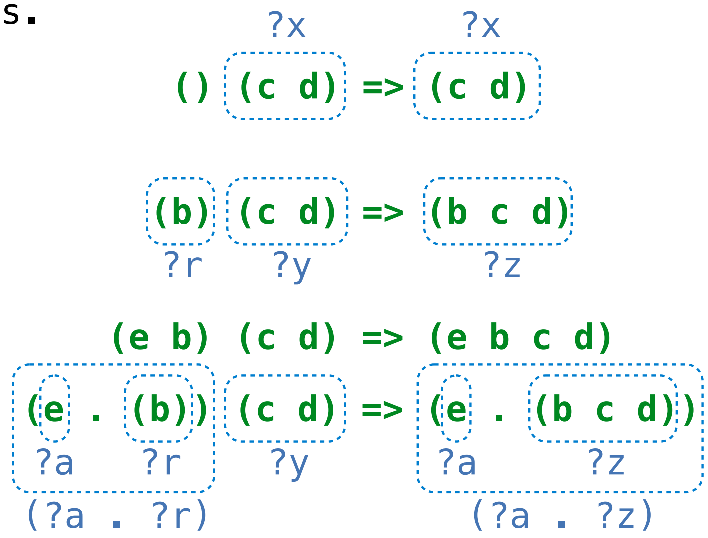
```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



Lists in Logic

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```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
(fact (app (?a . ?r) ?y (?a . ?z))
      (app       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

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left: (e b)
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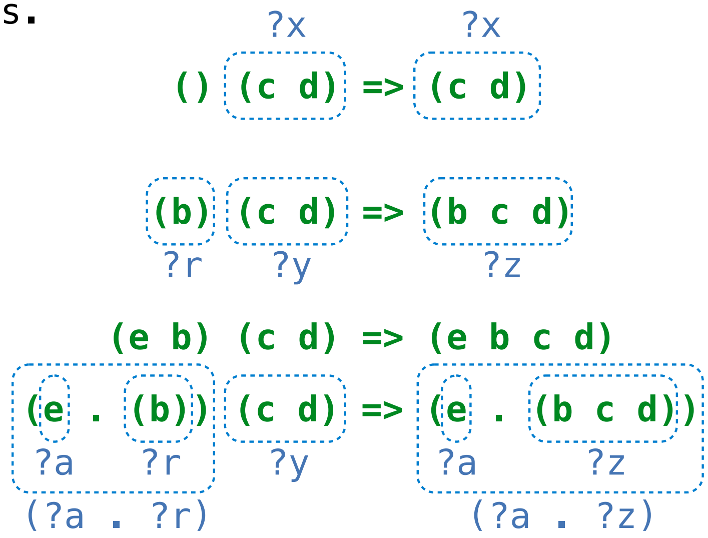
What ?left can append with (c d) to create (e b c d)



?x ?x

() (c d) => (c d)

(b) (c d) => (b c d)

?r ?y ?z

(e b) (c d) => (e b c d)

(e . (b)) (c d) => (e . (b c d))

?a ?r ?y ?a ?z

(?a . ?r) (?a . ?z)

The interpreter lists all bindings that it can find to satisfy the query.

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (app () ?x ?x))
```

Simple fact: Conclusion

```
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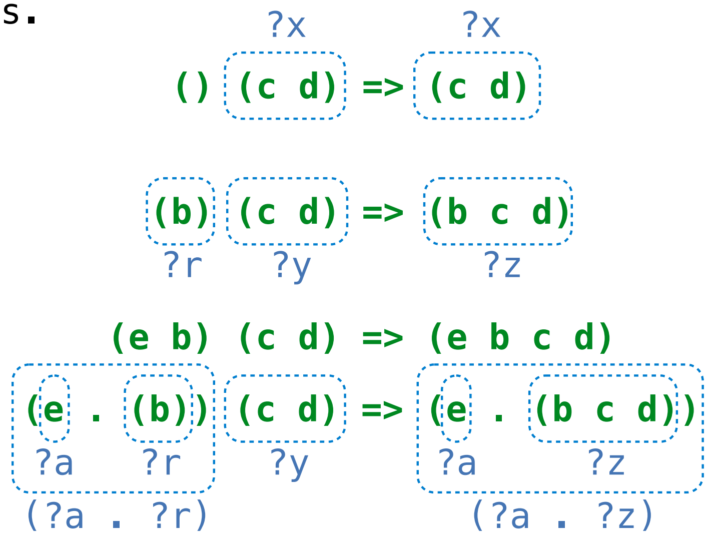
```

Conclusion

Hypothesis

```
(query (app ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



```
   ?x       ?x
() (c d) => (c d)

(b) (c d) => (b c d)
?r   ?y       ?z

(e b) (c d) => (e b c d)
(e . (b)) (c d) => (e . (b c d))
 ?a   ?r    ?y      ?a    ?z
(?a . ?r)          (?a . ?z)
```

The interpreter lists all bindings that it can find to satisfy the query.

(Demo)

Unification

Pattern Matching

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

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Unification is finding an assignment to variables that makes two relations the same.

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( (a  b) c  (a  b) )
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```
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(   ?x   c    ?x   )
```

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The basic operation of the Logic interpreter is to attempt to *unify* two relations.

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```
( (a  b) c  (a  b) )
                        True, {x: (a b)}
(   ?x   c    ?x    )
```

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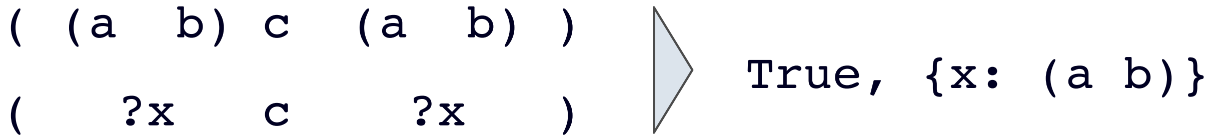

```
( (a  b) c  (a  b) )
                         True, {x: (a b)}
(   ?x   c    ?x   )

( (a  b) c  (a  b) )
```

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The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

```
( (a   b) c   (a   b) )
                            ▷   True, {x: (a b)}
(    ?x   c     ?x     )
```

```
( (a   b) c   (a   b) )

( (a ?y) ?z  (a   b) )
```

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

```
( (a  b) c  (a  b) )
                       ▷   True, {x: (a b)}
(   ?x   c     ?x   )

( (a  b) c  (a  b) )
                       ▷   True, {y: b, z: c}
( (a ?y) ?z (a  b) )
```

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Unification is finding an assignment to variables that makes two relations the same.

```
( (a  b) c  (a  b) )
                       ▷   True, {x: (a b)}
(    ?x   c    ?x    )

( (a  b) c  (a  b) )
                       ▷   True, {y: b, z: c}
( (a ?y) ?z (a  b) )

( (a  b) c  (a  b) )

(    ?x  ?x    ?x    )
```

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```
( (a  b) c  (a  b) )
                         True, {x: (a b)}
(   ?x   c    ?x   )

( (a  b) c  (a  b) )
                         True, {y: b, z: c}
( (a ?y) ?z (a  b) )

( (a  b) c  (a  b) )
                         False
(   ?x  ?x    ?x   )
```

Unification

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

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1. Look up variables in the current environment.

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1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

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```
( (a  b) c (a  b) )

(   ?x   c   ?x   )



  {            }
```

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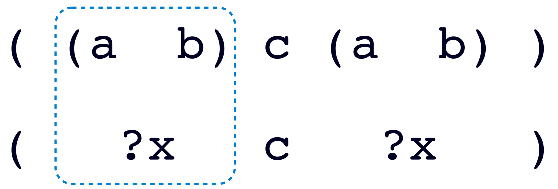

```
( (a   b) c (a   b) )

(   ?x    c    ?x    )
```

```
{                }
```

Unification

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```

```
{  x: (a b)  }
```

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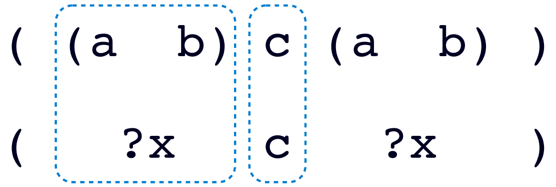

```
( (a   b) c (a   b) )
(   ?x    c    ?x    )
```

```
{  x: (a b)  }
```

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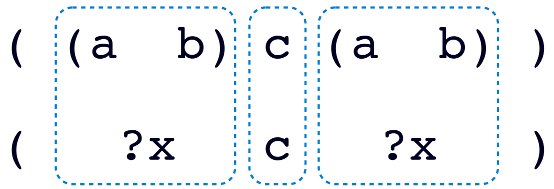

```
( (a   b) c (a   b) )
(    ?x   c    ?x   )
```

```
{  x: (a b)  }
```

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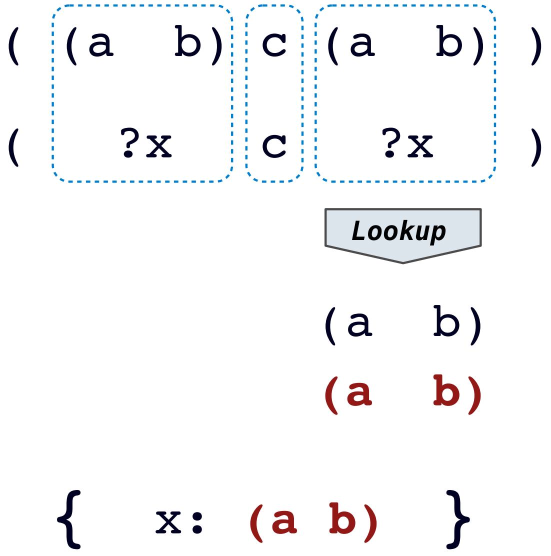

```
( (a   b) c (a   b) )

(   ?x    c   ?x    )
```

Lookup

```
(a   b)
(a   b)
```

```
{  x: (a b)  }
```

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

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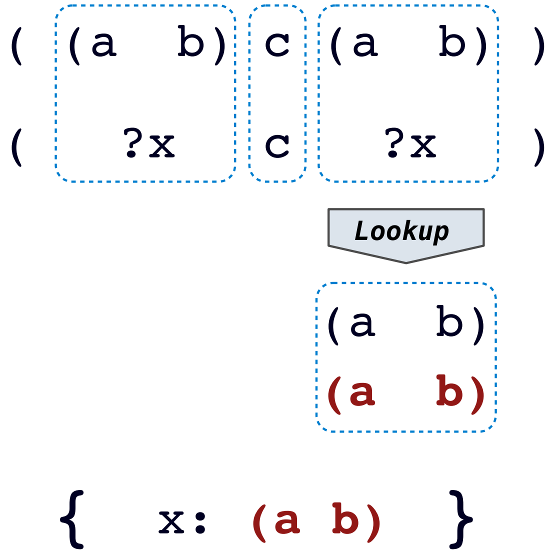
2. Establish new bindings to unify elements.



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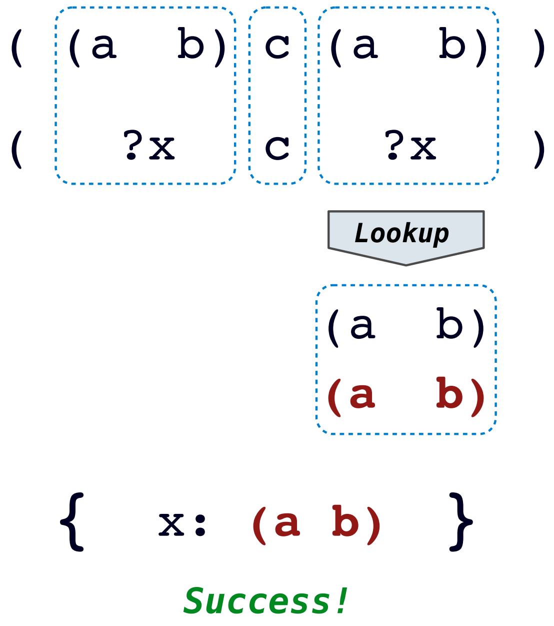

```
( (a   b) c (a   b) )

(   ?x    c   ?x    )
```

Lookup

```
(a   b)
(a   b)
```

{ x: (a b) }

Success!

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
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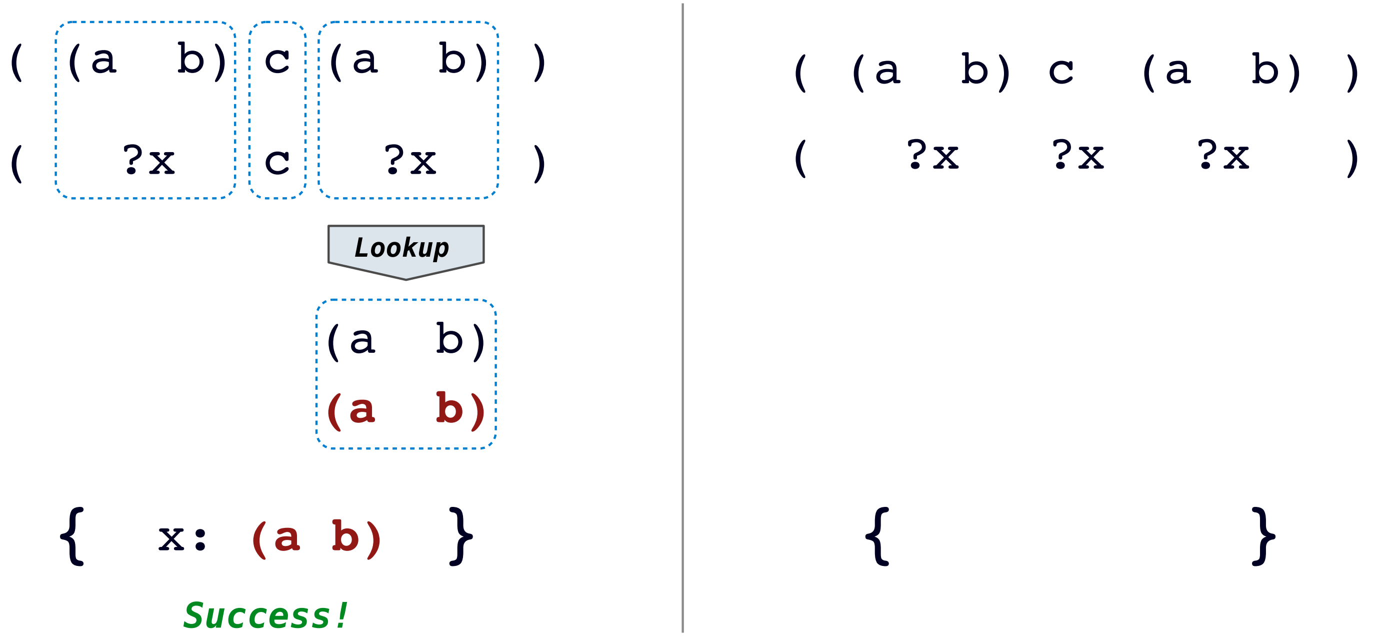

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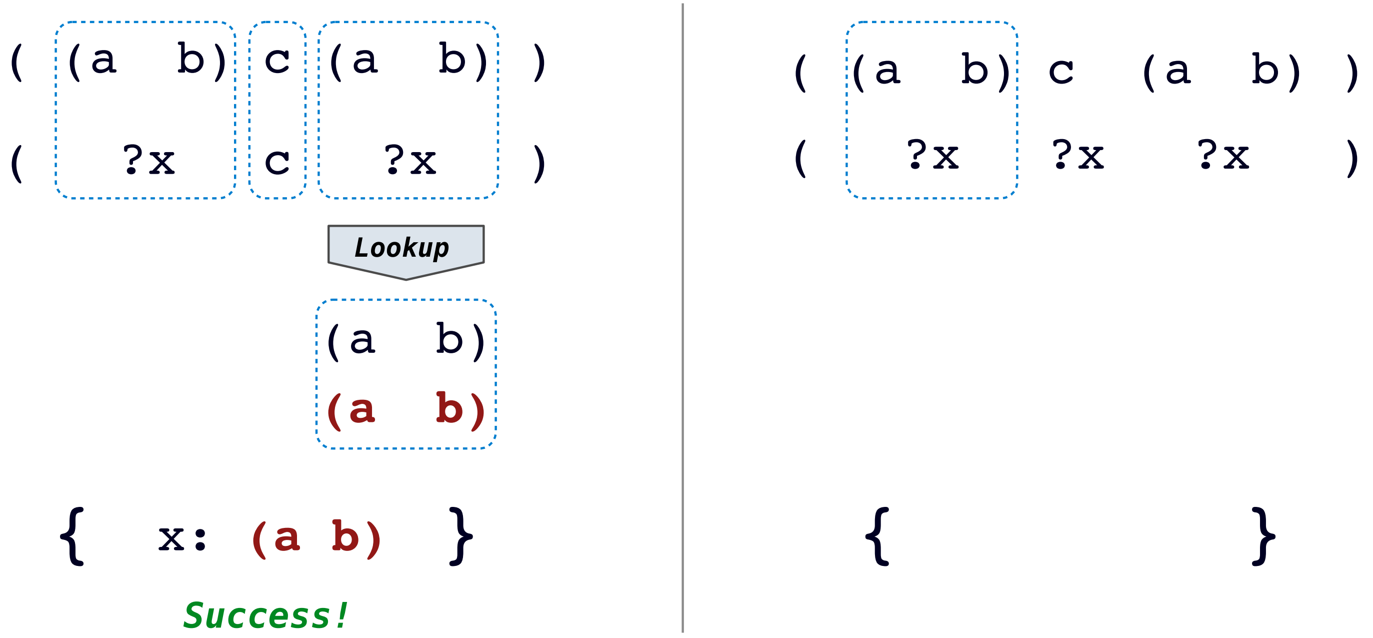

```
( (a   b) c (a   b) )
(   ?x    c   ?x    )
```

Lookup

```
(a   b)
(a   b)
```

```
{  x: (a b)  }
```

Success!

```
( (a   b) c  (a   b) )
(   ?x    ?x   ?x    )
```

```
{           }
```

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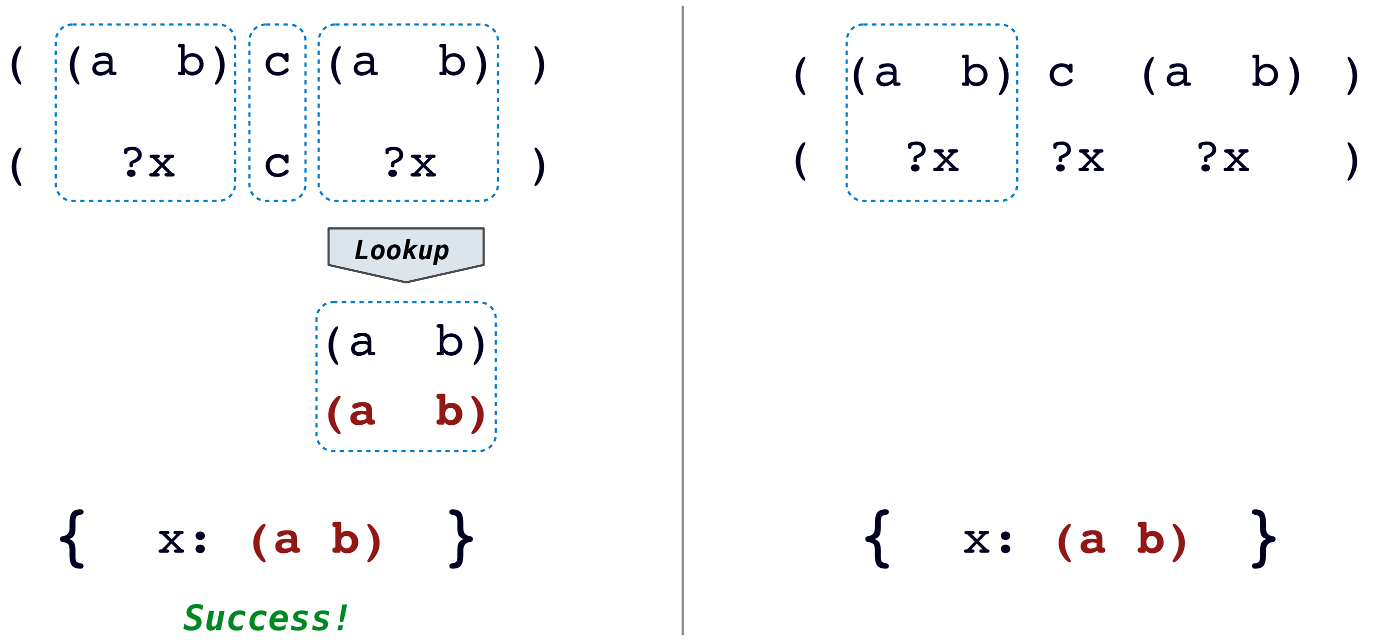

```
( (a   b)  c  (a   b) )
(   ?x     c    ?x    )
```

Lookup

```
(a   b)
(a   b)
```

```
{  x: (a b)  }
```

Success!

```
( (a   b)  c   (a   b) )
(   ?x    ?x     ?x    )
```

```
{  x: (a b)  }
```

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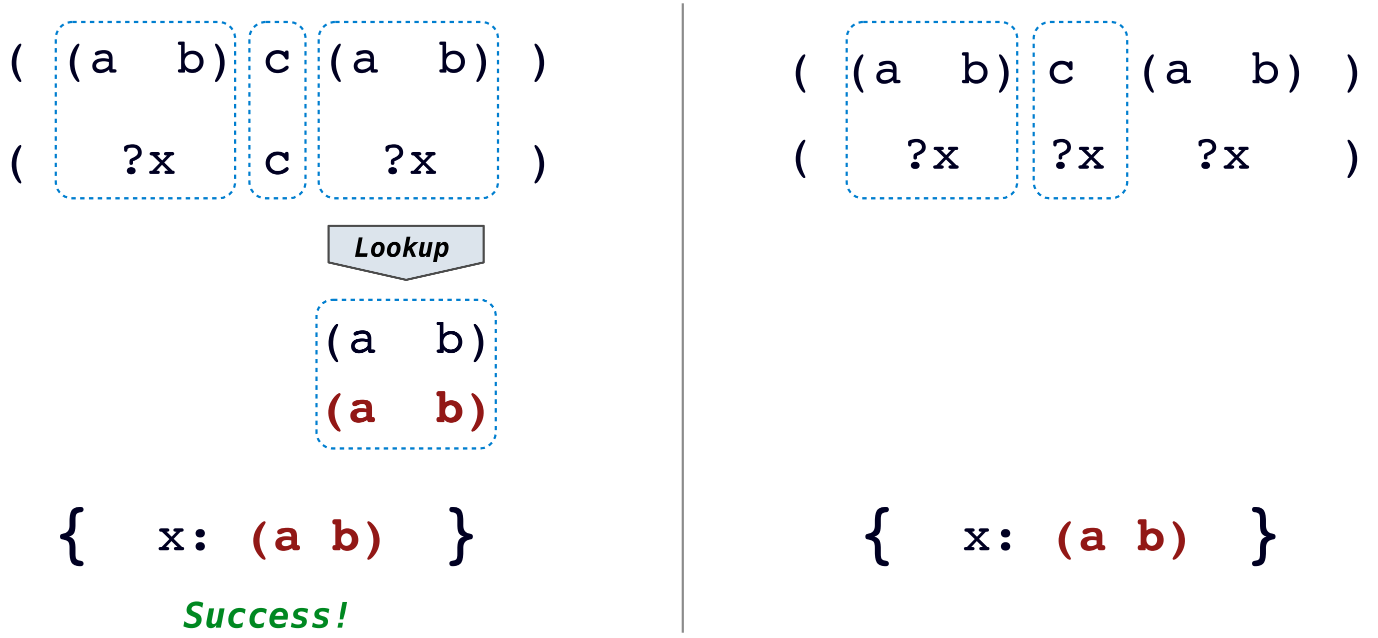

```
( (a  b) c (a  b) )
(   ?x   c   ?x   )
```

Lookup

```
(a  b)
(a  b)
```

{ x: (a b) }

Success!

```
( (a  b) c  (a  b) )
(   ?x   ?x   ?x   )
```

{ x: (a b) }

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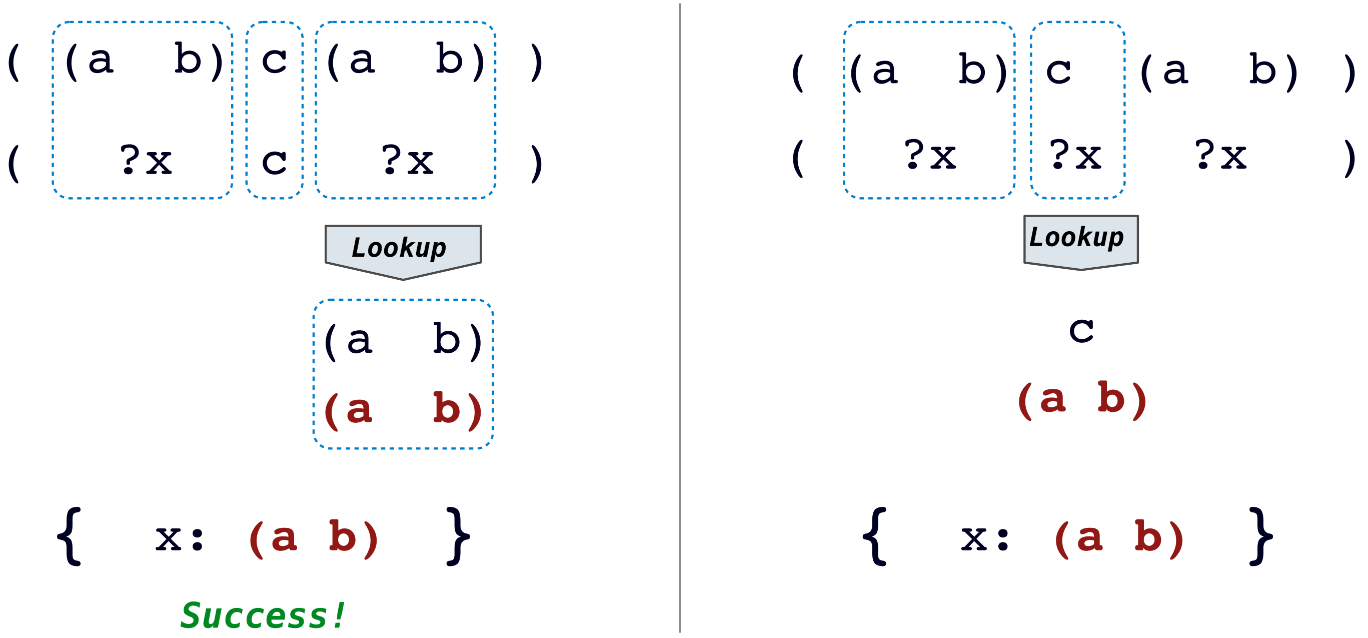

((a b) c (a b))

(?x c ?x)

Lookup

(a b)

(a b)

{ x: **(a b)** }

Success!

((a b) c (a b))

(?x ?x ?x)

Lookup

c

(a b)

{ x: **(a b)** }

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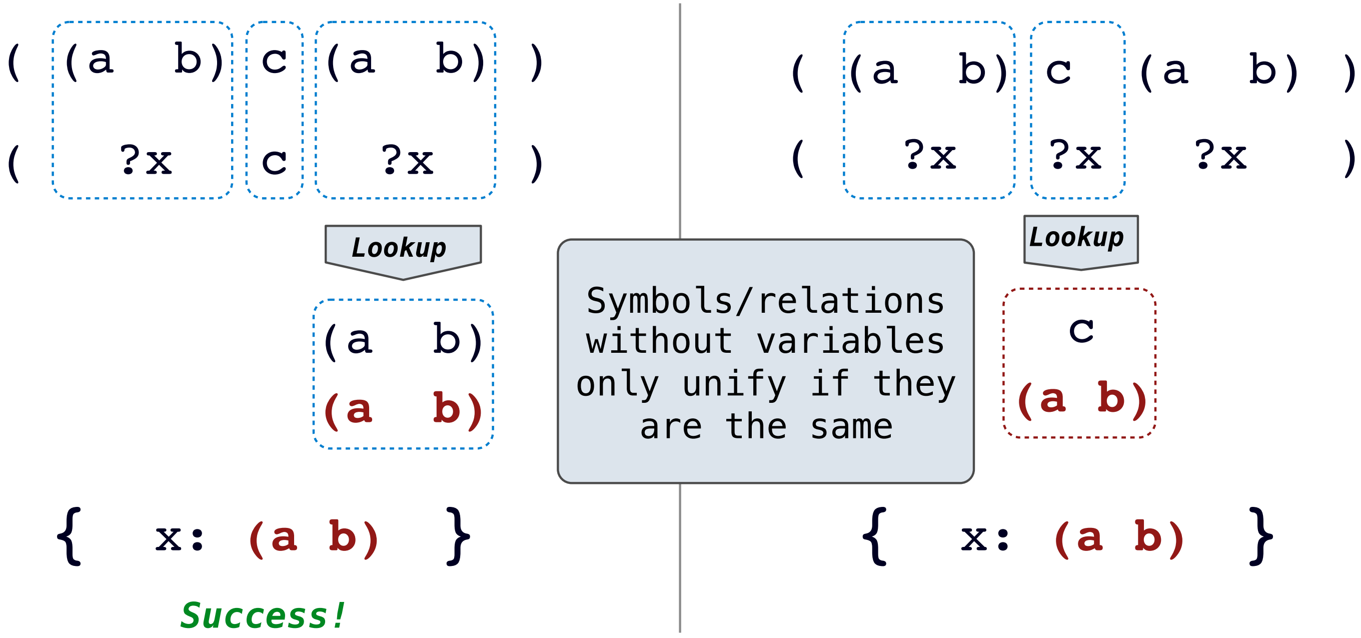
1. Look up variables in the current environment.
2. Establish new bindings to unify elements.



Symbols/relations without variables only unify if they are the same

Unification

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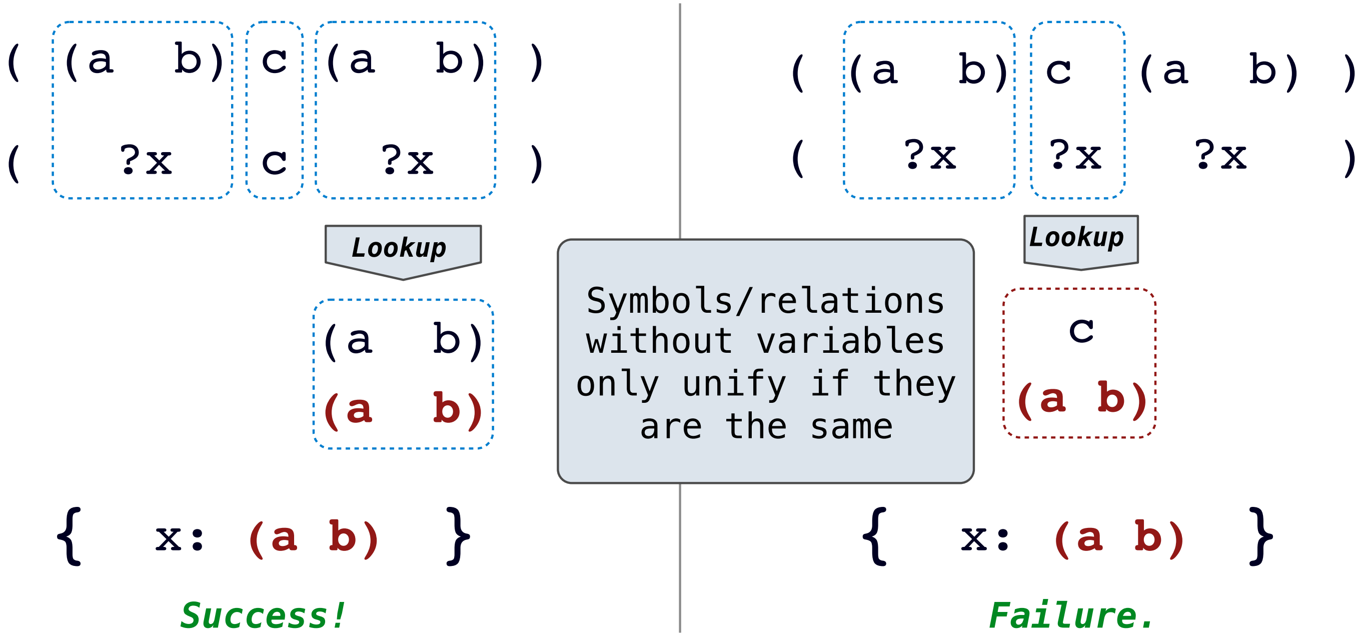
2. Establish new bindings to unify elements.



```
( (a   b) c (a   b) )
(   ?x    c   ?x    )
```

Lookup

```
(a   b)
(a   b)
```

{ x: (a b) }

Success!

Symbols/relations without variables only unify if they are the same

```
( (a   b) c  (a   b) )
(   ?x    ?x   ?x    )
```

Lookup

```
  c
(a b)
```

{ x: (a b) }

Failure.

Unifying Variables

Unifying Variables

Two relations that contain variables can be unified as well.

Unifying Variables

Two relations that contain variables can be unified as well.

```
(   ?x       ?x   )

((a ?y c) (a b ?z))
```

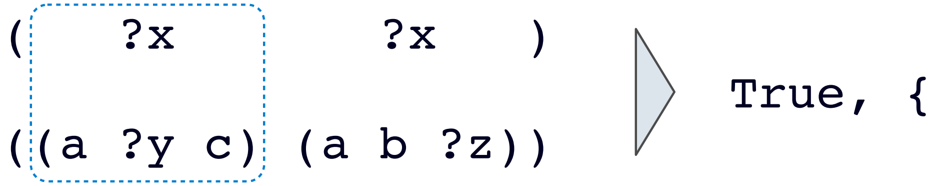
Unifying Variables

Two relations that contain variables can be unified as well.

```
(    ?x         ?x    )
                            True, {
((a ?y c) (a b ?z))
```

Unifying Variables

Two relations that contain variables can be unified as well.



```
(    ?x          ?x     )
((a ?y c)  (a b ?z))
```

True, {

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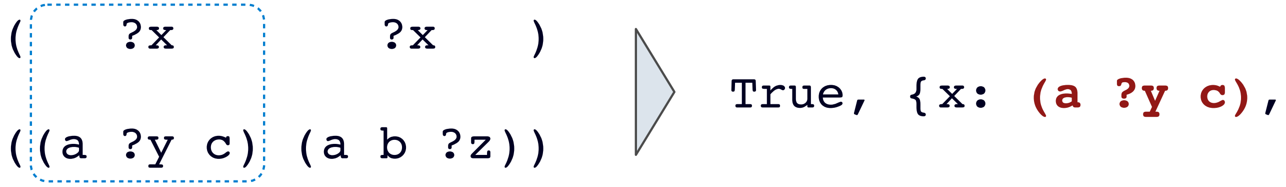

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(    ?x          ?x     )
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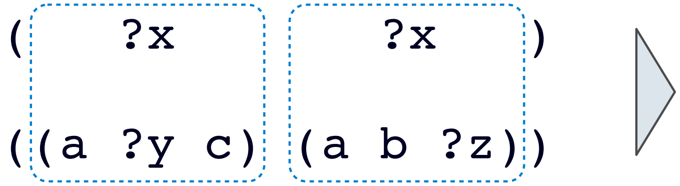

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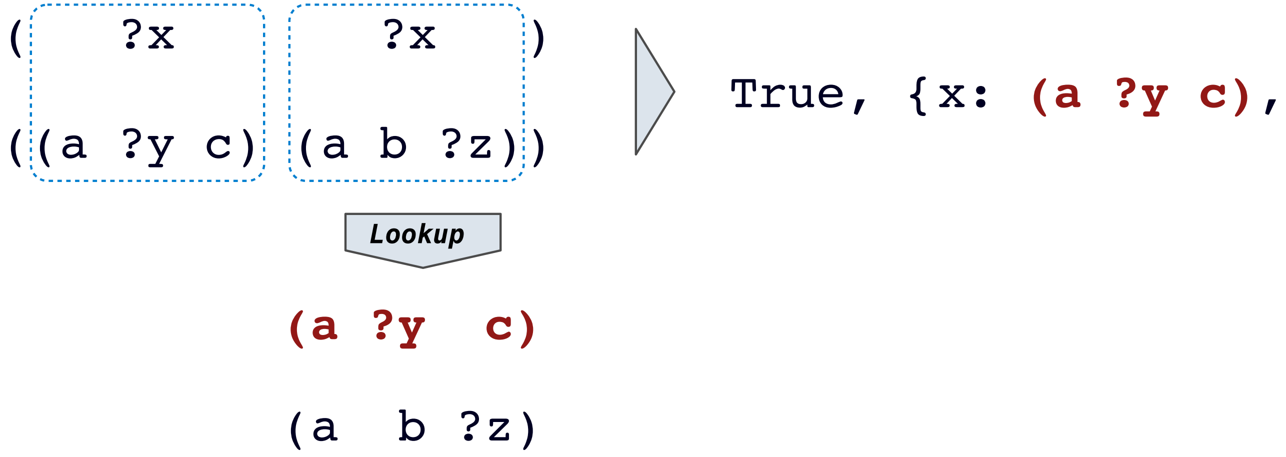

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(    ?x          ?x     )

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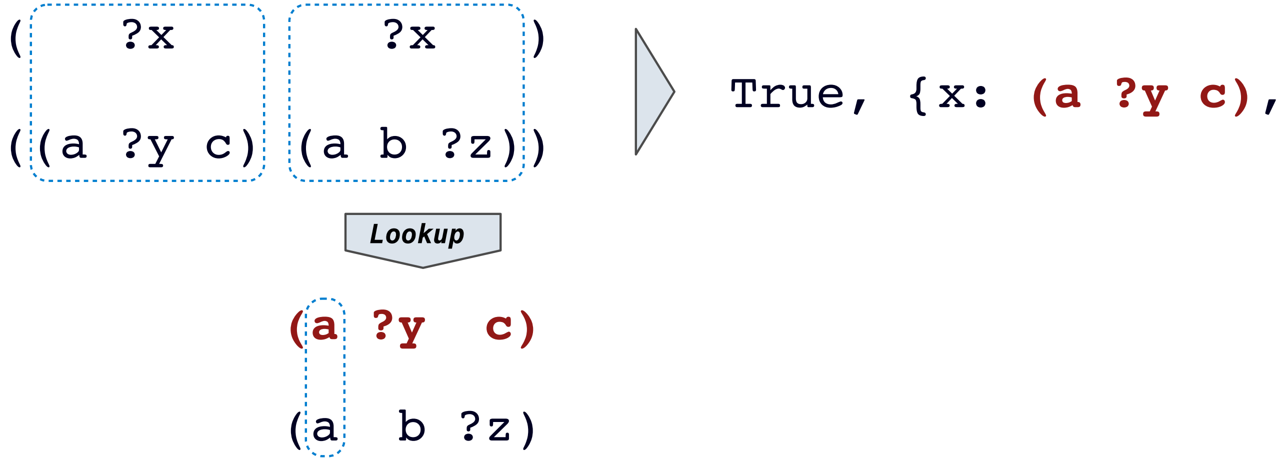

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( ?x        ?x       )
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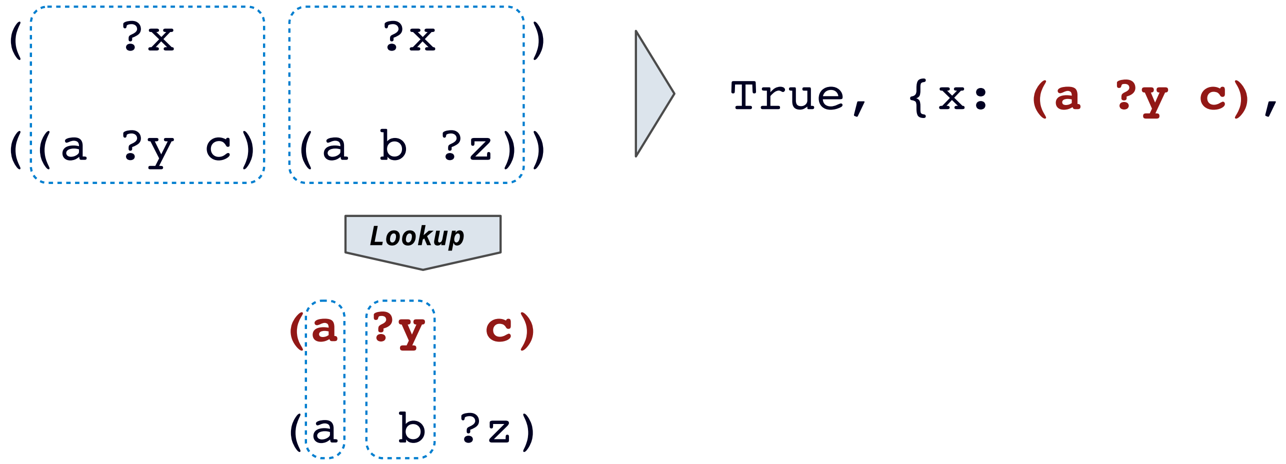

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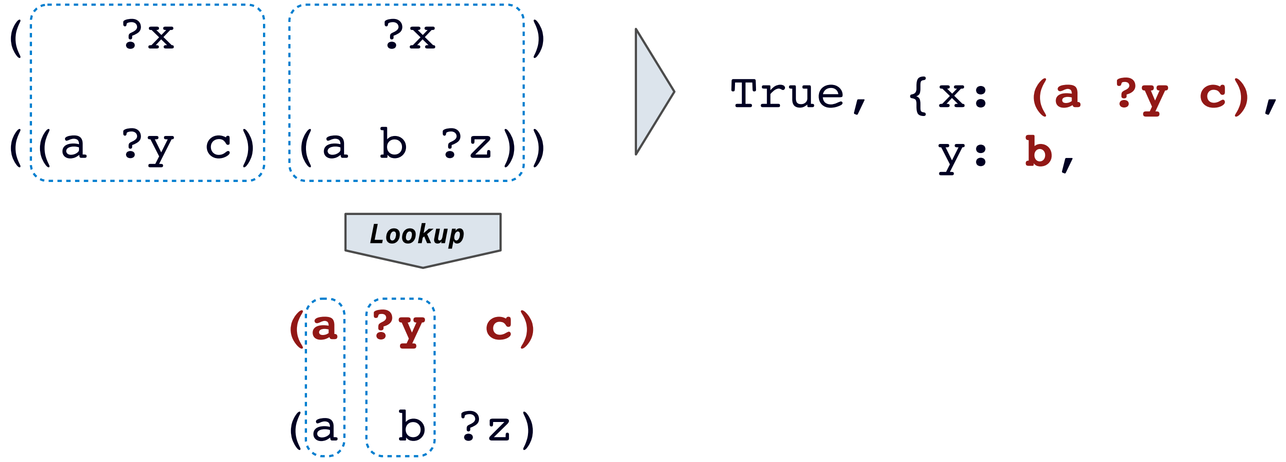

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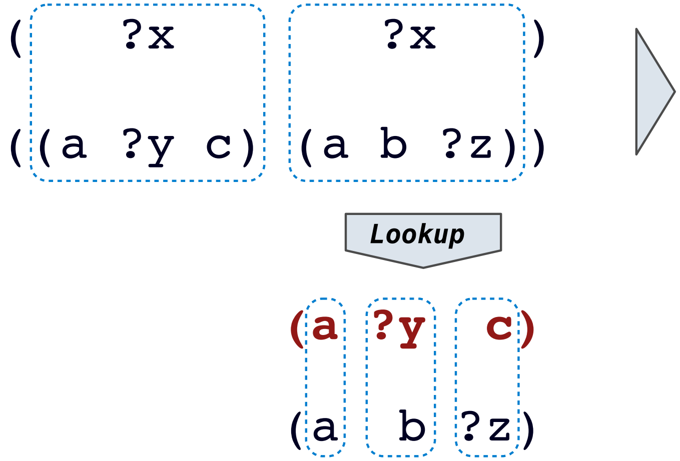

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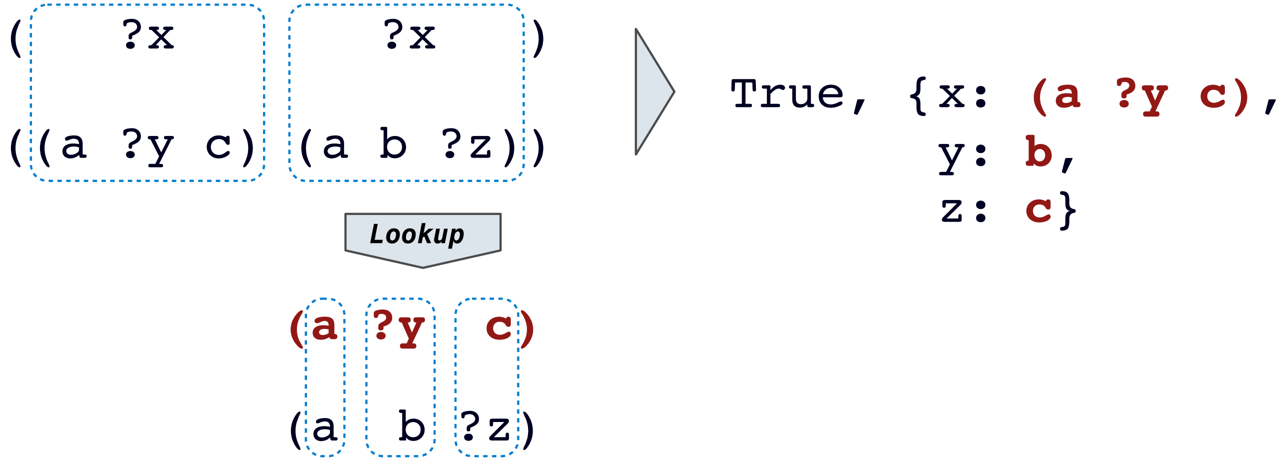

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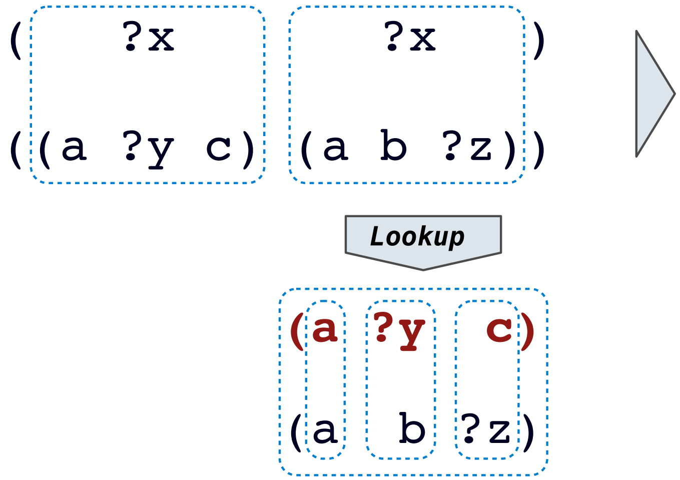

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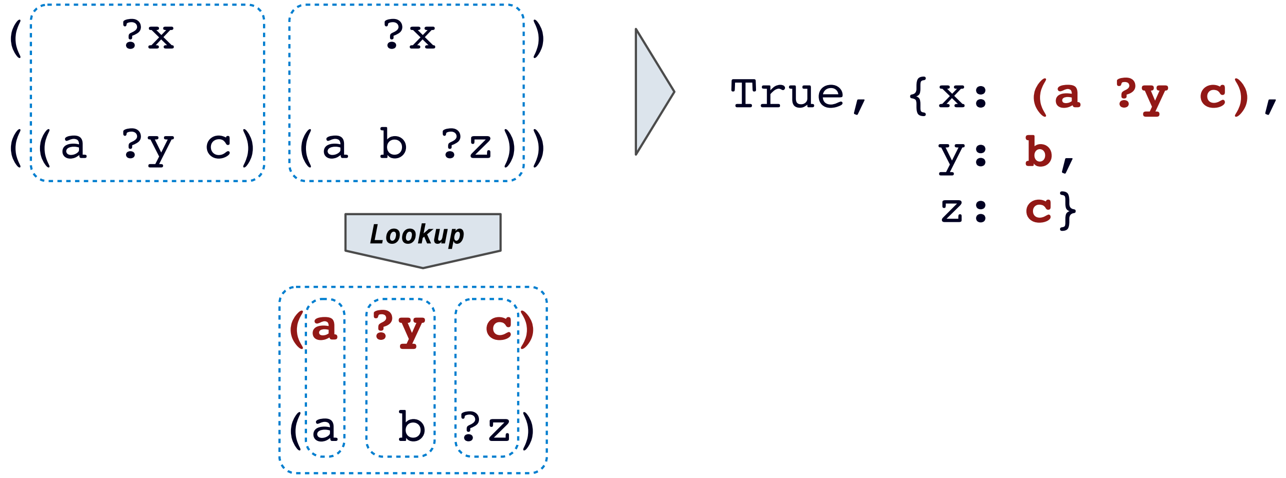

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Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

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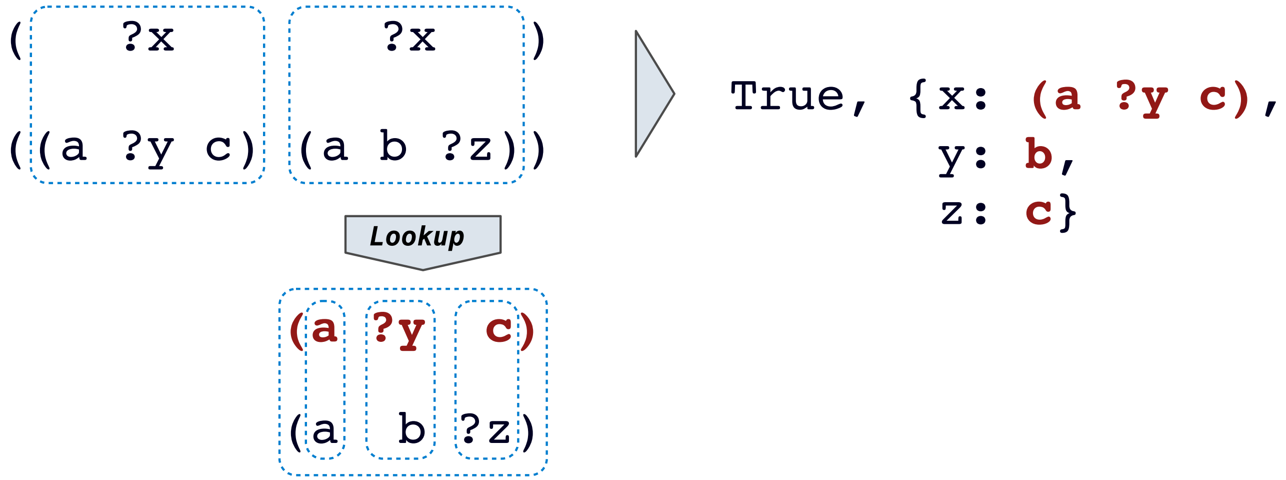

```
(   ?x          ?x     )
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```

Lookup

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(a ?y c)
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`lookup('?x')`

Unifying Variables

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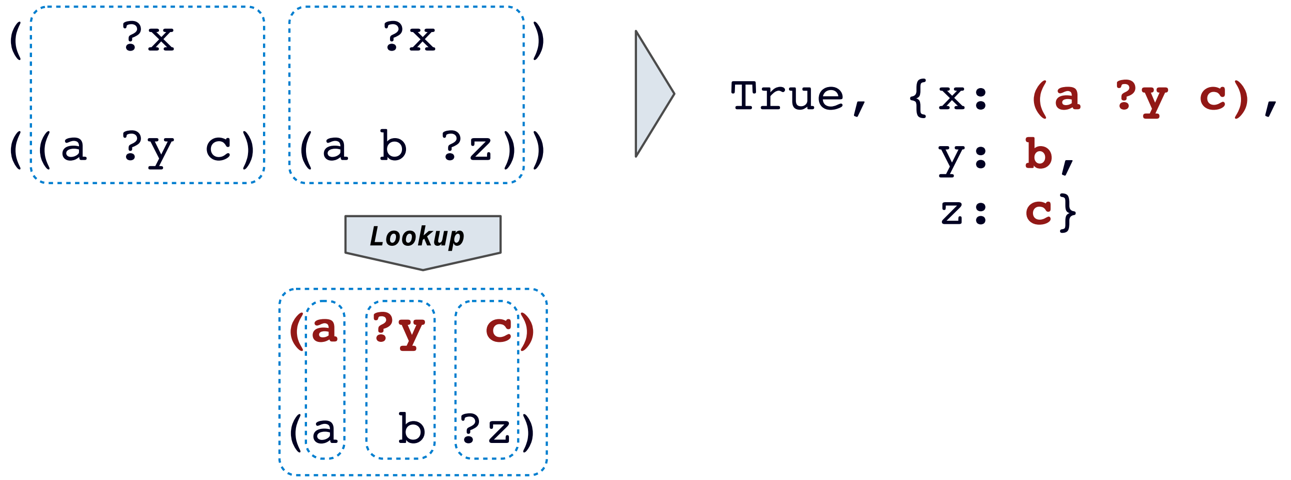

```
(   ?x          ?x    )

((a ?y c)  (a b ?z))
```

Lookup

```
(a ?y c)

(a  b ?z)
```

```
True, {x: (a ?y c),
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lookup('?x') ⇨ **(a ?y c)**

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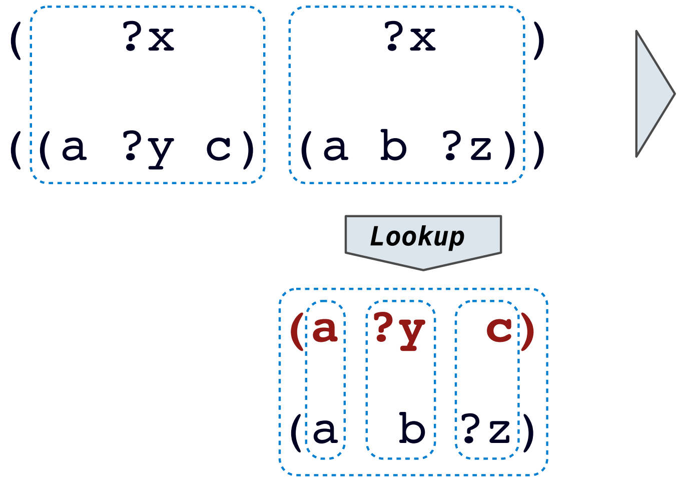

```
(    ?x          ?x     )

( (a ?y c)  (a b ?z) )
```

Lookup

```
(a ?y  c)

(a  b ?z)
```

```
True, {x: (a ?y c),
       y: b,
       z: c}
```

Substituting values for variables may require multiple steps.

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lookup('?x') ⇨ **(a ?y c)**   **lookup('?y')**

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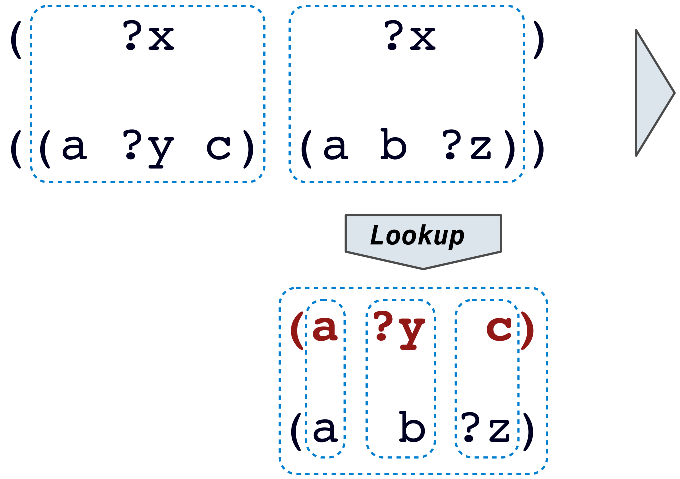

```
True, {x: (a ?y c),
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```

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This process is called *grounding*. Two unified expressions have the same grounded form.

lookup('?x') ⇨ **(a ?y c)**   **lookup('?y')** ⇨ **b**

Unifying Variables

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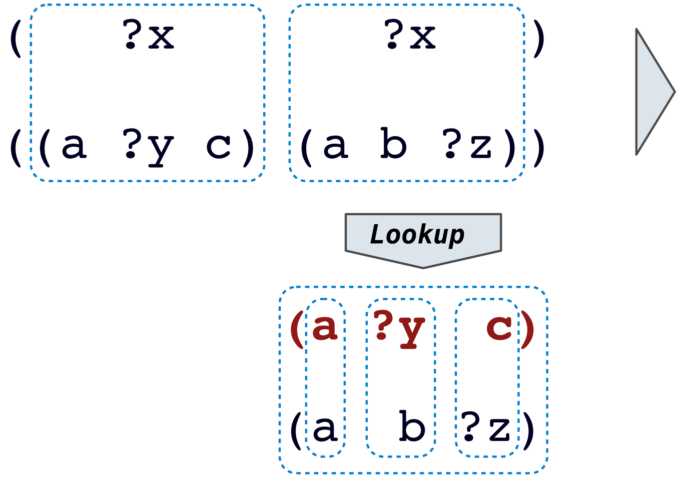

```
(    ?x          ?x     )
((a ?y c)   (a b ?z))
```

Lookup

```
(a ?y  c)
(a  b ?z)
```

```
True, {x: (a ?y c),
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lookup('?x') ⇨ **(a ?y c)** **lookup('?y')** ⇨ **b** **ground('?x')**

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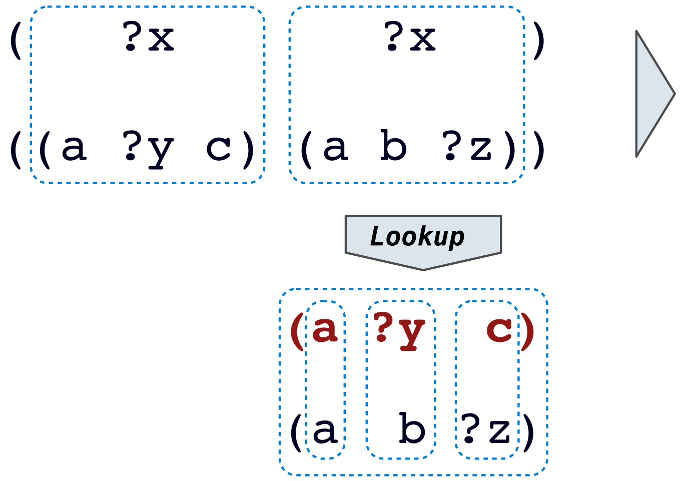

```
(  ?x          ?x      )
((a ?y c)  (a b ?z))
```

Lookup

```
(a ?y c)
(a  b ?z)
```

```
True, {x: (a ?y c),
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This process is called *grounding*. Two unified expressions have the same grounded form.

lookup('?x') ⇨ **(a ?y c)** **lookup('?y')** ⇨ **b** **ground('?x')** ⇨ **(a b c)**

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

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1. Look up variables in the current environment

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```

1. Look up variables in the current environment

2. Establish new bindings to unify elements.

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Symbols/relations without variables only unify if they are the same

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Recursively unify the first and rest of any lists.

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( (a  b) c (a  b) )

(   ?x   c   ?x   )
```

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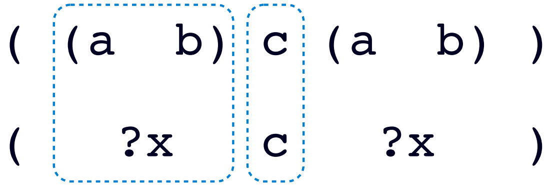

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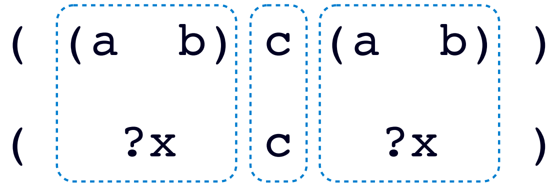

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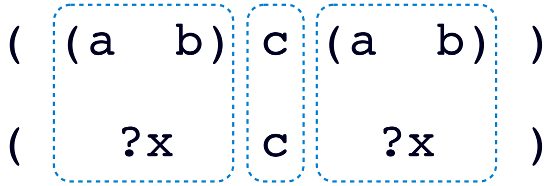

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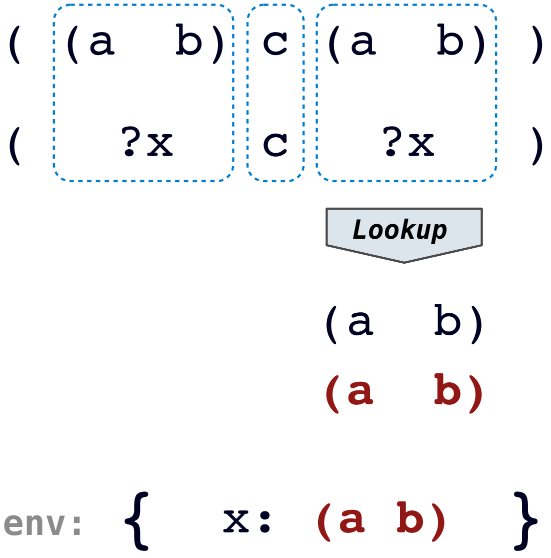

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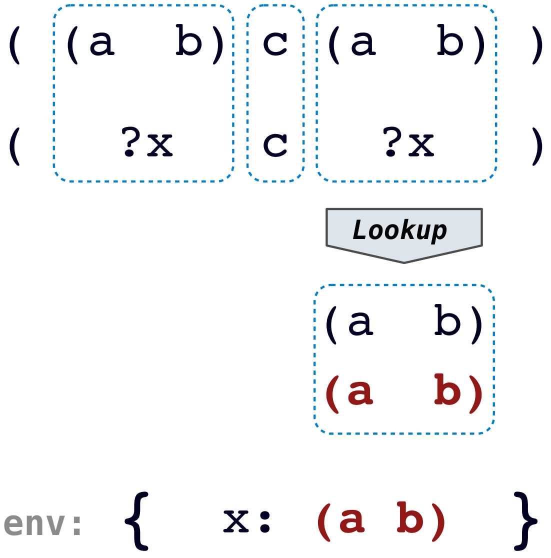

Search

Searching for Proofs

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The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

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(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y        ?z ))

(query (app ?left (c d) (e b c d)))
```

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```

```
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```

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

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(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))


(app (?a . ?r) ?y (?a . ?z))
```

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y        ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
```

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       (app       ?r  ?y       ?z ))

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```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
```

▷ (app (e . ?r) (c d) (e b c d))

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(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
```

▷ `(app (e . ?r) (c d) (e b c d))`

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```
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(fact  (app (?a . ?r) ?y (?a . ?z))
       (app        ?r  ?y        ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))


(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

```
(app (e . ?r) (c d) (e b c d))
```

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

Variables are local to facts & queries

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

```
(app (e . ?r) (c d) (e b c d))
```

Variables are local to facts & queries

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
```

```
(app (e . ?r) (c d) (e b c d))
```

conclusion <- hypothesis

```
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

```
(app (b . ?r2) (c d) (b c d))
```

Variables are local to facts & queries

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
```

Variables are local to facts & queries

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
```

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}              (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}              (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))

(app () ?x ?x)
```

Variables are local to facts & queries

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}          (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}          (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

Variables are local to facts & queries

Searching for Proofs

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```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}            ▷ (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}            ▷ (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}   ▷ (app () (c d) (c d))
(app () ?x ?x)
```

Variables are local to facts & queries

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
```

Variables are local to facts & queries

```
(app () (c d) (c d))
```

?left:

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}        (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}        (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}        (app () (c d) (c d))
(app () ?x ?x)
```

Variables are local to facts & queries

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```
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(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}      ▷ (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}      ▷ (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}      ▷ (app () (c d) (c d))
(app () ?x ?x)
```

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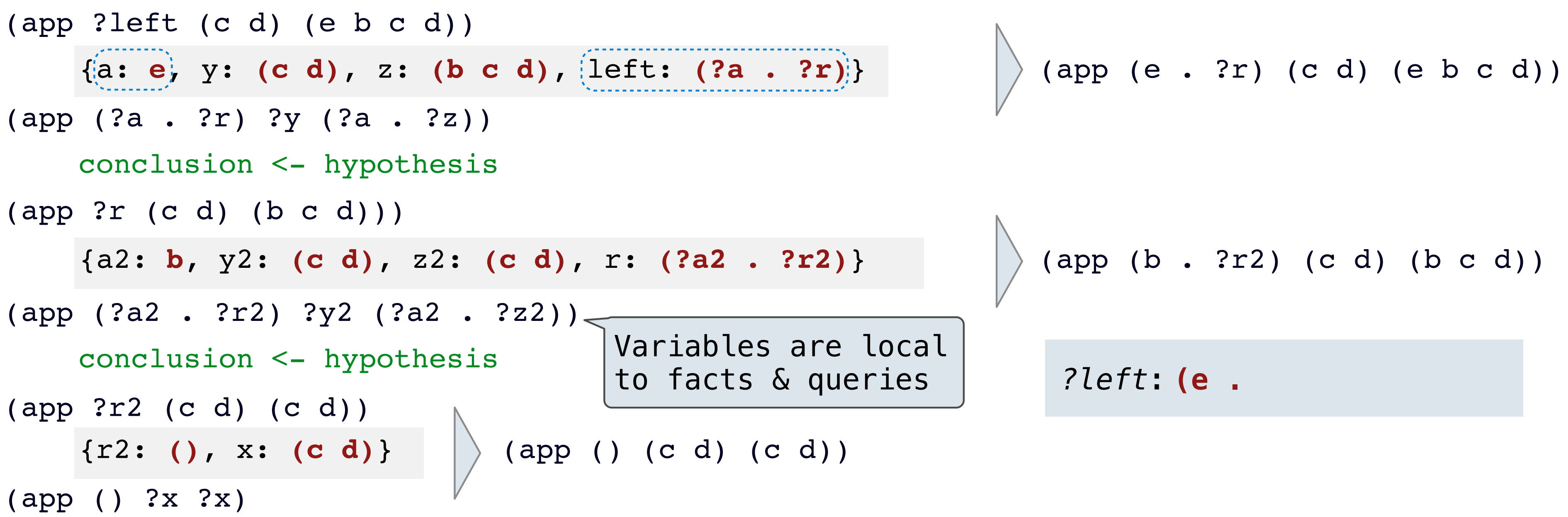
(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```



```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}          (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}          (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}          (app () (c d) (c d))
(app () ?x ?x)
```

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```
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       (app       ?r  ?y       ?z ))

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```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}        (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}        (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}       (app () (c d) (c d))
(app () ?x ?x)
```

Variables are local to facts & queries

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?r:

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(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

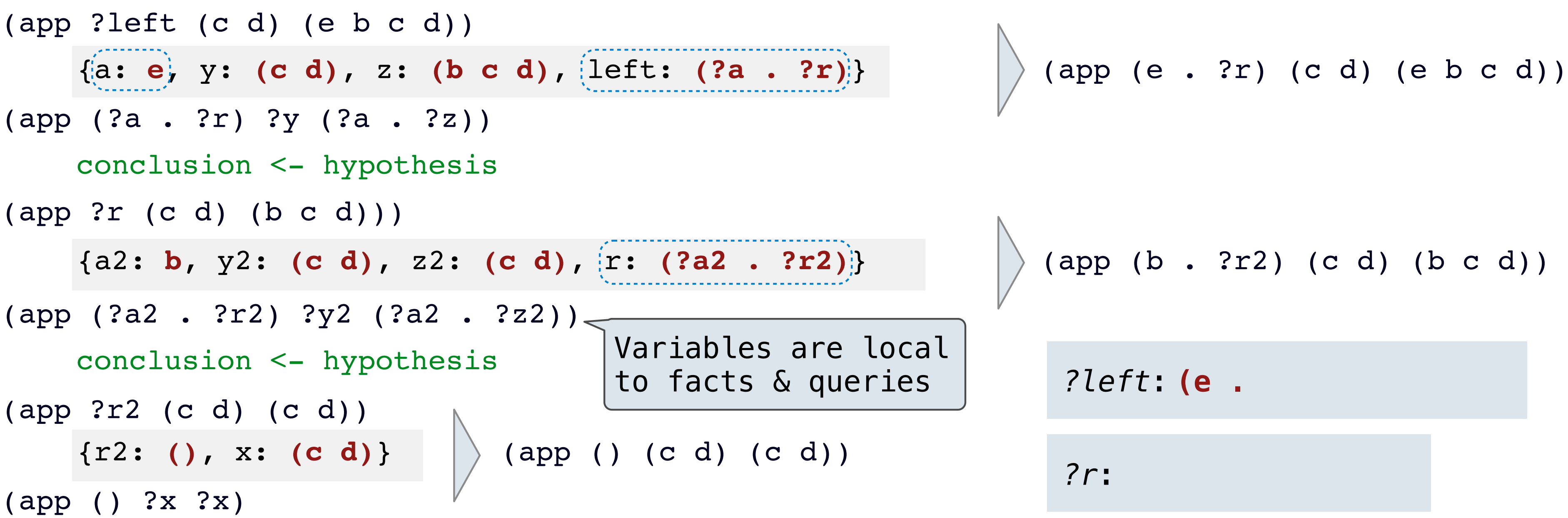

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}      (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}      (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}      (app () (c d) (c d))
(app () ?x ?x)
```

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?r:

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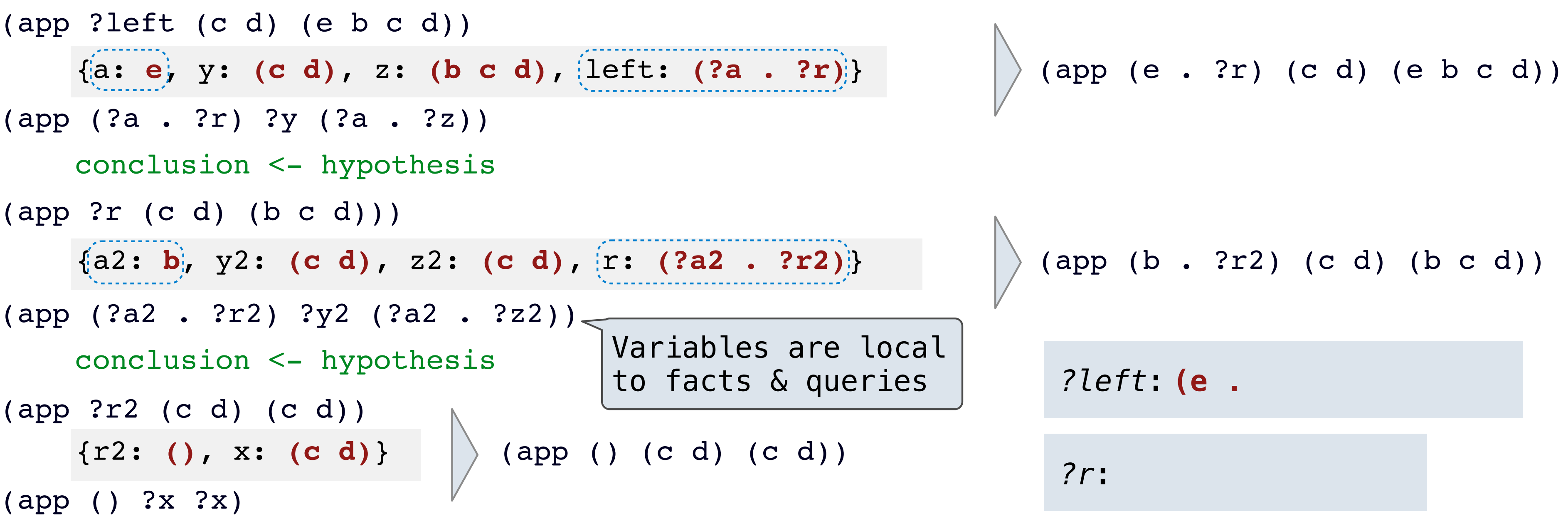
```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```



```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

```
(app (e . ?r) (c d) (e b c d))

(app (b . ?r2) (c d) (b c d))

(app () (c d) (c d))
```

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?r:

Searching for Proofs

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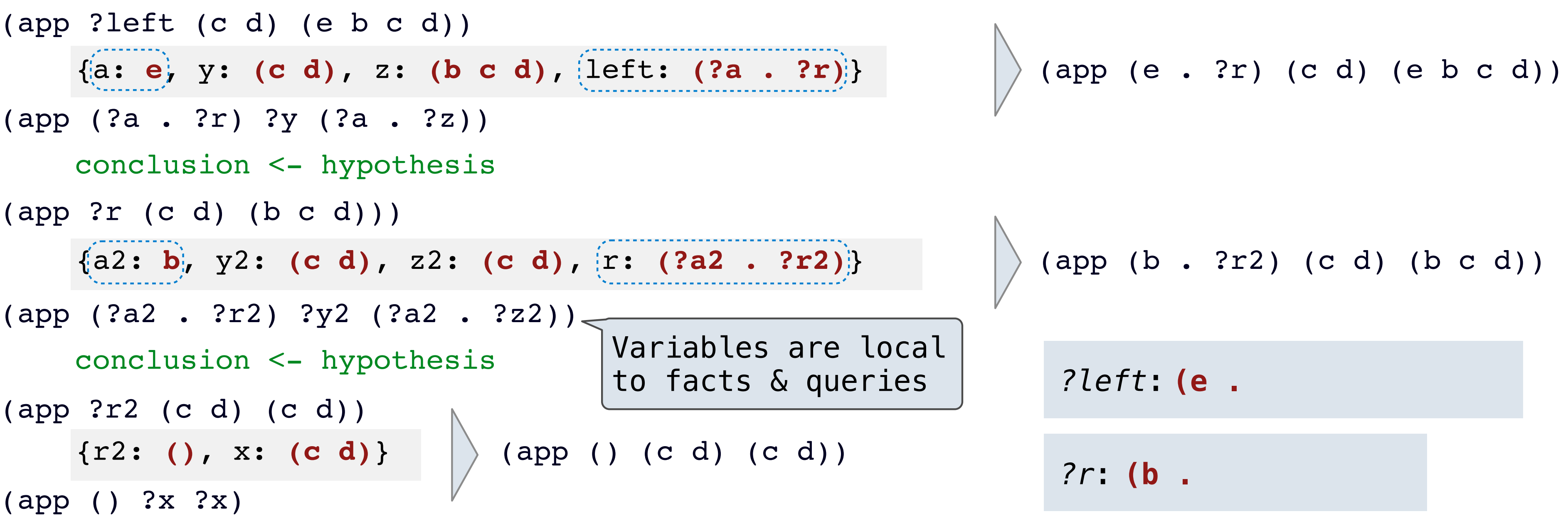
```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```



```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}           (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}           (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}      (app () (c d) (c d))
(app () ?x ?x)
```

Variables are local to facts & queries

?left: (e .

?r: (b .

Searching for Proofs

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```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y        ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}        (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}        (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}        (app () (c d) (c d))
(app () ?x ?x)
```

Variables are local to facts & queries

?left: **(e .**

?r: **(b .**

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}          ▷ (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}          ▷ (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}   ▷ (app () (c d) (c d))
(app () ?x ?x)
```

Variables are local to facts & queries

?left: **(e .**

?r: **(b . ())**

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

```
(app (e . ?r) (c d) (e b c d))

(app (b . ?r2) (c d) (b c d))

(app () (c d) (c d))
```

Variables are local to facts & queries

```
?left: (e .

?r: (b . ()) ⇨ (b)
```

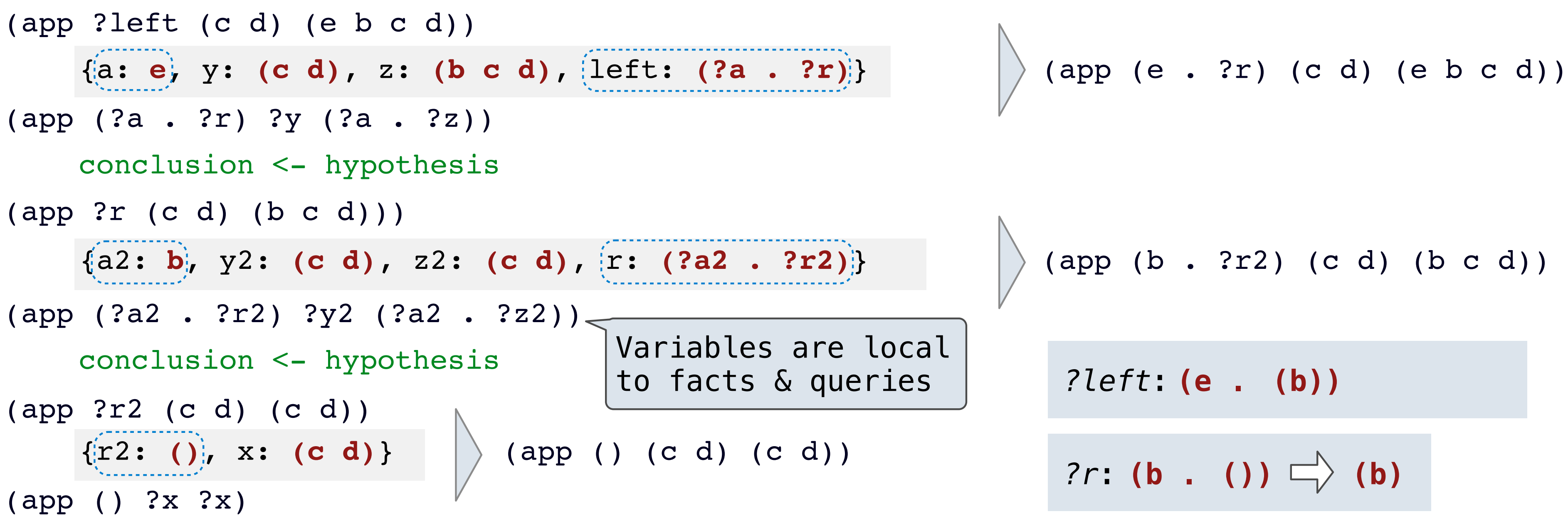
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```



```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}        (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}        (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}        (app () (c d) (c d))
(app () ?x ?x)
```

Variables are local to facts & queries

?left: **(e . (b))**

?r: **(b . ())** ⇨ **(b)**

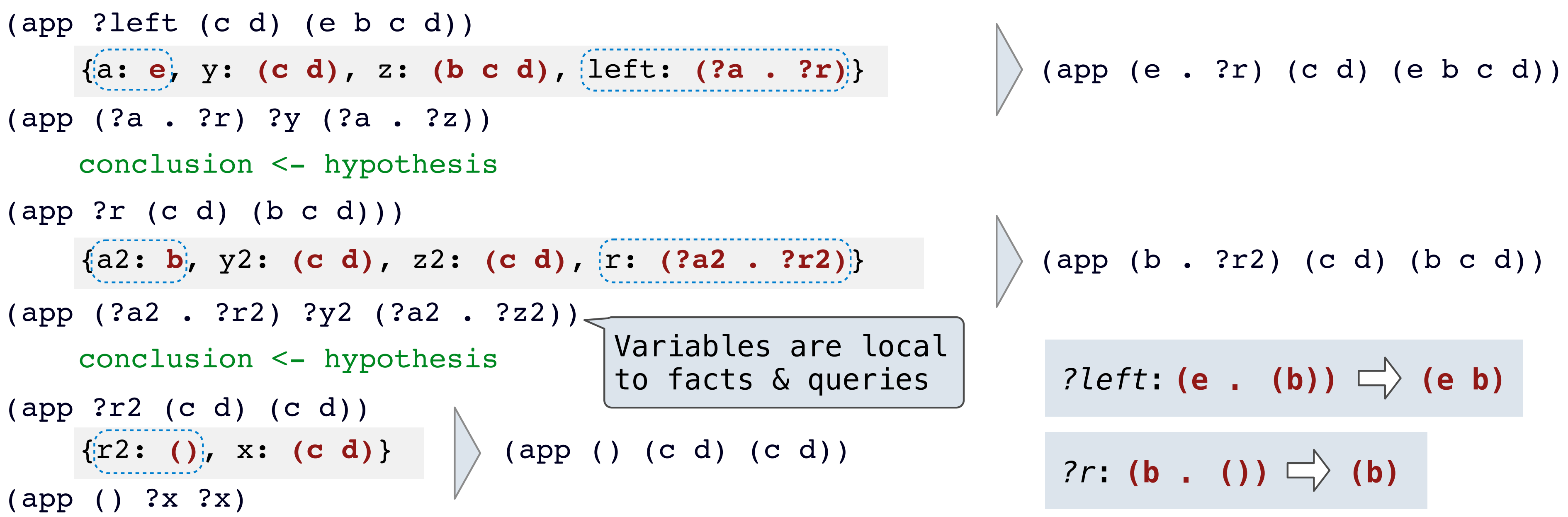
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```



```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

(app (e . ?r) (c d) (e b c d))

(app (b . ?r2) (c d) (b c d))

(app () (c d) (c d))

Variables are local to facts & queries

?left: (e . (b)) ⇨ (e b)

?r: (b . ()) ⇨ (b)

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(Demo)

Addition

(Demo)