Environments

Announcements

Environments for Higher-Order Functions

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Environment diagrams describe how higher-order functions work!

(Demo)

Names can be Bound to Functional Arguments

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1  def apply_twice(f, x):
2      return f(f(x))
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5      return x * x
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7  result = apply_twice(square, 2)
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Global frame

apply_twice → func apply_twice(f, x) [parent=Global]

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pythontutor.com/composingprograms.html#code=def%20apply_twice%28f,%20x%29%3A%0A%20%20%20%20return%20f%28f%28x%29%29%0A%20%20%20%20%0Adef%20square%28x%29%3A%0A%20%20%20%20return%20x%20*%20x%0A%20%20%20%20%0Aresult%20%3D%20apply_twice%28square,%202%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

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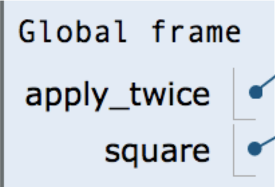

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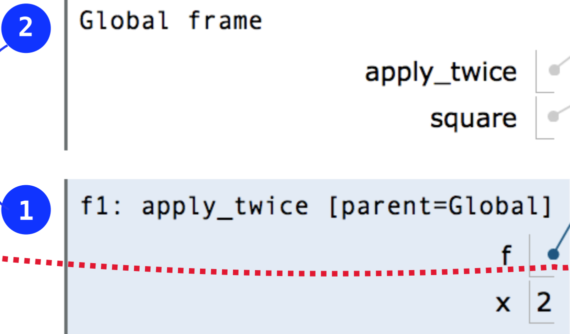

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Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements

```
1  def make_adder(n):
2      def adder(k):
3          return k + n
4      return adder
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6  add_three = make_adder(3)
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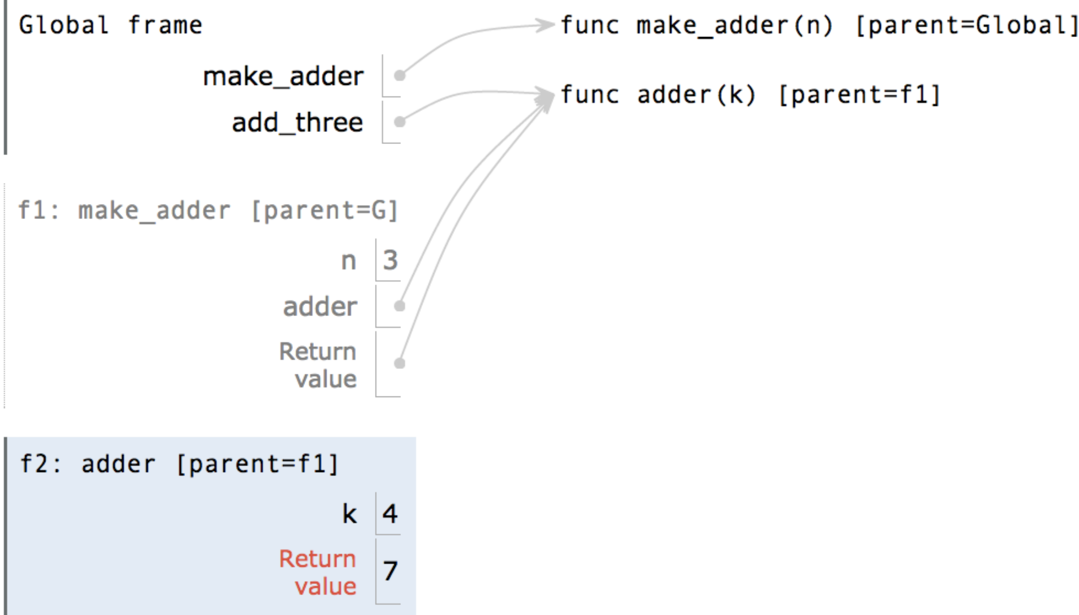

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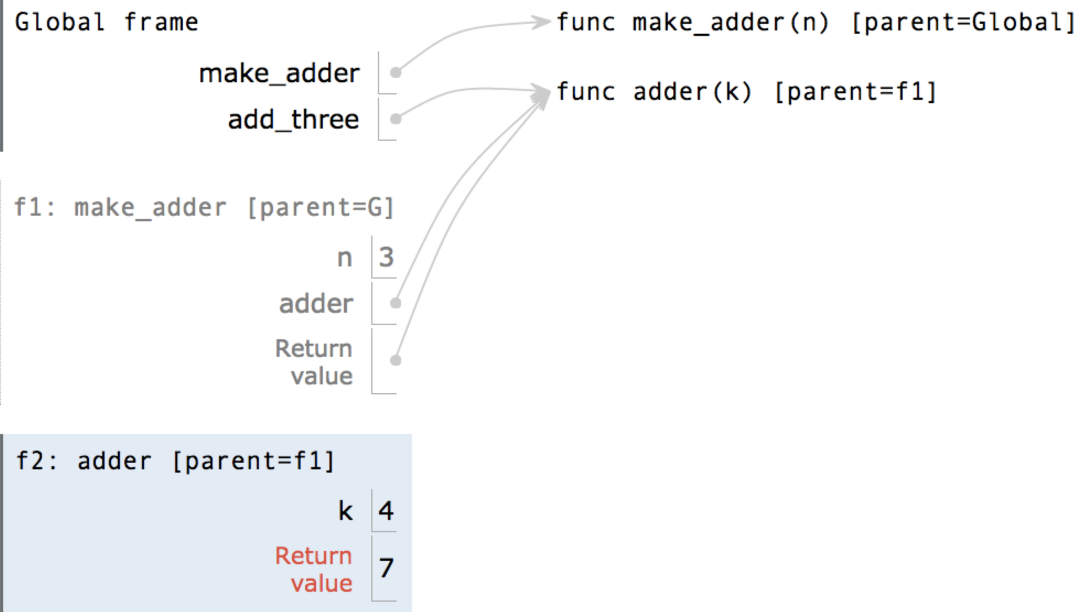

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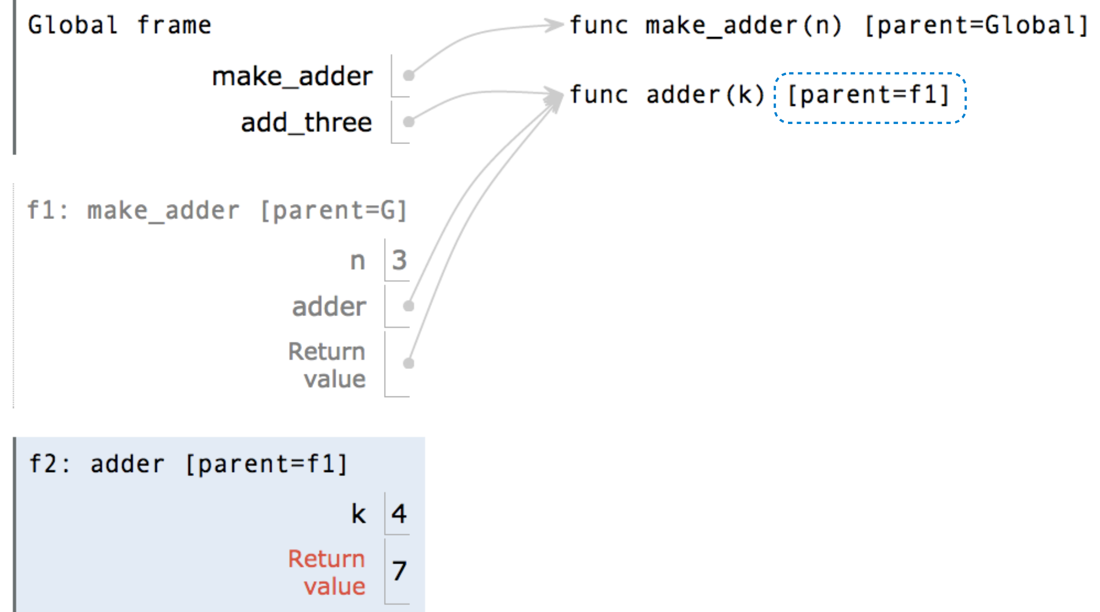

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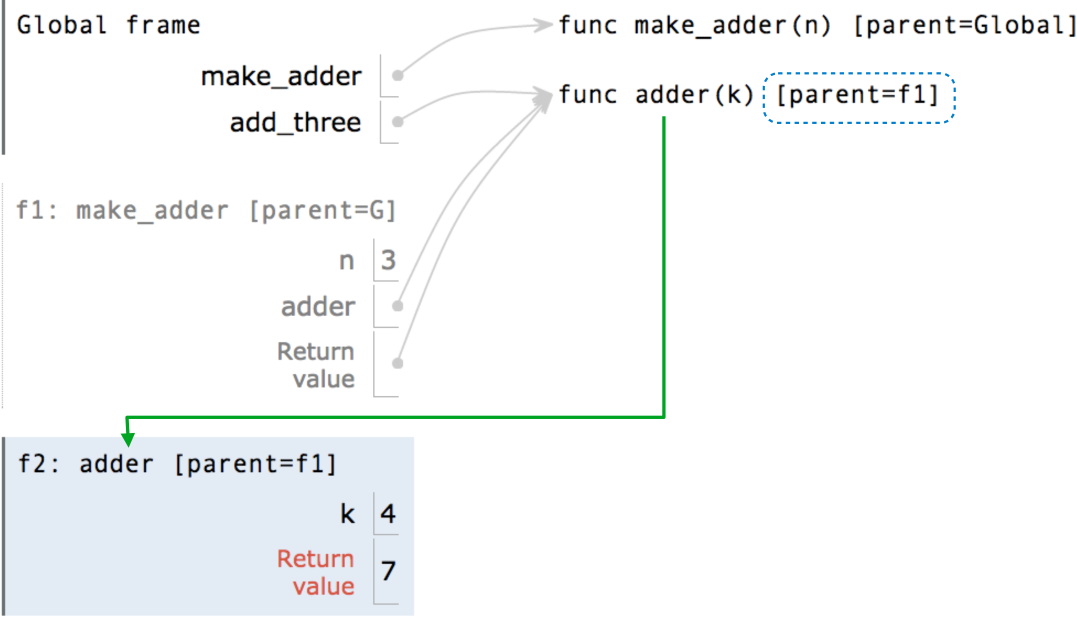

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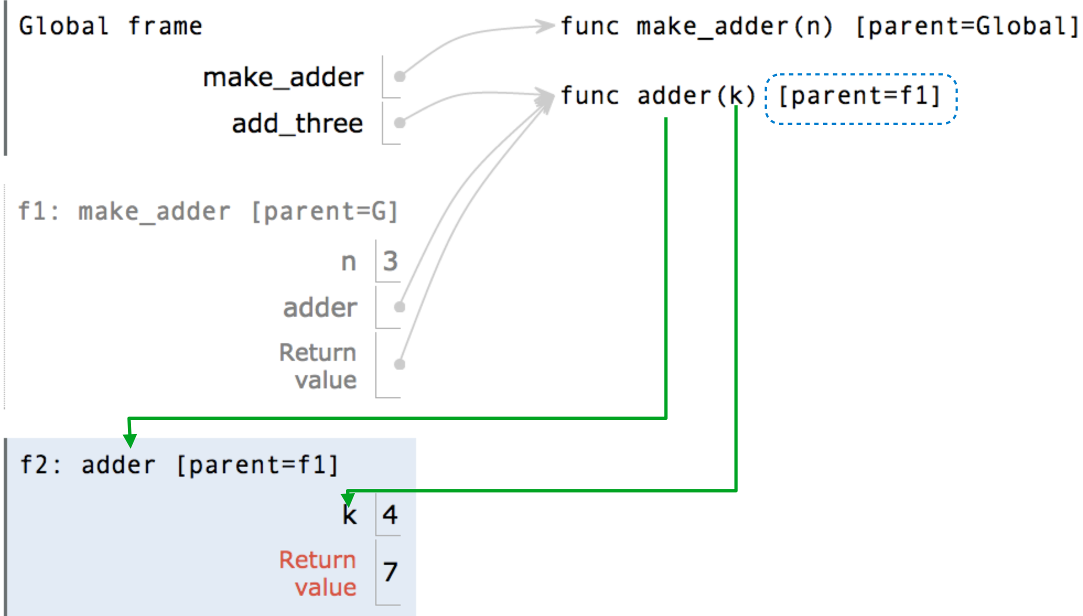

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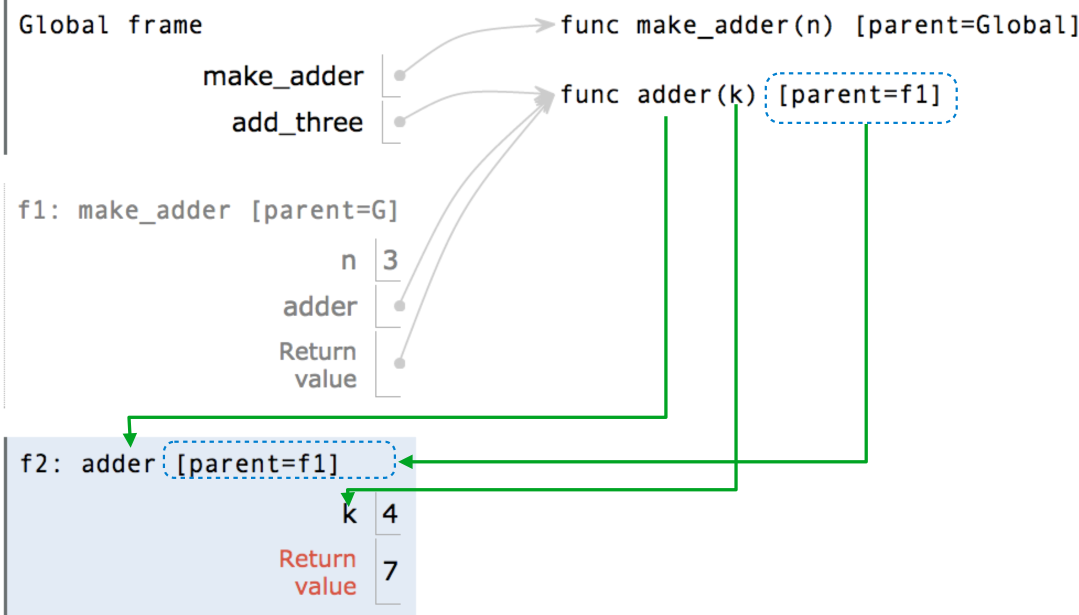

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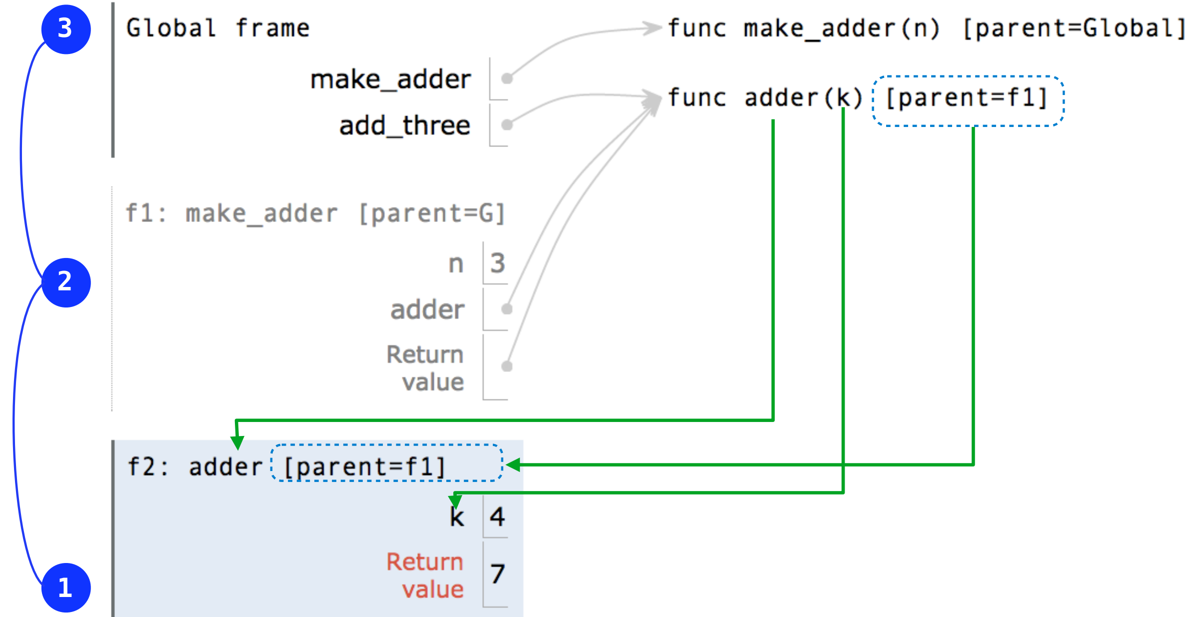

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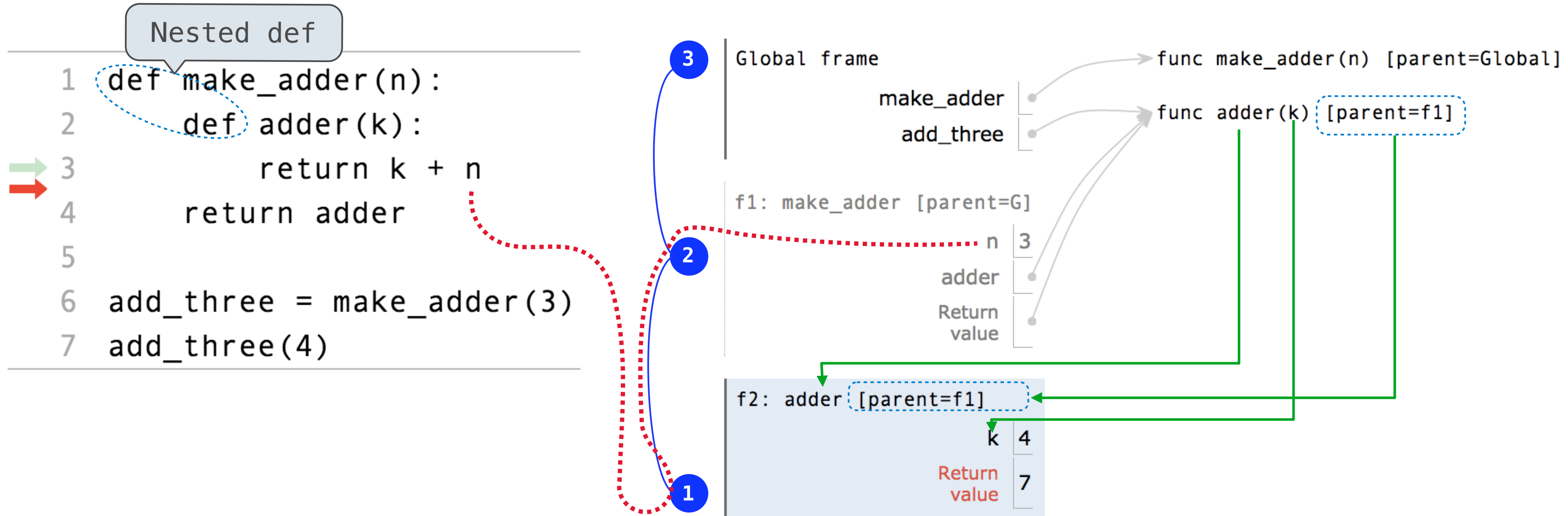

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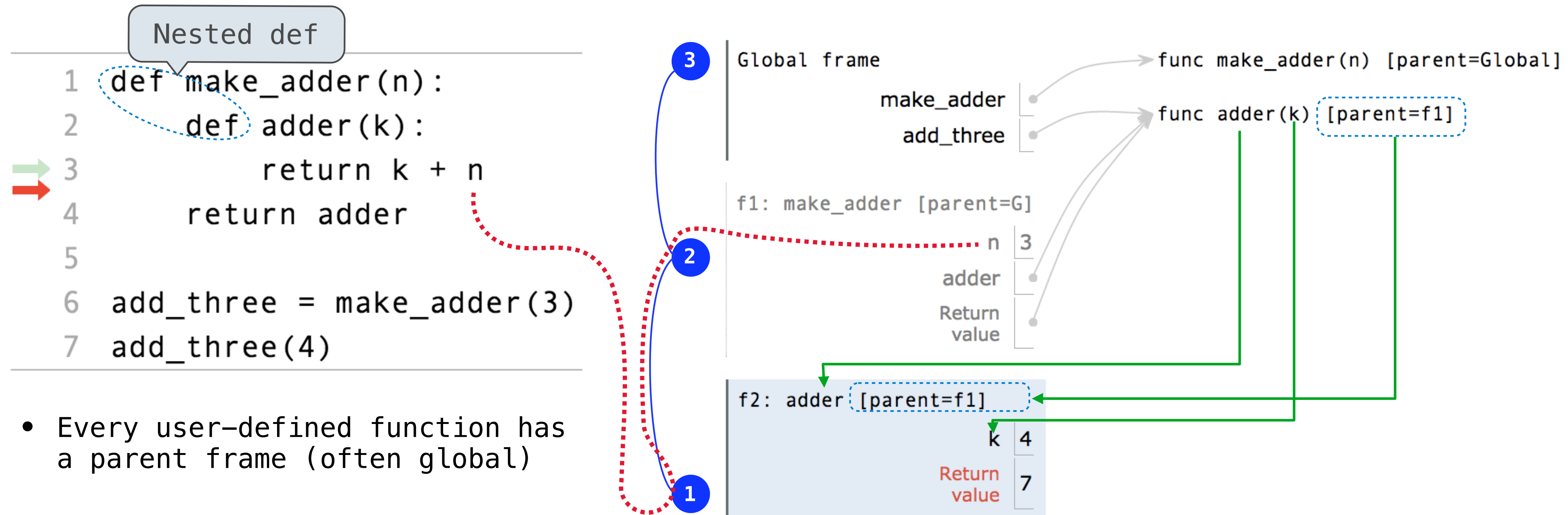

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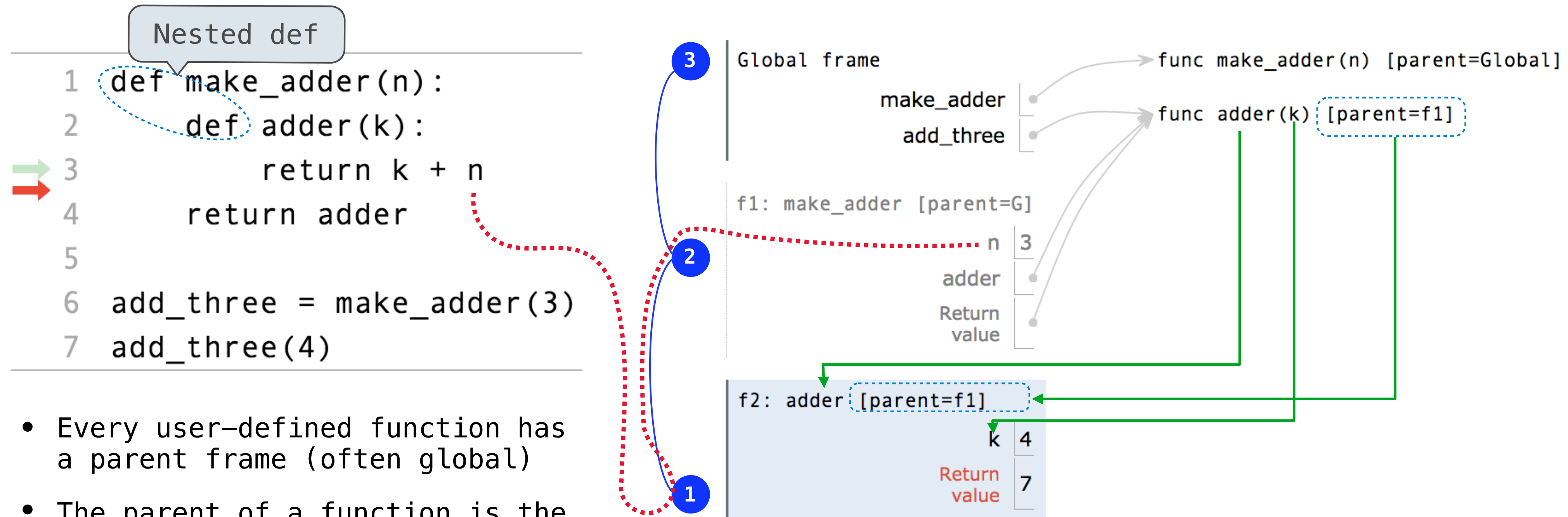

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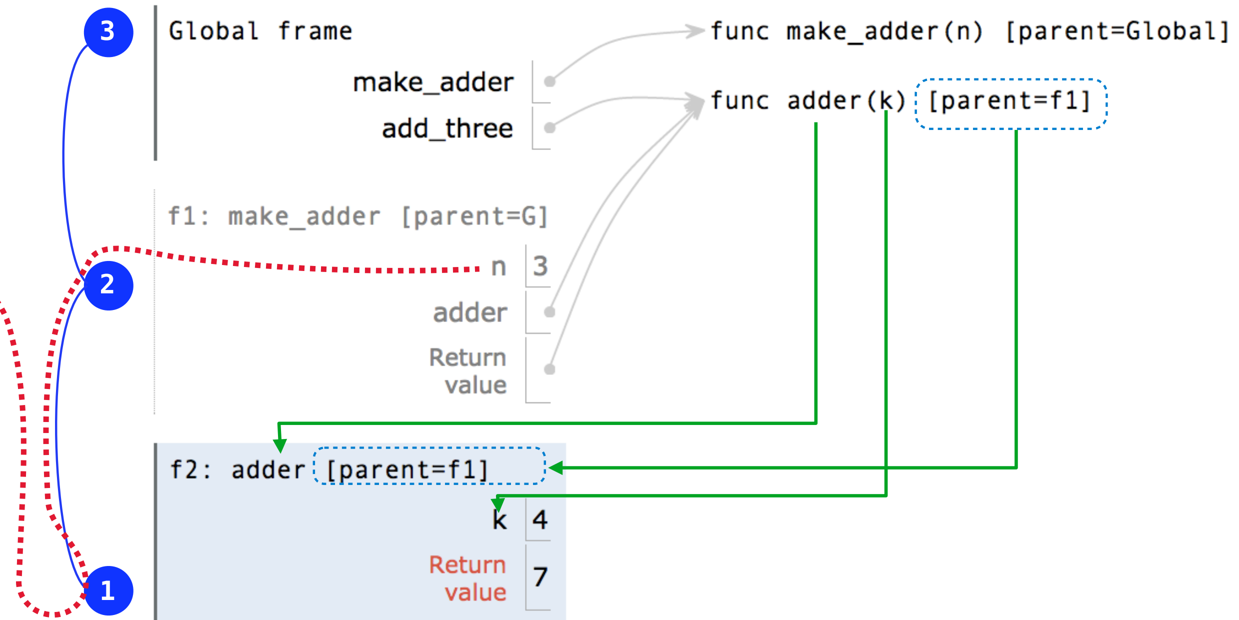

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- Every local frame has a parent frame (often global)

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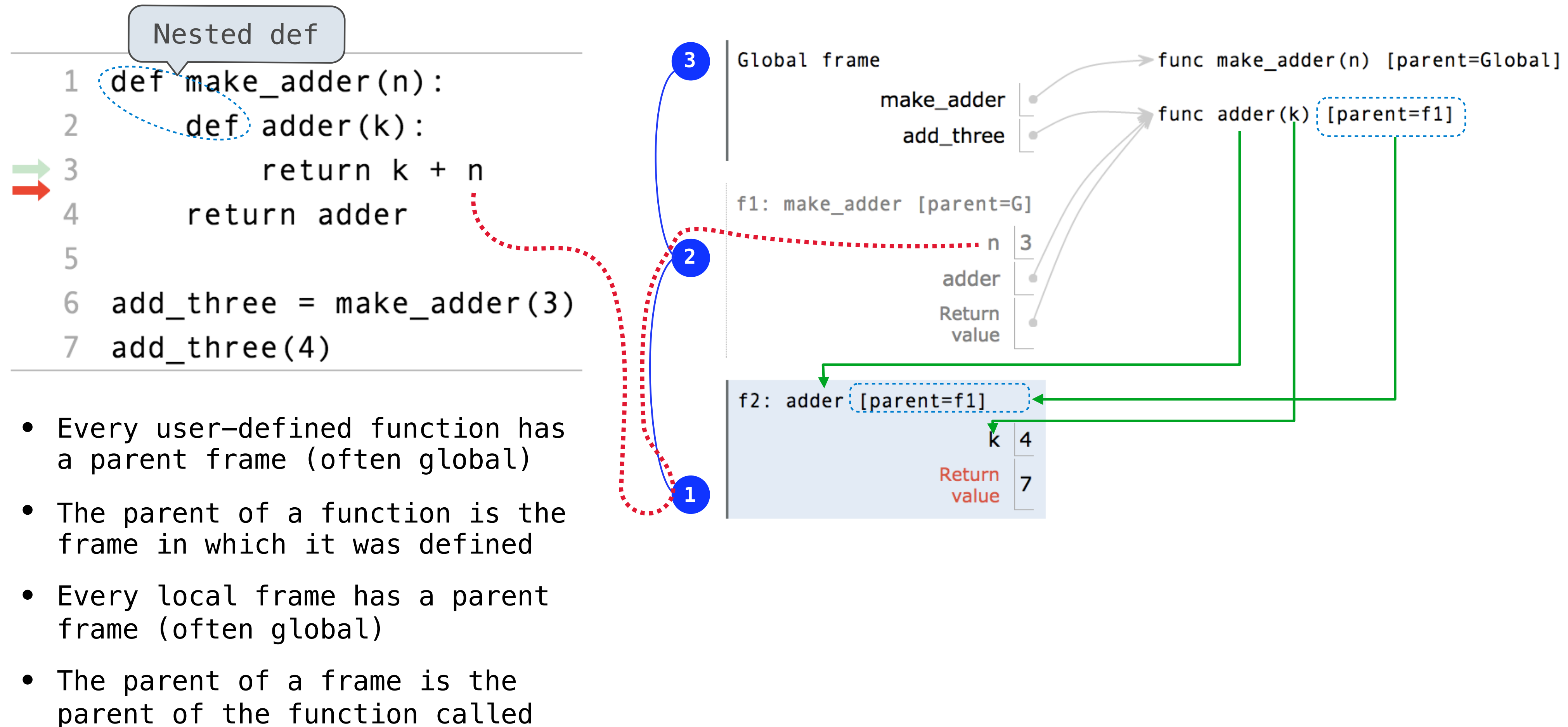

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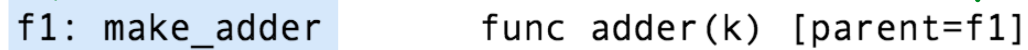

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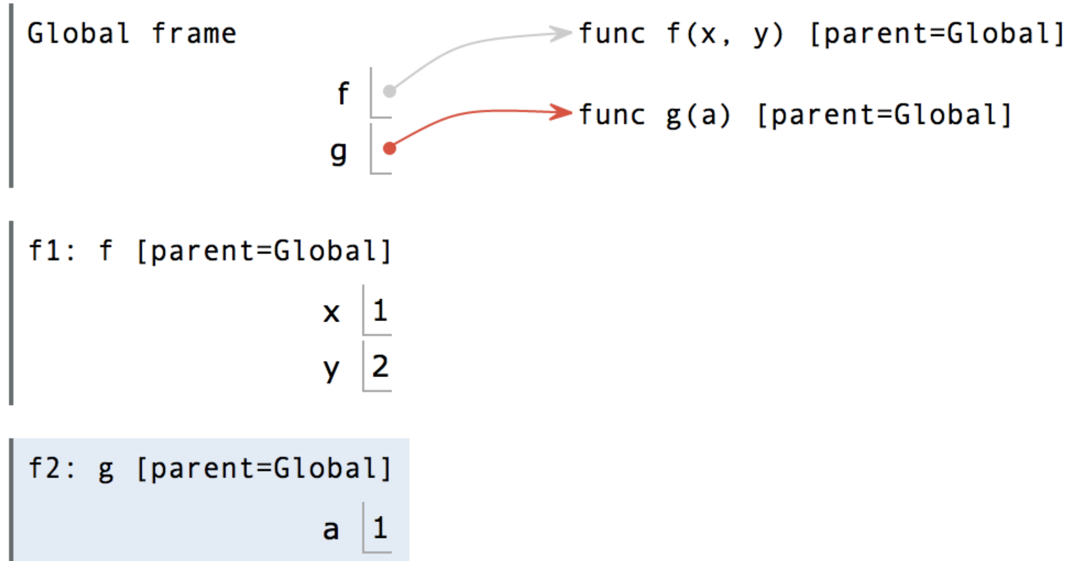
3. Bind the <formal parameters> to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

(Demo)

Local Names are not Visible to Other (Non-Nested) Functions

```
1  def f(x, y):
2      return g(x)
3
4  def g(a):
5      return a + y
6
7  result = f(1, 2)
```



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%20y%29%3A%0A%20%20%20%20return%20g%28x%29%0A%0Adef%20g%28a%29%3A%0A%20%20%20%20return%20a%20%2B%20y%0A%20%20%20%20%0Aresult%20%3D%20f%281,%202%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

Local Names are not Visible to Other (Non-Nested) Functions

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1  def f(x, y):
2      return g(x)
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4  def g(a):
5      return a + y
6
7  result = f(1, 2)
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Global frame
f → func f(x, y) [parent=Global]
g → func g(a) [parent=Global]

f1: f [parent=Global]
x 1
y 2

f2: g [parent=Global]
a 1

1
2

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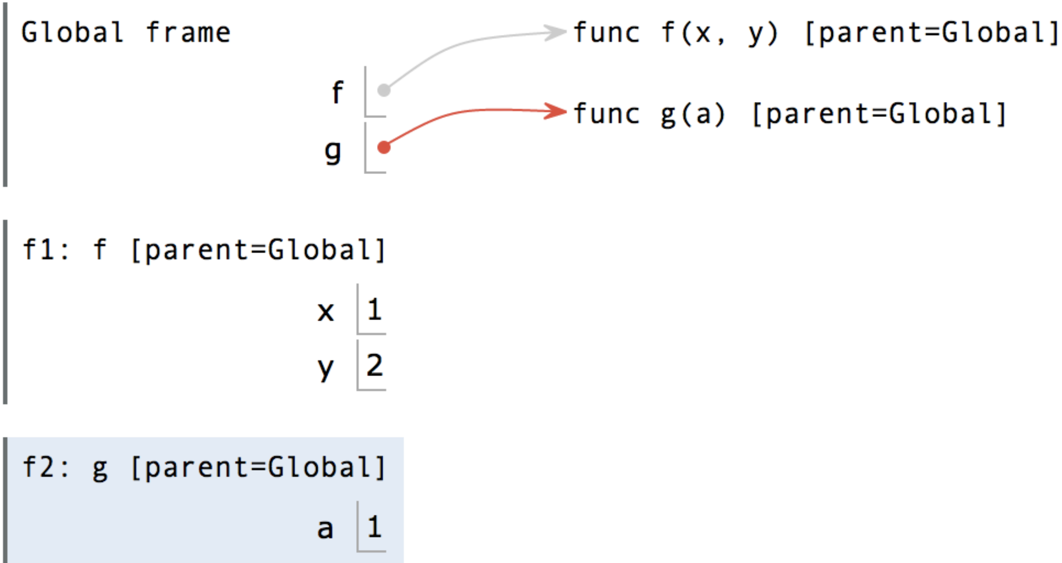

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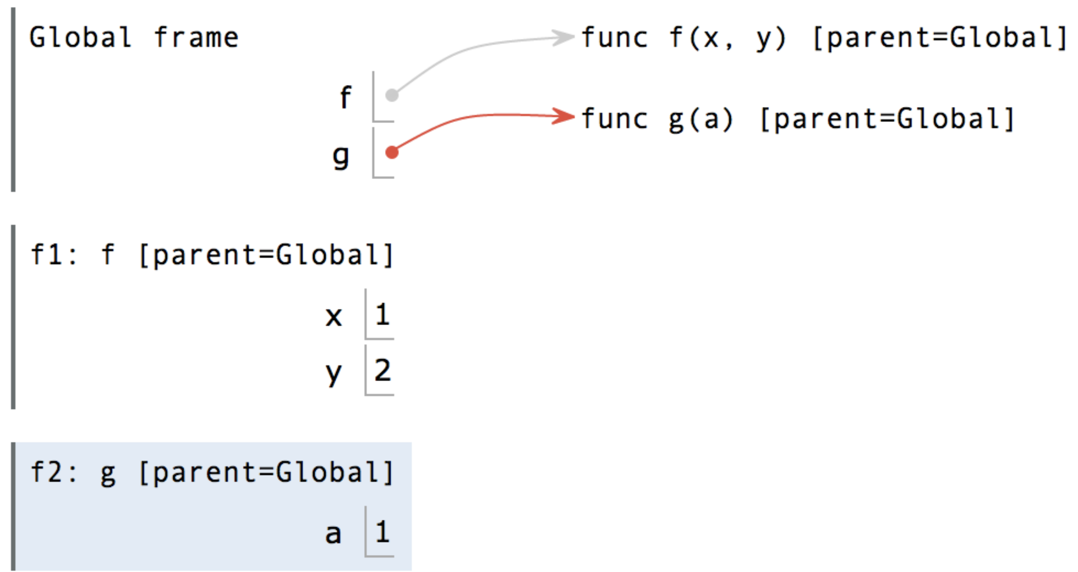
“y” is not found



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f1: f [parent=Global]
x 1
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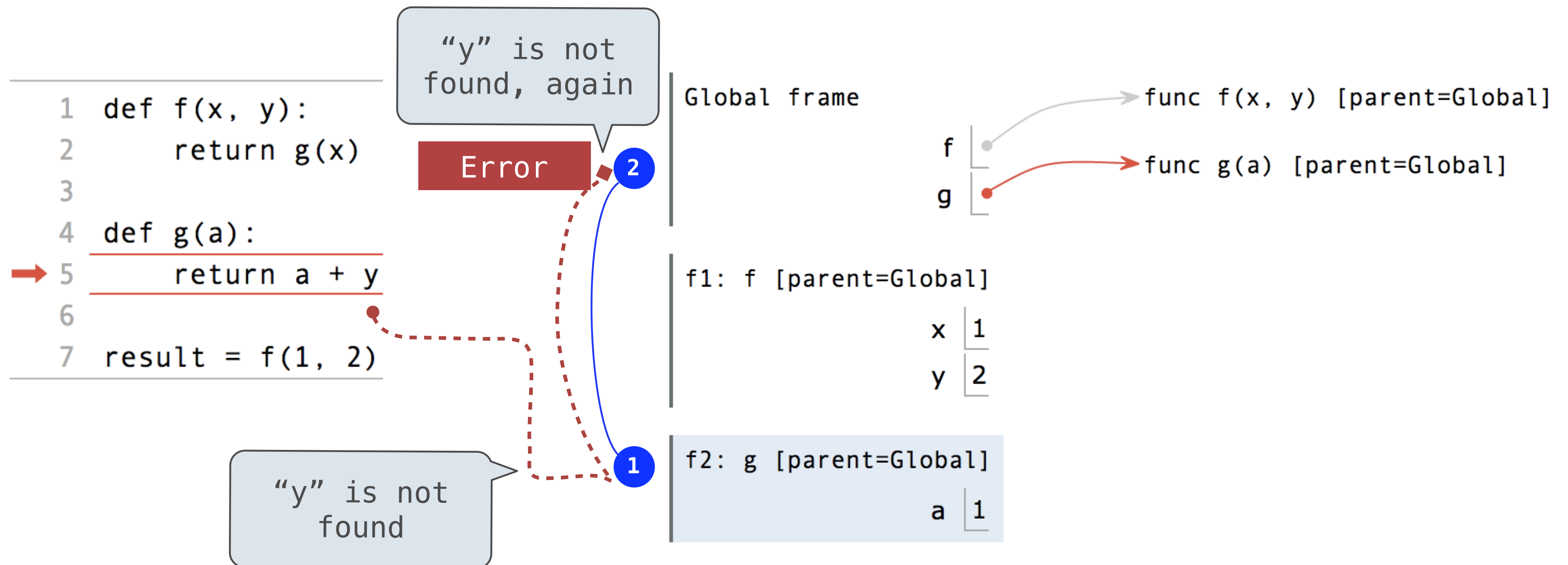

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"y" is not found, again

Error

Global frame

f → func f(x, y) [parent=Global]

g → func g(a) [parent=Global]

f1: f [parent=Global]

x 1

y 2

f2: g [parent=Global]

a 1

"y" is not found

1

2

- An environment is a sequence of frames.

http://pythontutor.com/composingprograms.html#code=def%20f%28x,
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“y” is not found, again

Error

2

Global frame

f → func f(x, y) [parent=Global]

g → func g(a) [parent=Global]

f1: f [parent=Global]

x 1

y 2

f2: g [parent=Global]

a 1

1

“y” is not found

- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

http://pythontutor.com/composingprograms.html#code=def%20f%28x,%20y%29%3A%0A%20%20%20%20return%20g%28x%29%0A%0Adef%20g%28a%29%3A%0A%20%20%20%20return%20a%20%2B%20y%0A%20%20%20%20%0Aresult%20%3D%20f%281,%202%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

Function Composition

(Demo)

The Environment Diagram for Function Composition

```
 1  def square(x):
 2      return x * x
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 4  def make_adder(n):
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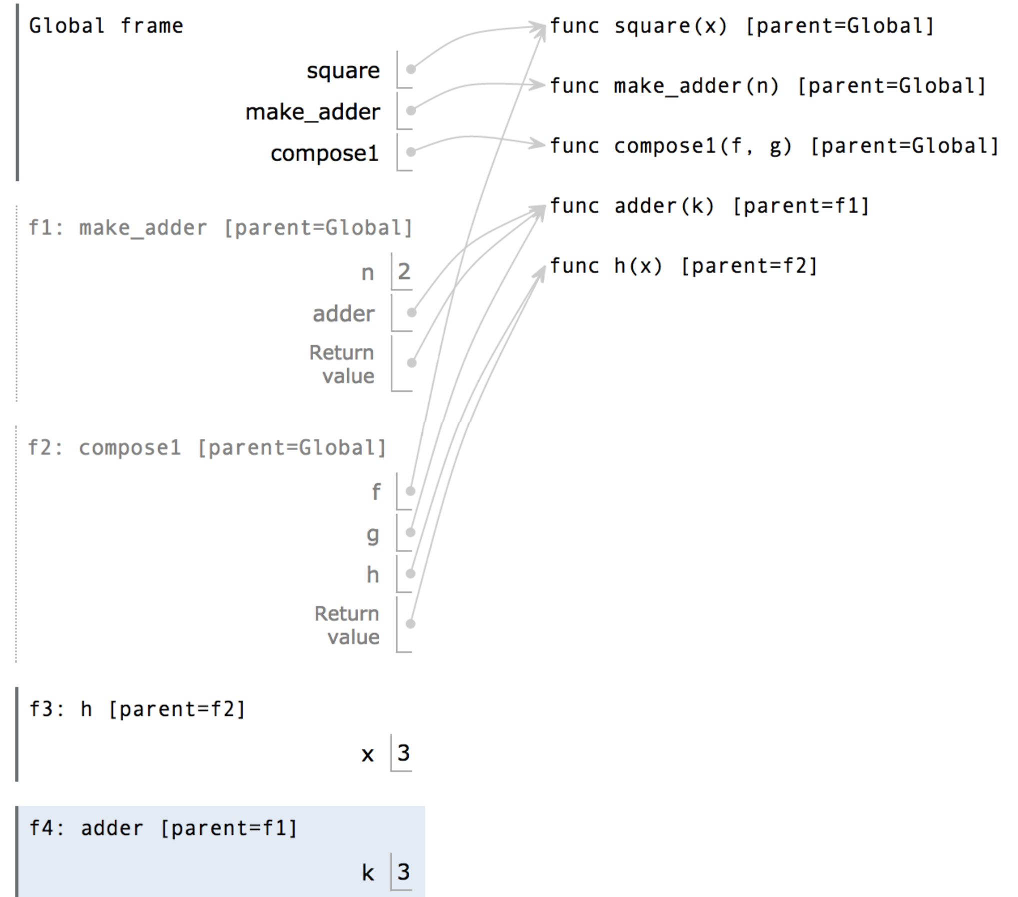

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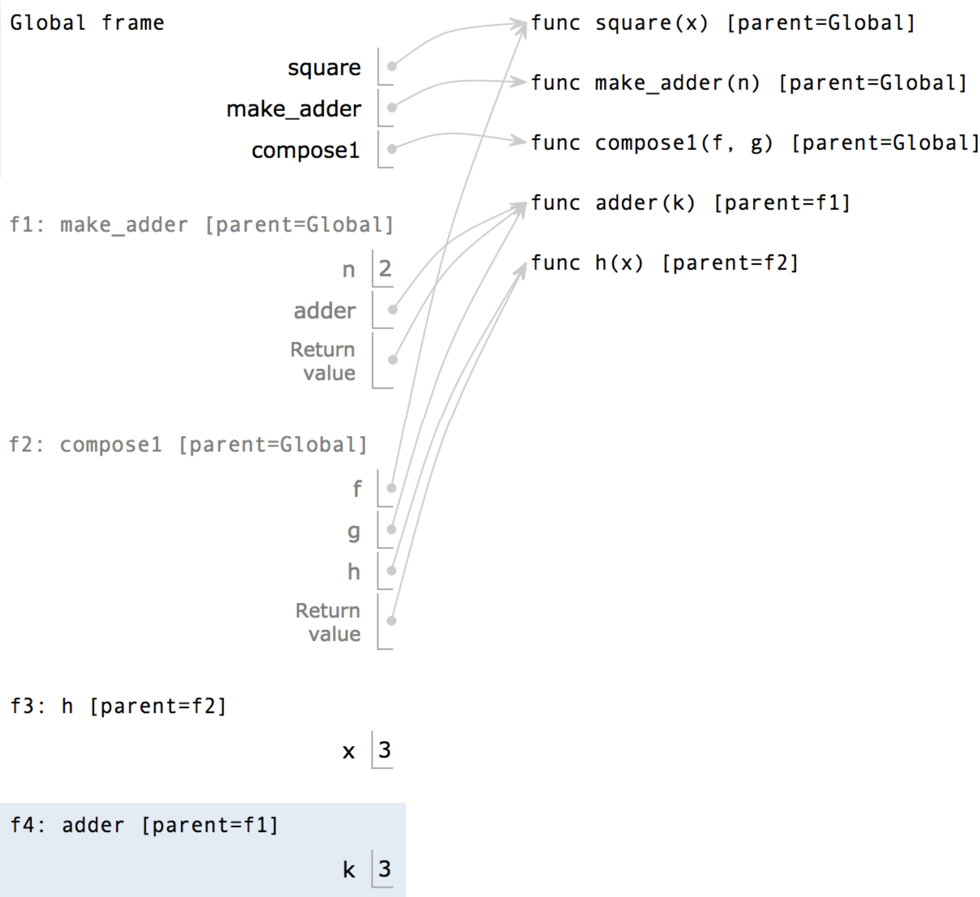

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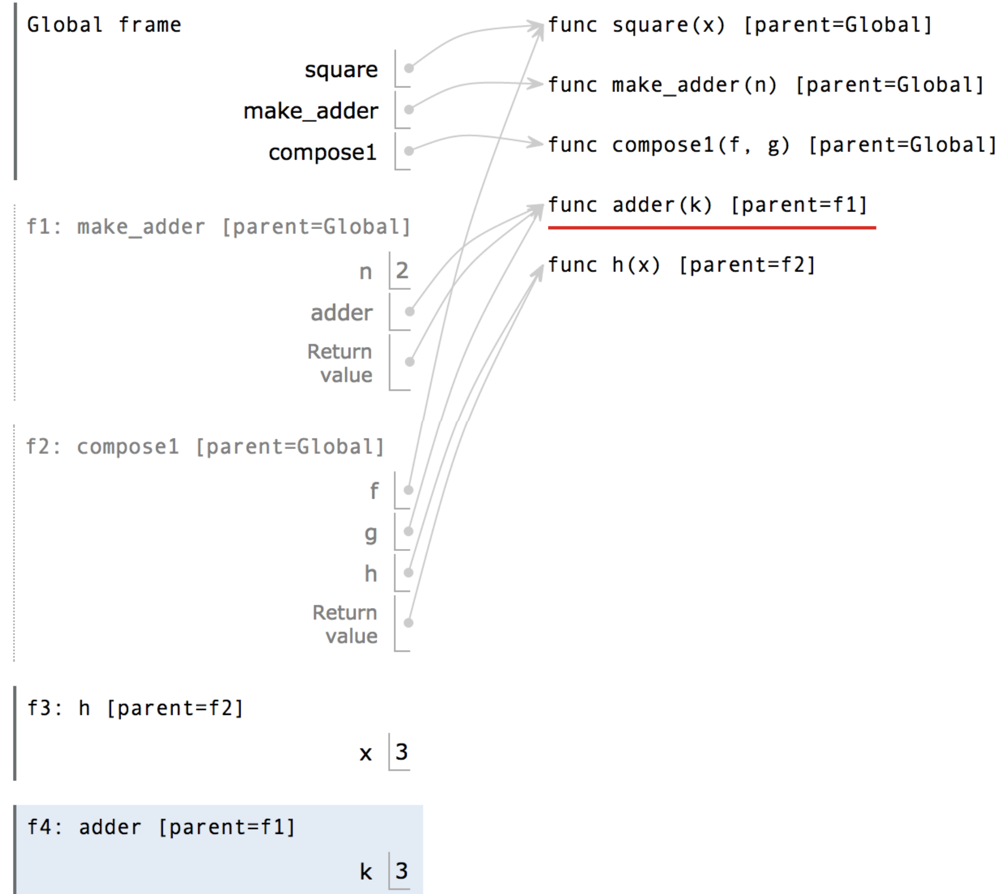

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```

Global frame
square
make_adder
compose1

func square(x) [parent=Global]
func make_adder(n) [parent=Global]
func compose1(f, g) [parent=Global]
func adder(k) [parent=f1]
func h(x) [parent=f2]

f1: make_adder [parent=Global]
n 2
adder
Return value

f2: compose1 [parent=Global]
f
g
h
Return value

f3: h [parent=f2]
x 3

f4: adder [parent=f1]
k 3

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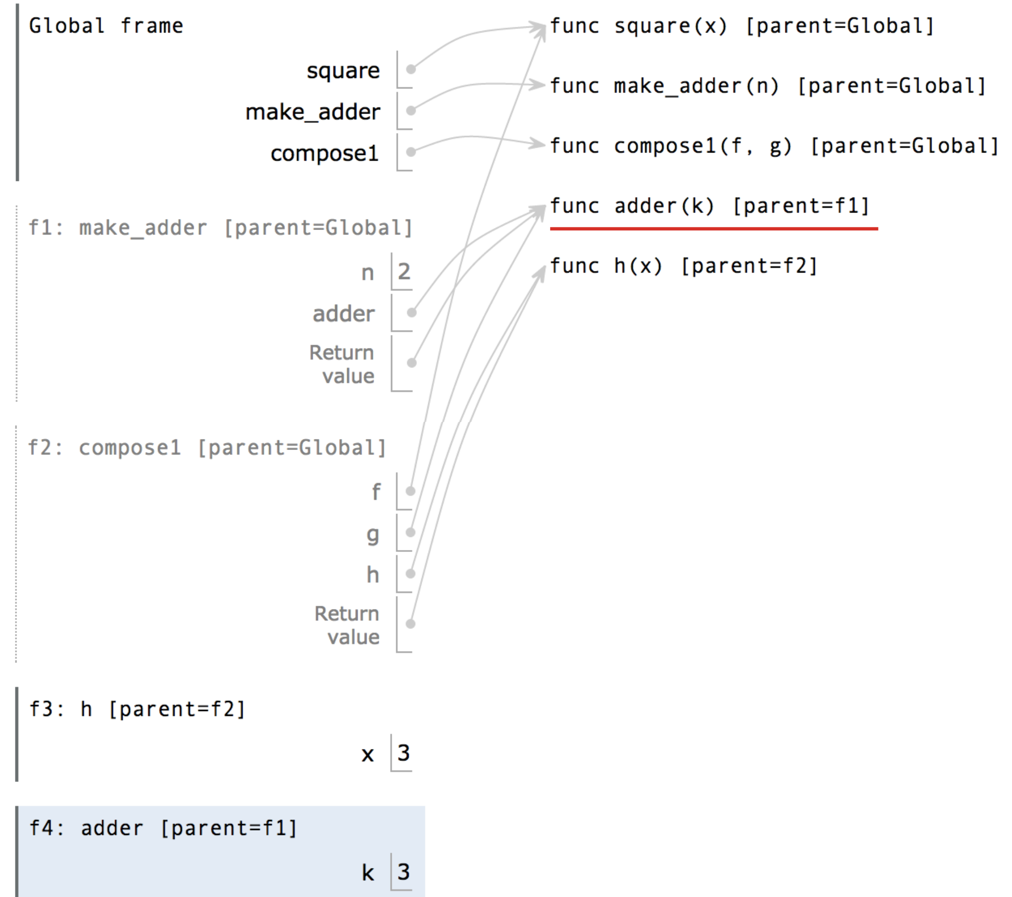
Return value of make_adder is an argument to compose1



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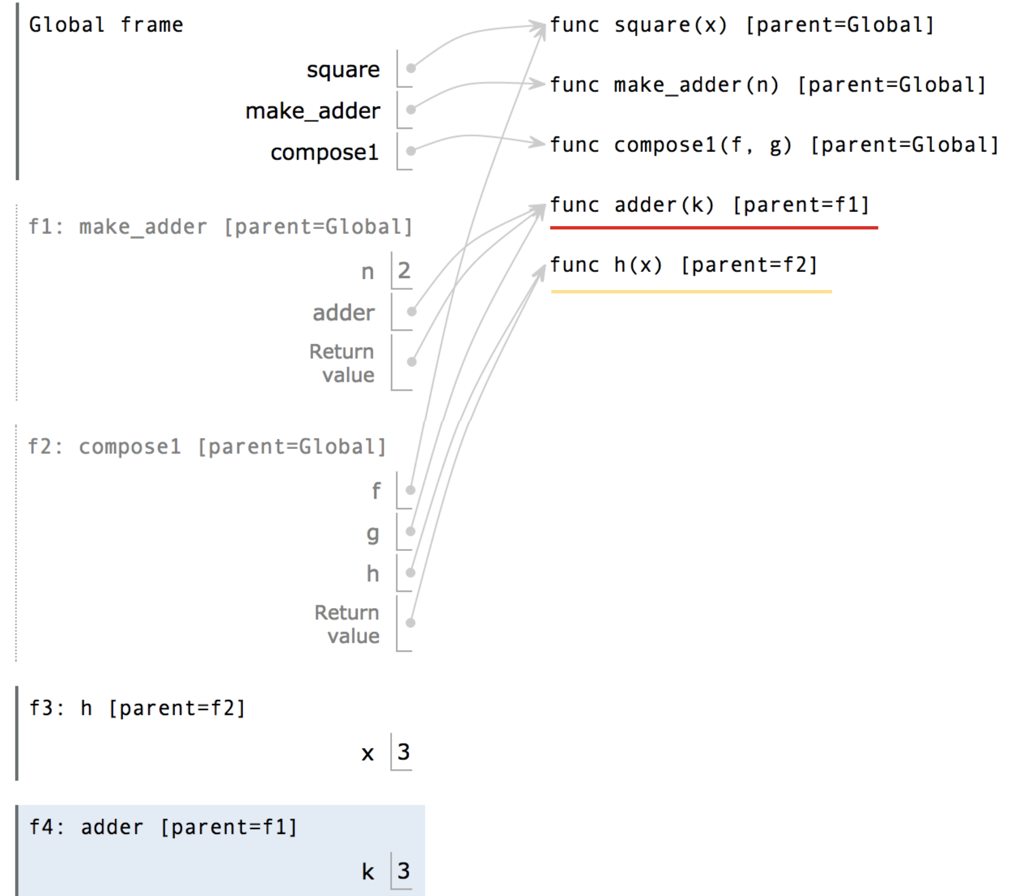

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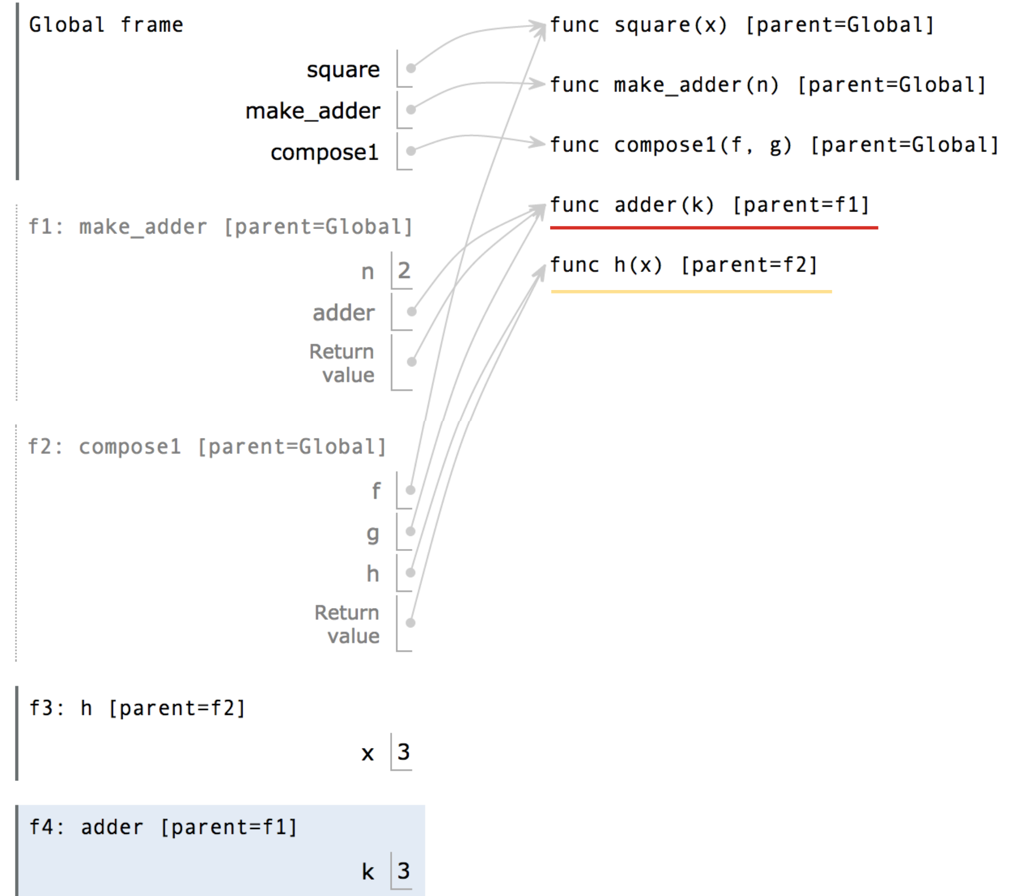

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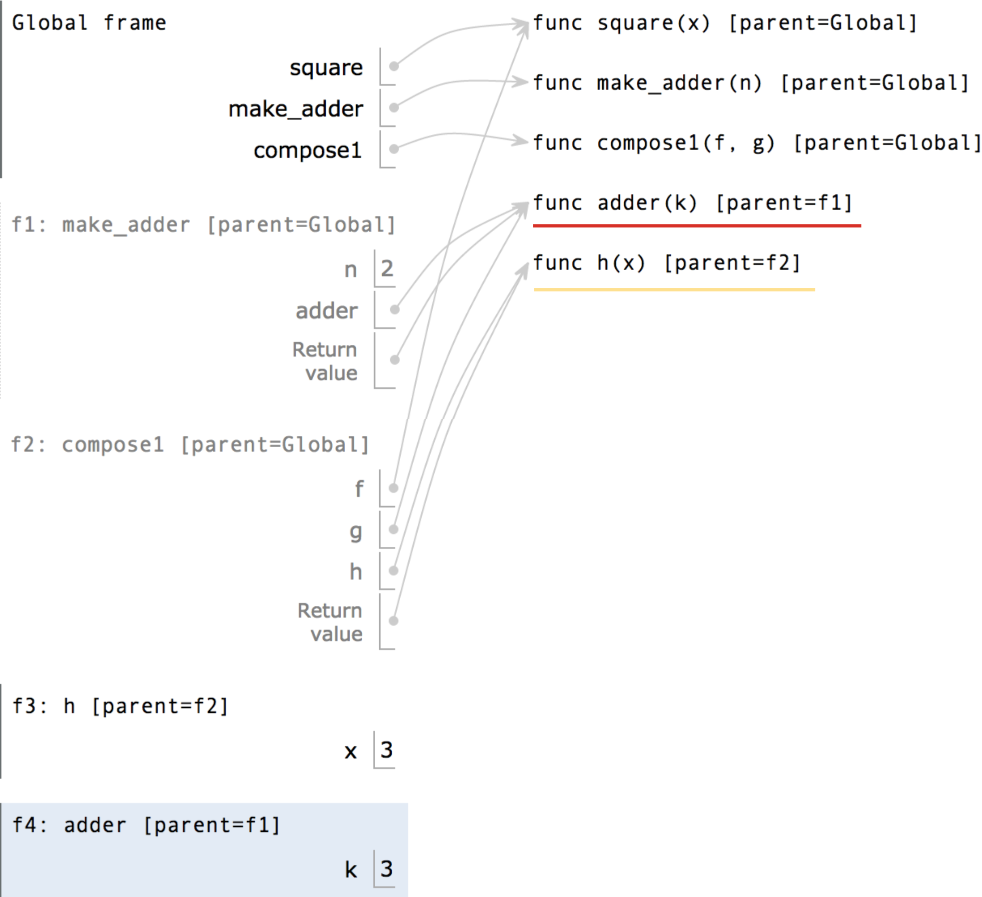

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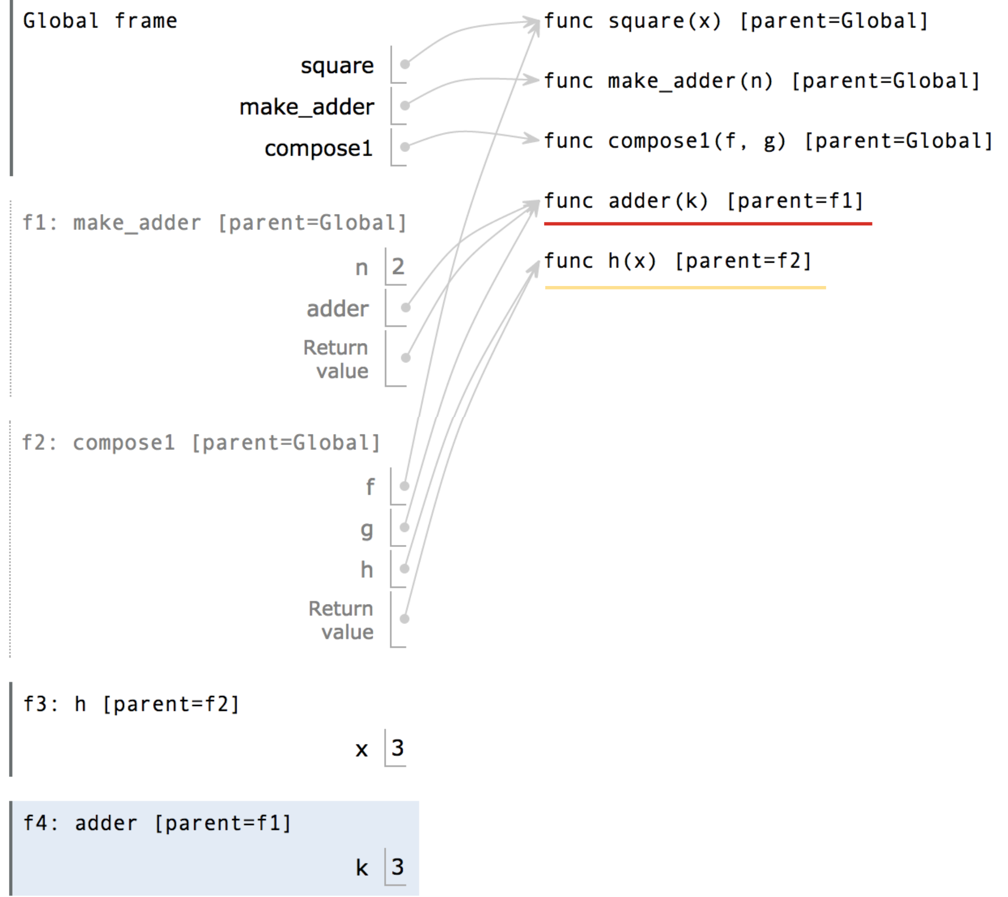

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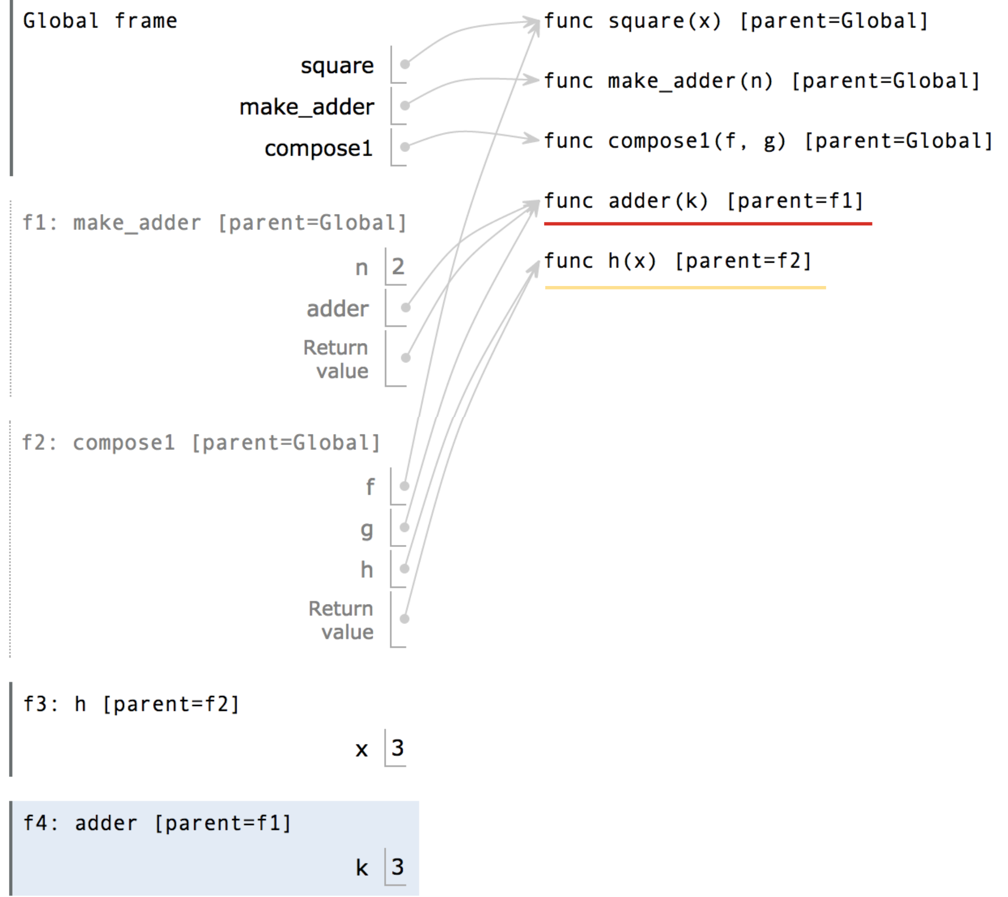

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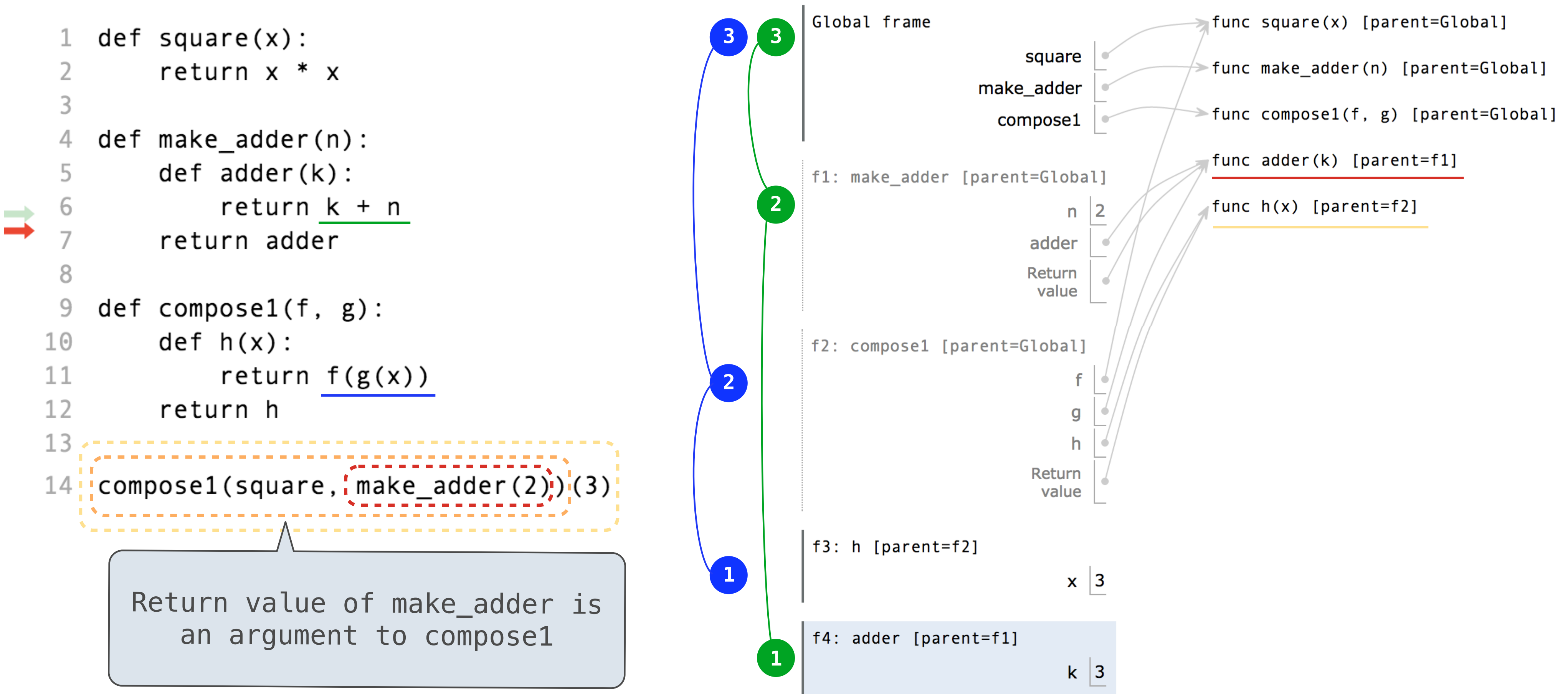

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Self-Reference

(Demo)

Returning a Function Using Its Own Name

```
1  def print_sums(n):
2      print(n)
3      def next_sum(k):
4          return print_sums(n+k)
5      return next_sum
6
7  print_sums(1)(3)(5)
```

Frames

Objects

Global frame

print_sums

func print_sums(n) [parent=Global]

func next_sum(k) [parent=f1]

func next_sum(k) [parent=f3]

func next_sum(k) [parent=f5]

f1: print_sums [parent=Global]

n 1

next_sum

Return value

f2: next_sum [parent=f1]

k 3

Return value

f3: print_sums [parent=Global]

n 4

next_sum

Return value

f4: next_sum [parent=f3]

k 5

f5: print_sums [parent=Global]

n 9

next_sum

Return value

http://pythontutor.com/composingprograms.html#code=def%20print_all%28k%29%3A%0A%20%20%20%20print%28k%29%0A%20%20%20%20return%20print_all%0A%20%20%20%20%0Aprint_all%281%29%283%29%285%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

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