Tree Recursion

Announcements

Order of Recursive Calls

The Cascade Function

(Demo)

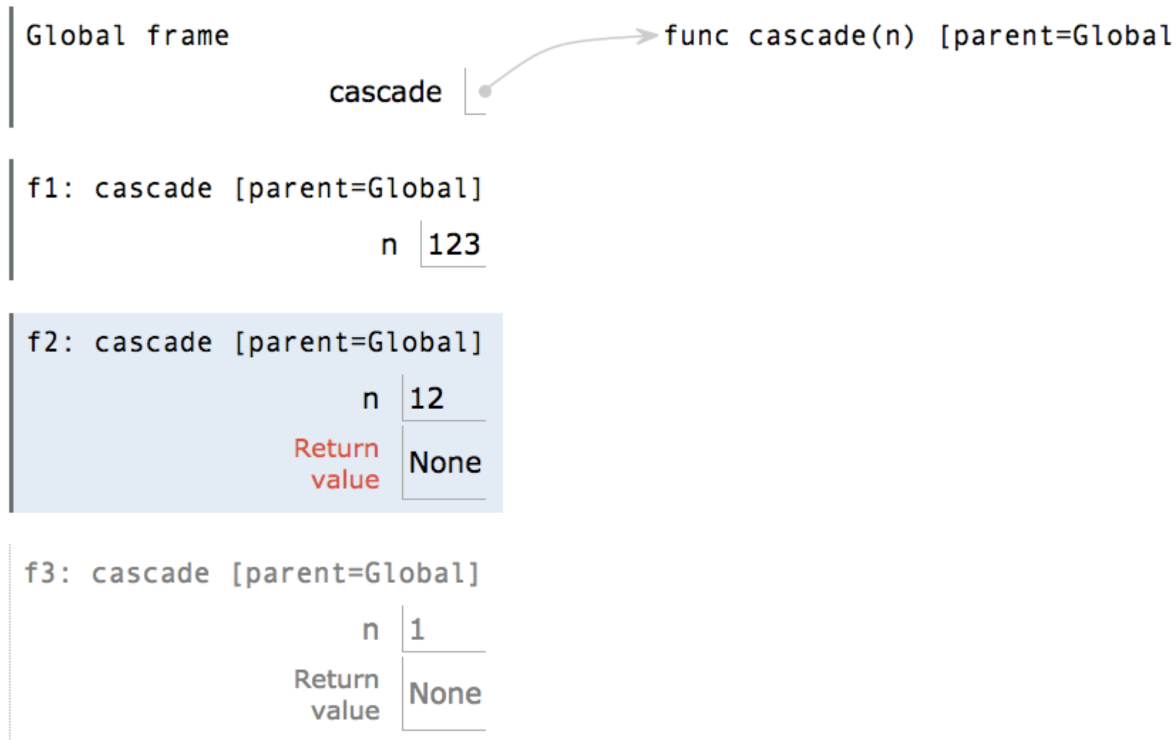
Interactive Diagram

The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



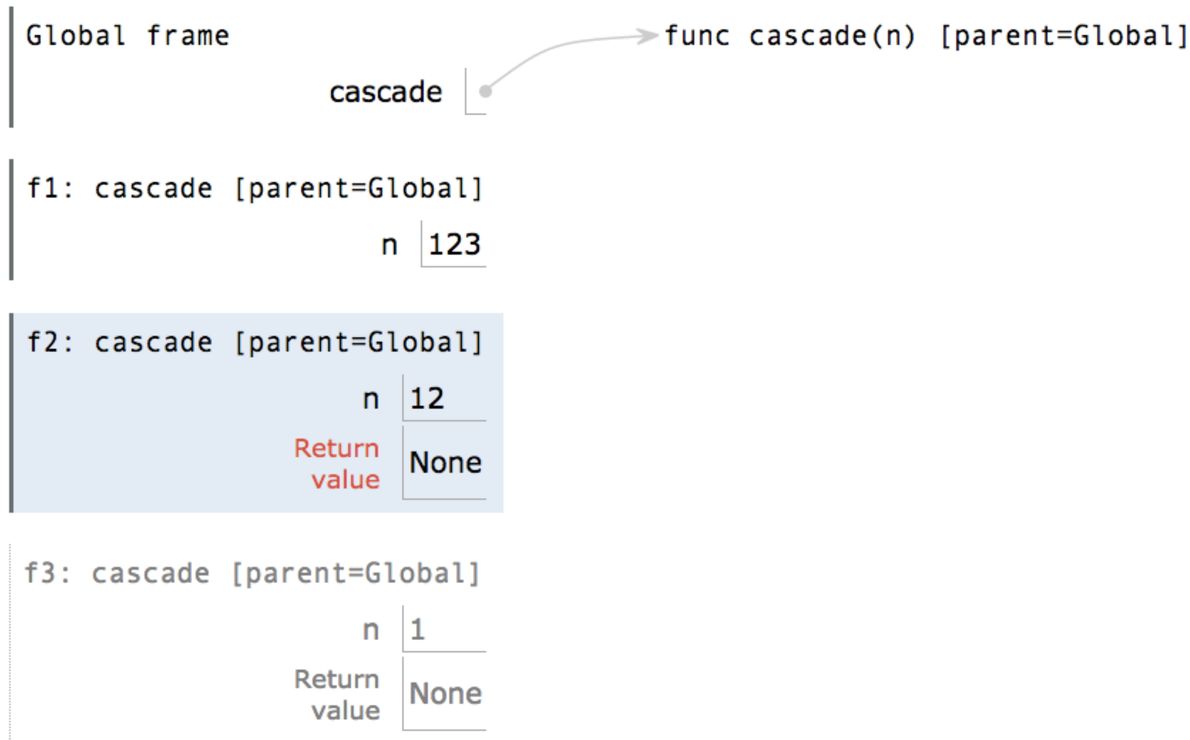
Interactive Diagram

The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



Program output:

```
123
12
1
12
```

Interactive Diagram

The Cascade Function

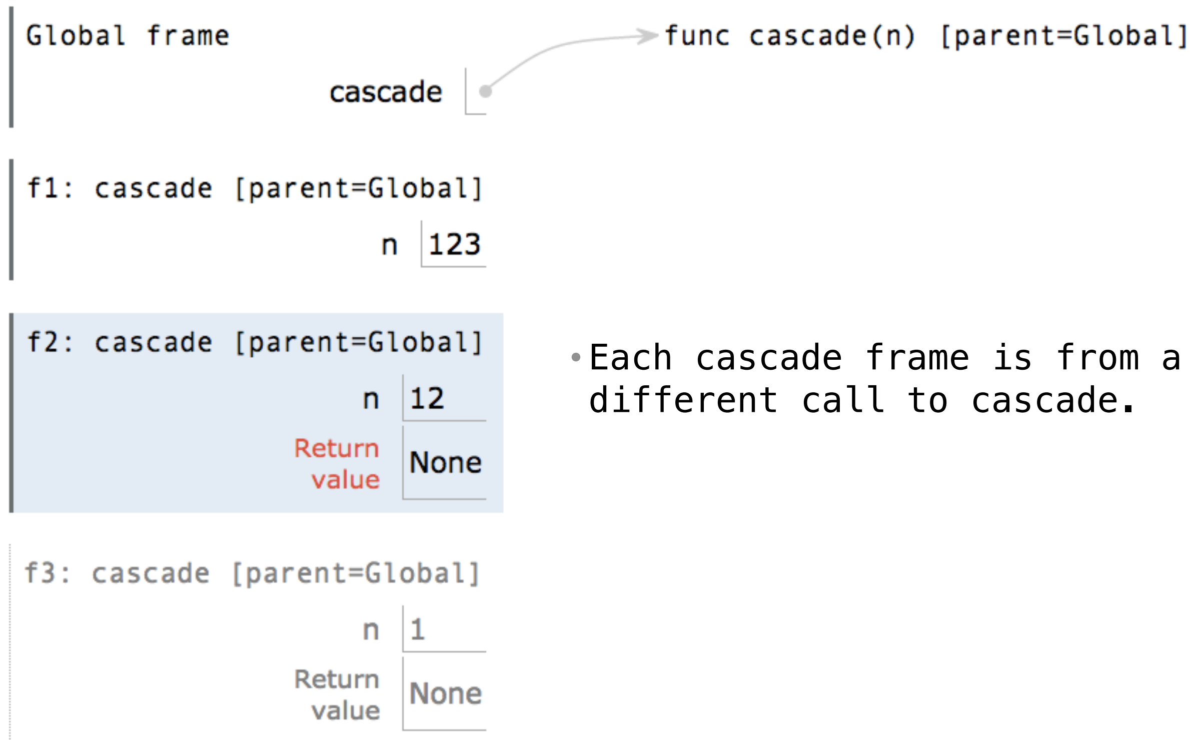
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each cascade frame is from a different call to cascade.

Interactive Diagram

The Cascade Function

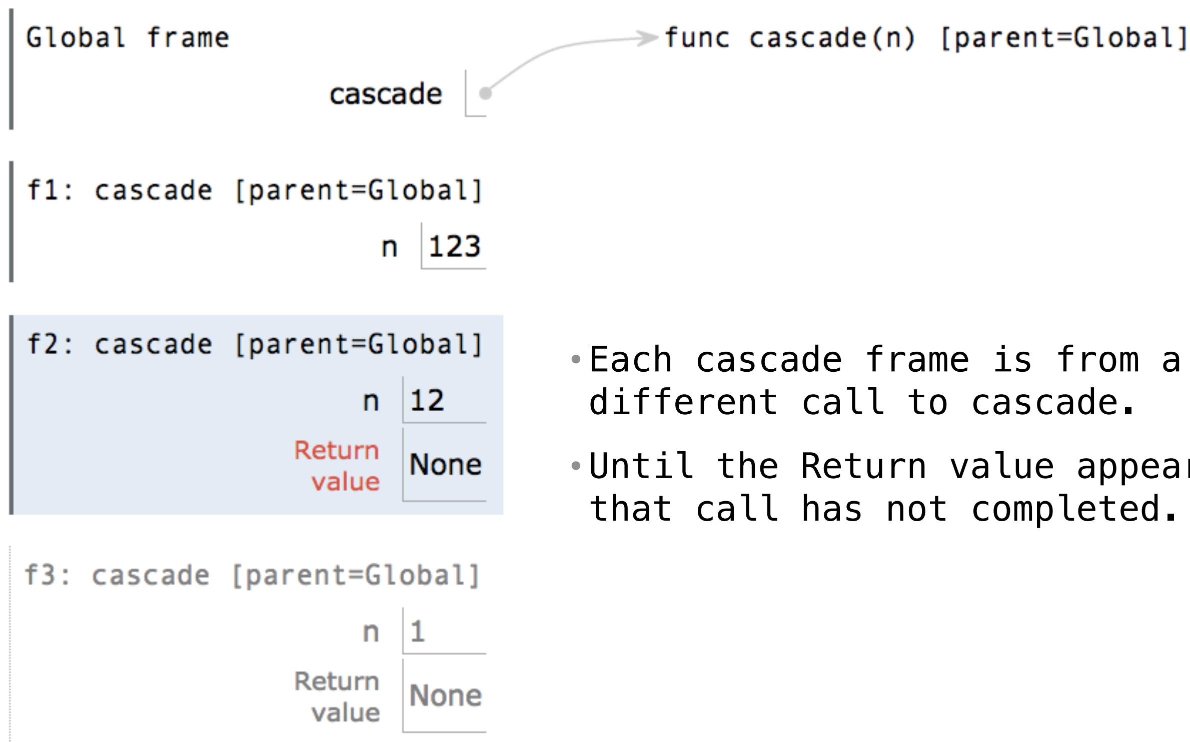
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.

Interactive Diagram

The Cascade Function

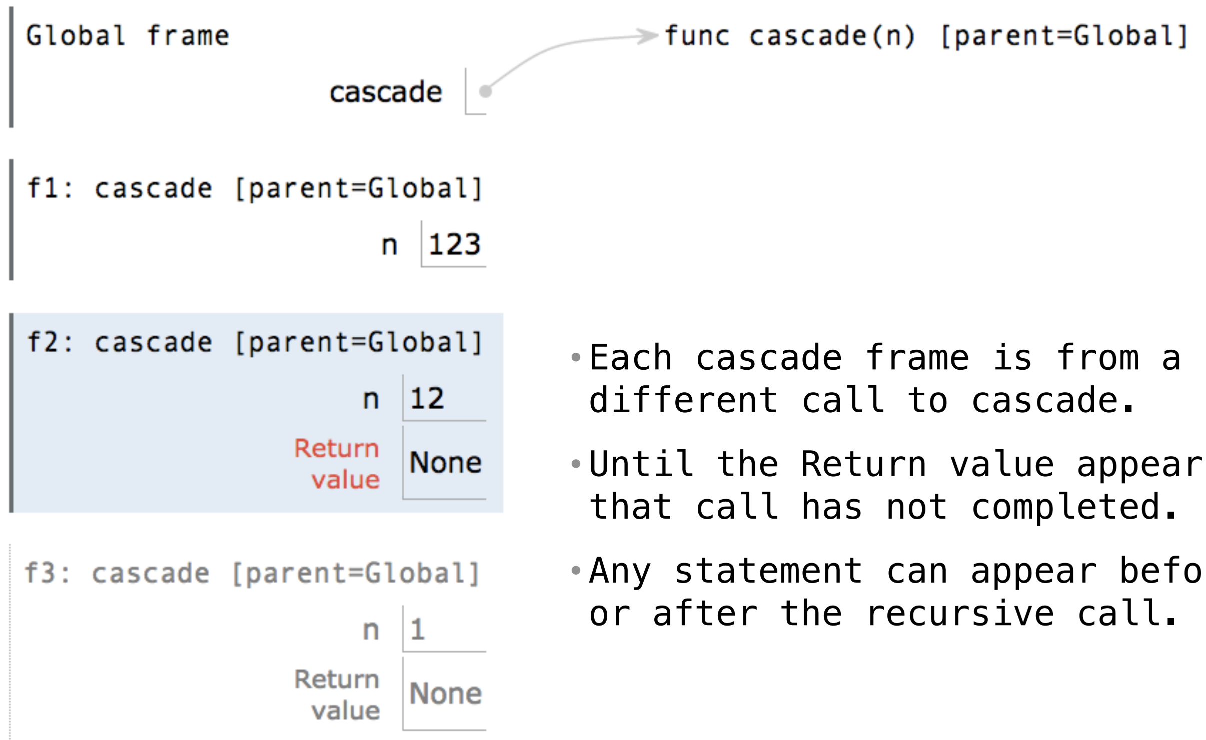
(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Interactive Diagram

The Cascade Function

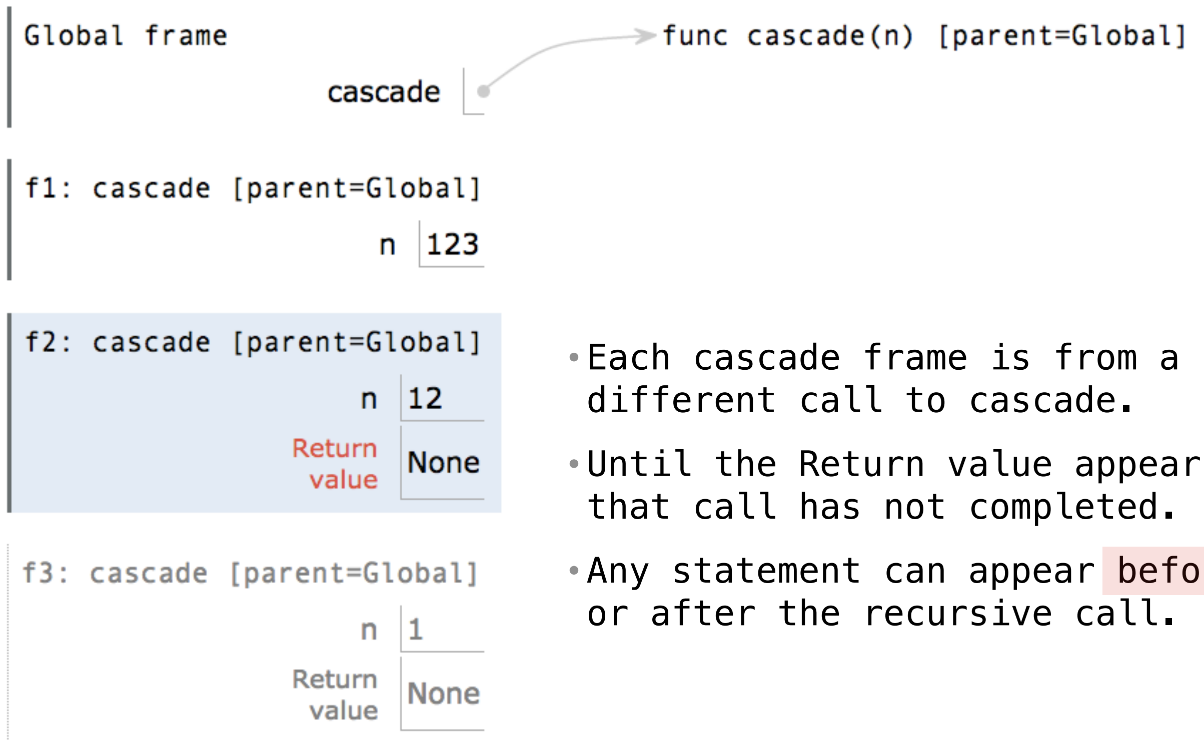
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Interactive Diagram

The Cascade Function

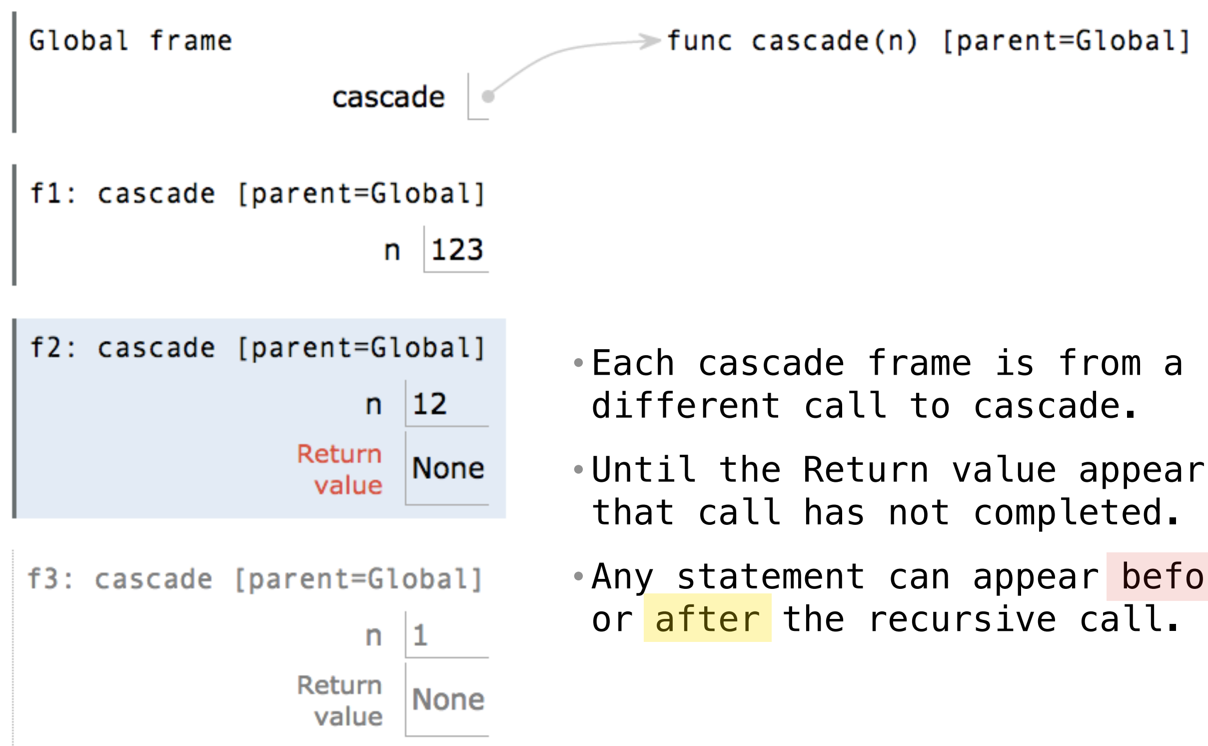
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Interactive Diagram

The Cascade Function

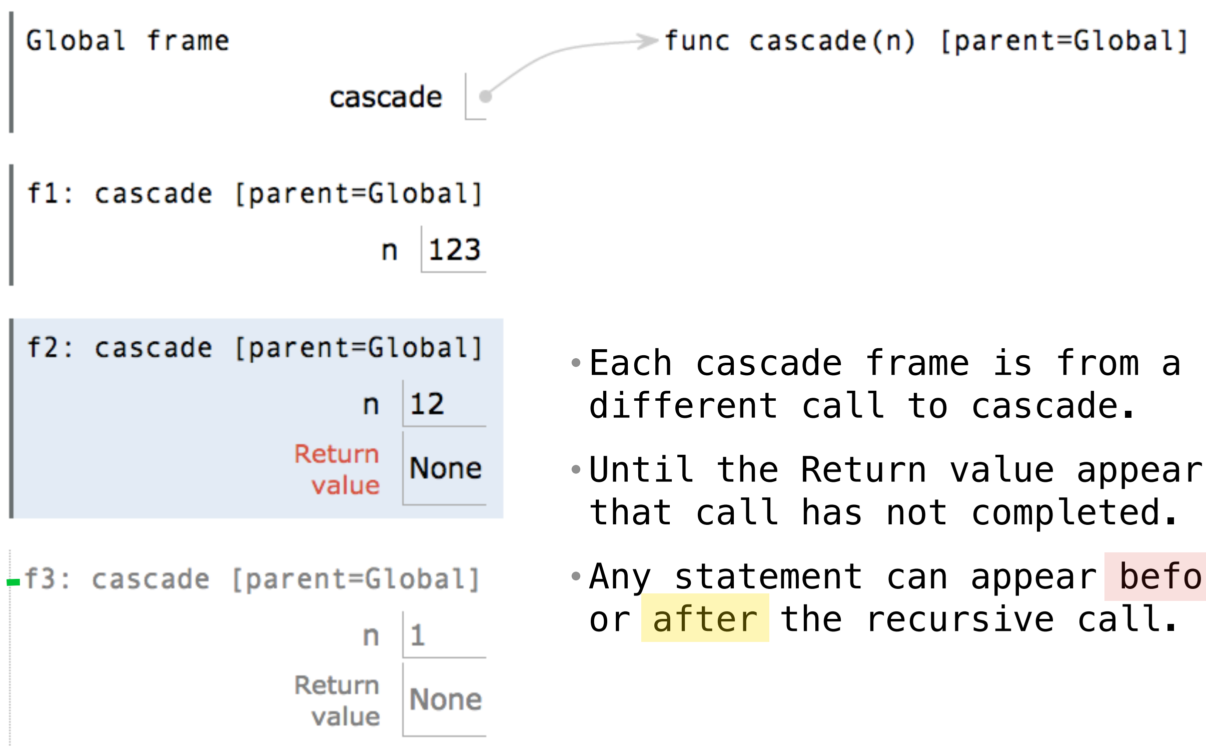
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

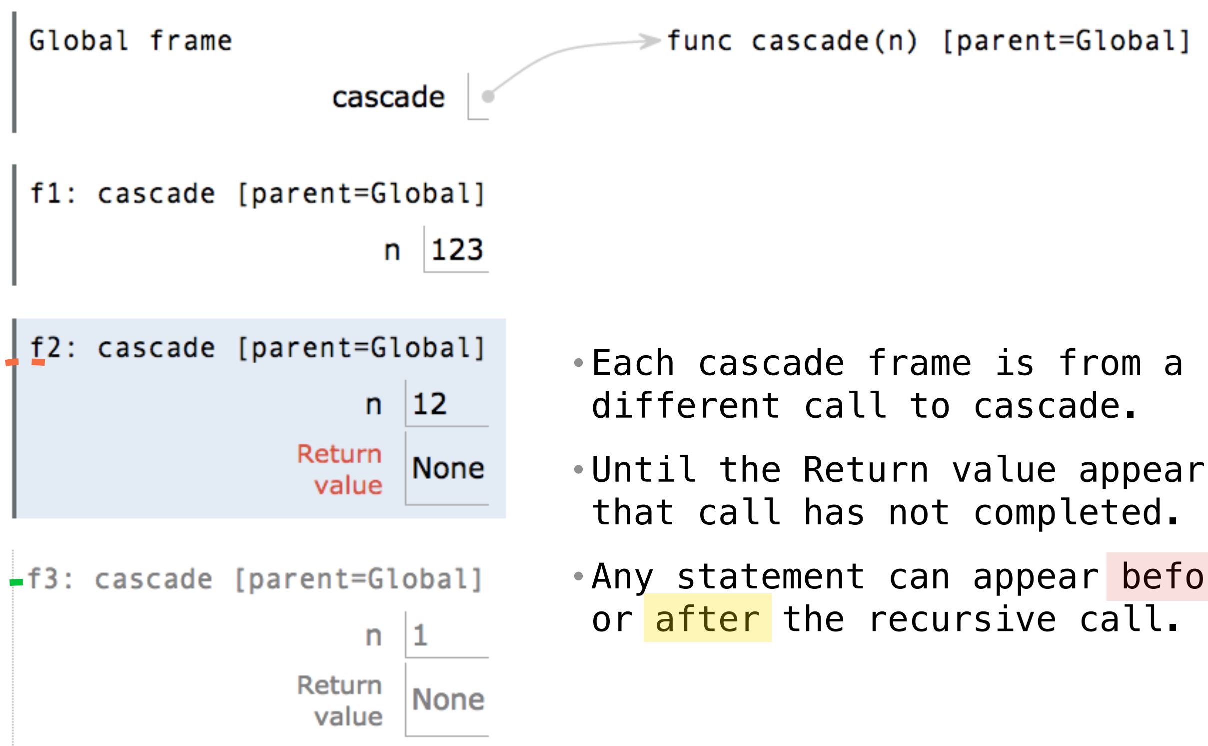
Interactive Diagram

The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



Program output:

```
123
12
1
12
```

- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Interactive Diagram

Two Definitions of Cascade

(Demo)

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure

Example: Inverse Cascade

Inverse Cascade

Write a function that prints an inverse cascade:

Inverse Cascade

```
Write a function that prints an inverse cascade:
```

```
1
12
123
1234
123
12
1
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow =   lambda n: f_then_g(                    )
shrink = lambda n: f_then_g(                    )
```

Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow =   lambda n: f_then_g(grow,  print,  n//10)
shrink = lambda n: f_then_g(print, shrink, n//10)
```

Tree Recursion

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,		

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 9,227,465

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	9,227,465

```
def fib(n):
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 9,227,465

```
def fib(n):
    if n == 0:
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	9,227,465

```
def fib(n):
    if n == 0:
        return 0
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 9,227,465

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 9,227,465

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	9,227,465

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	9,227,465

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

A Tree-Recursive Process

The computational process of fib evolves into a tree structure

A Tree-Recursive Process

The computational process of fib evolves into a tree structure

fib(5)

A Tree-Recursive Process

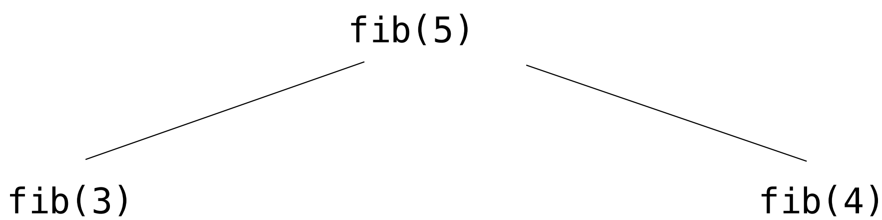
The computational process of fib evolves into a tree structure

fib(5)

fib(3)

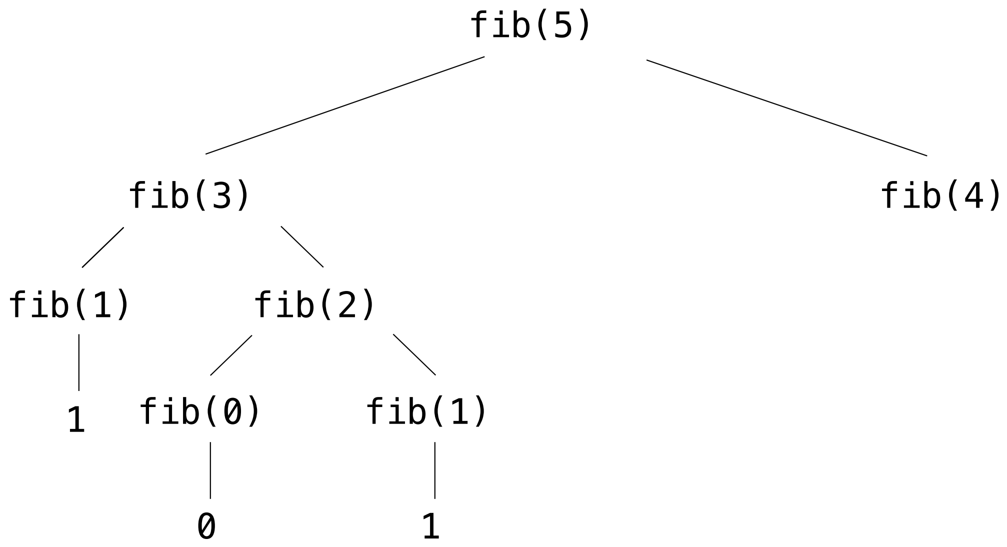
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



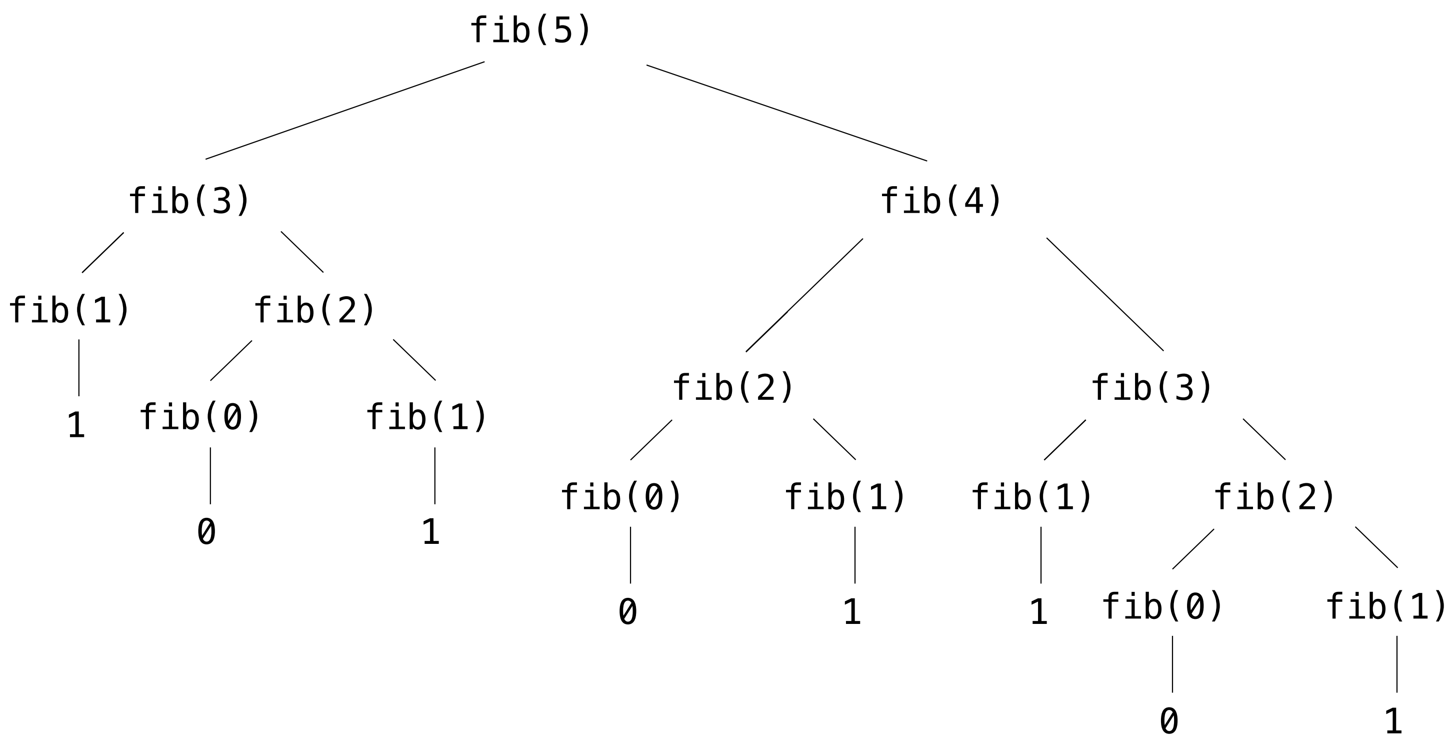
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



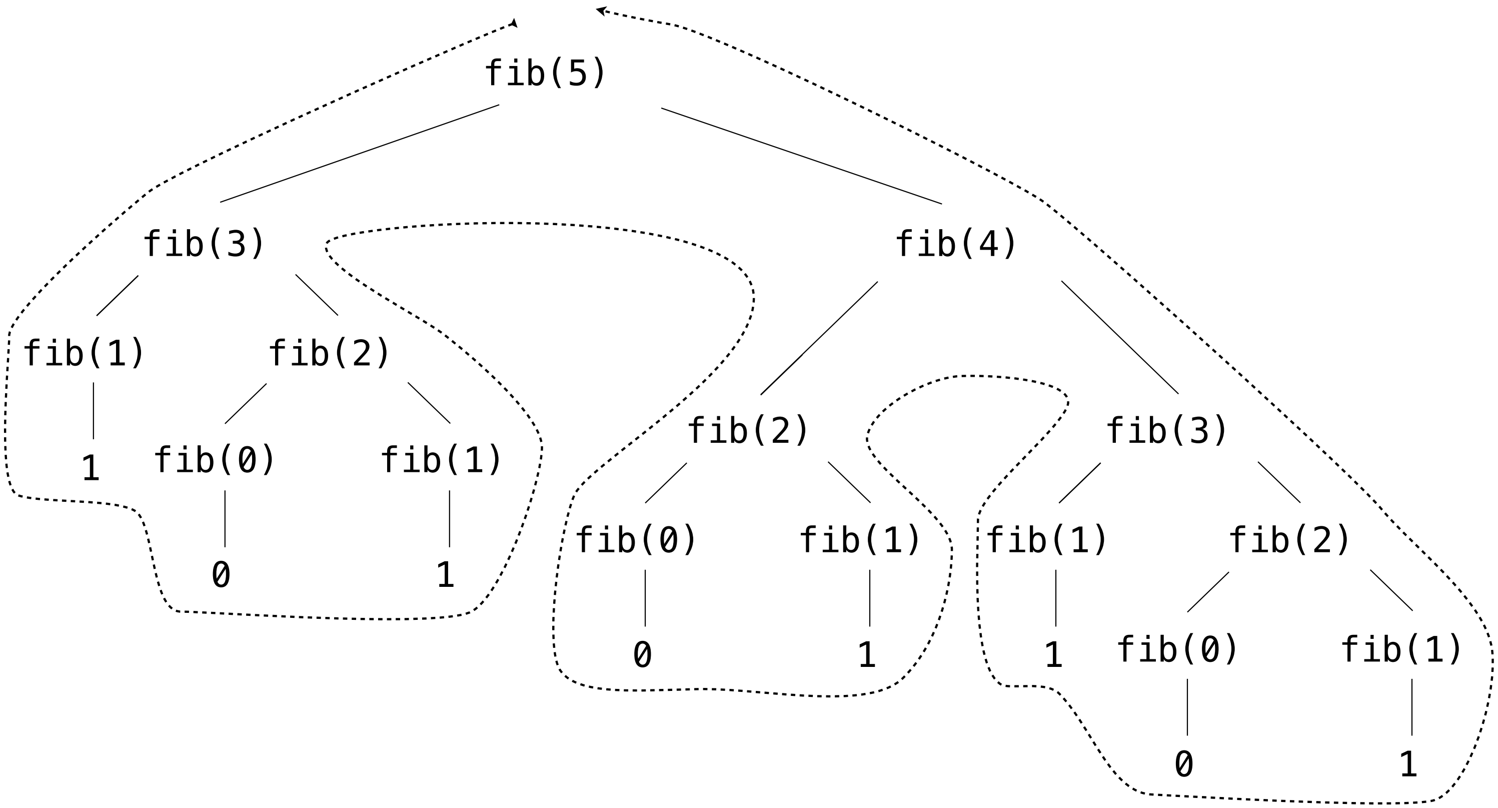
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



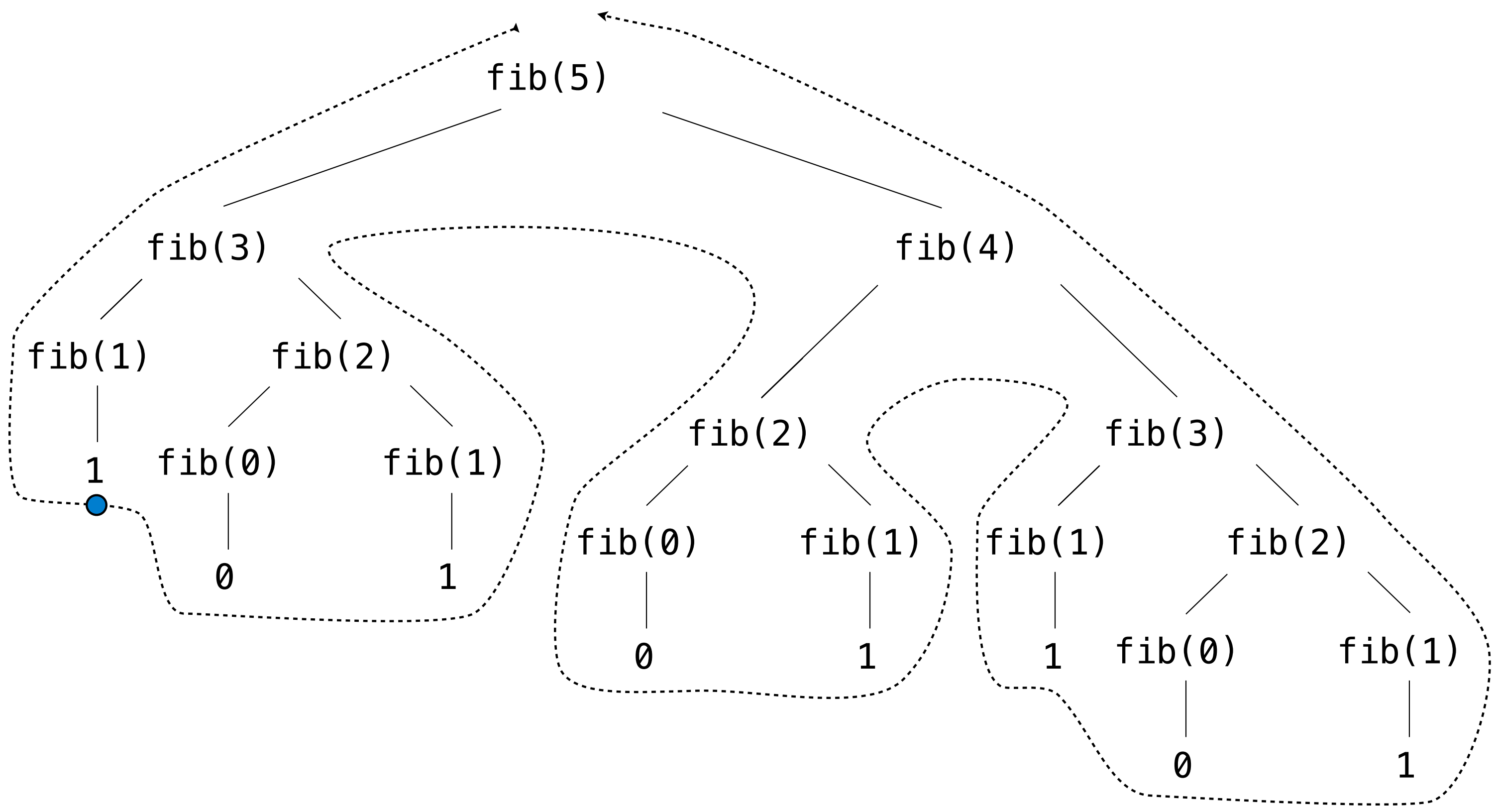
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



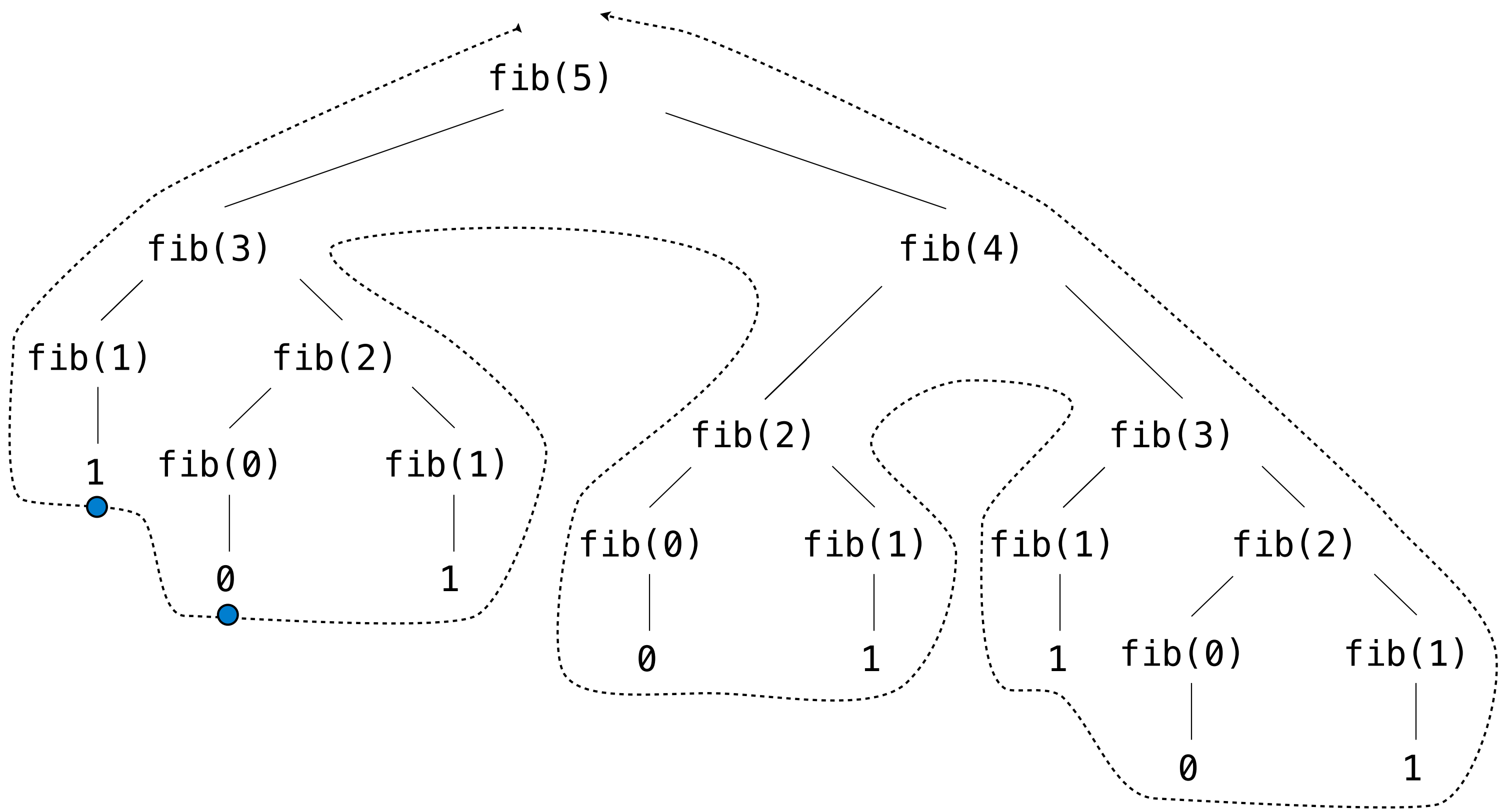
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



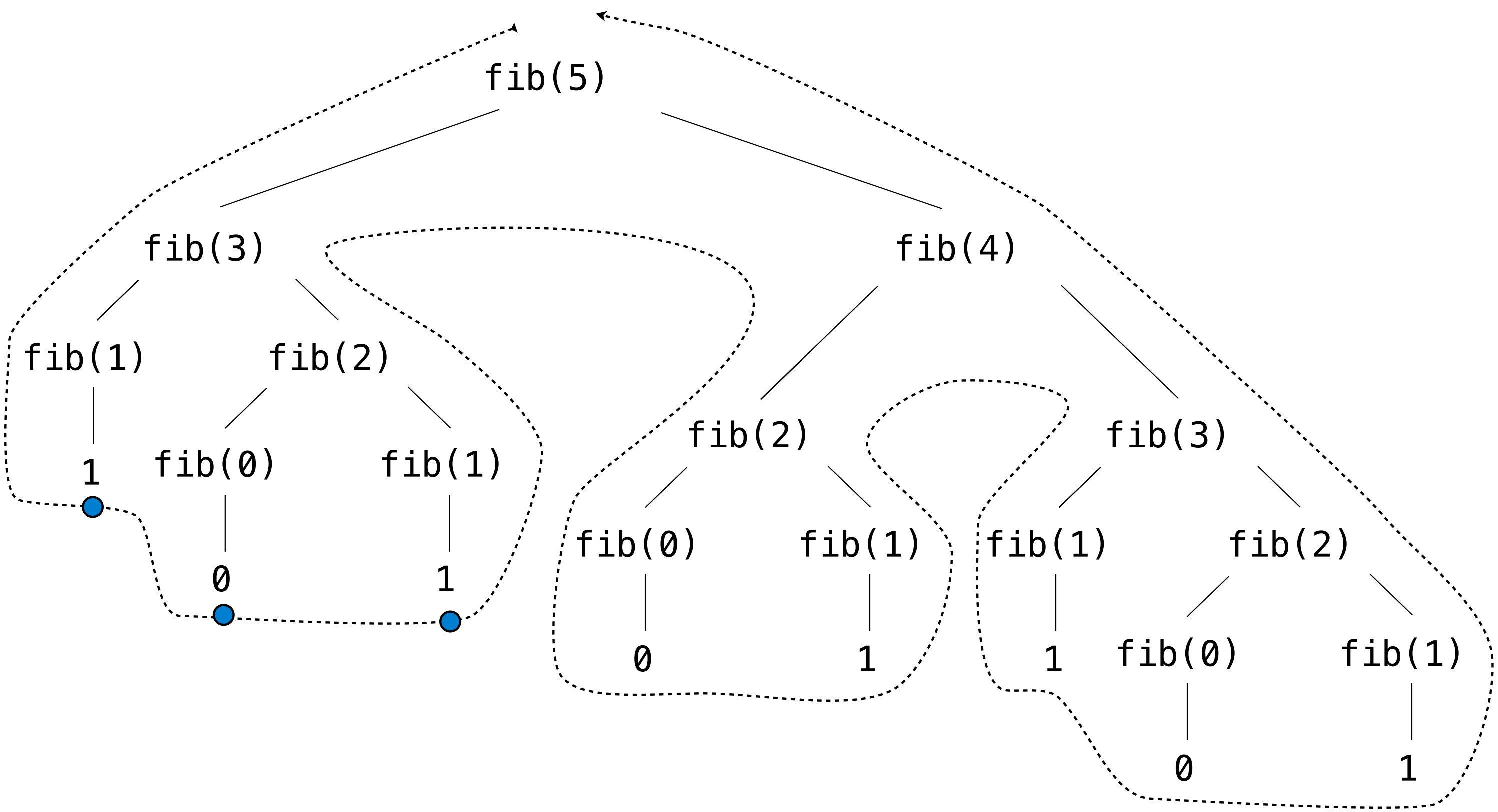
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



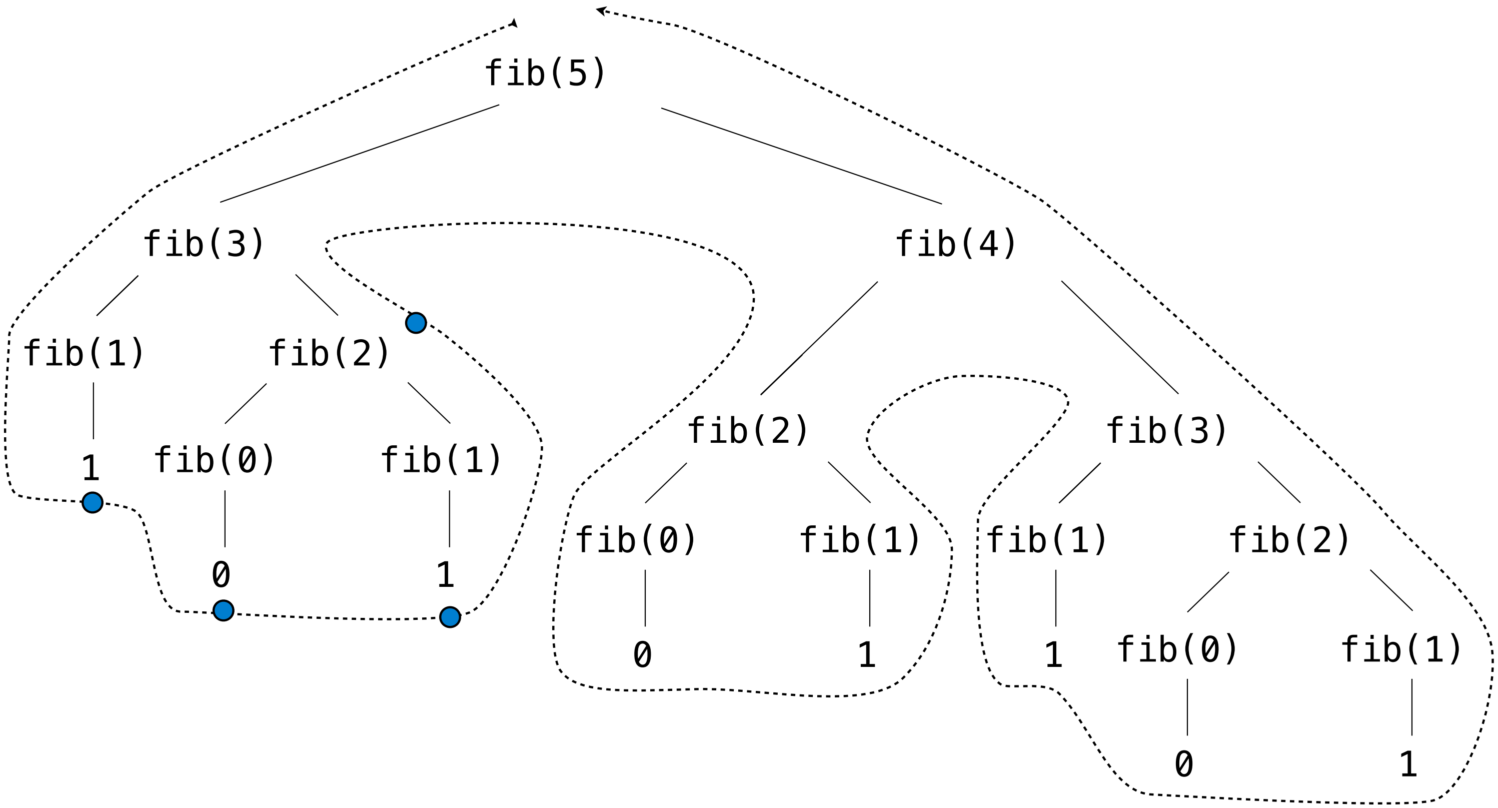
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



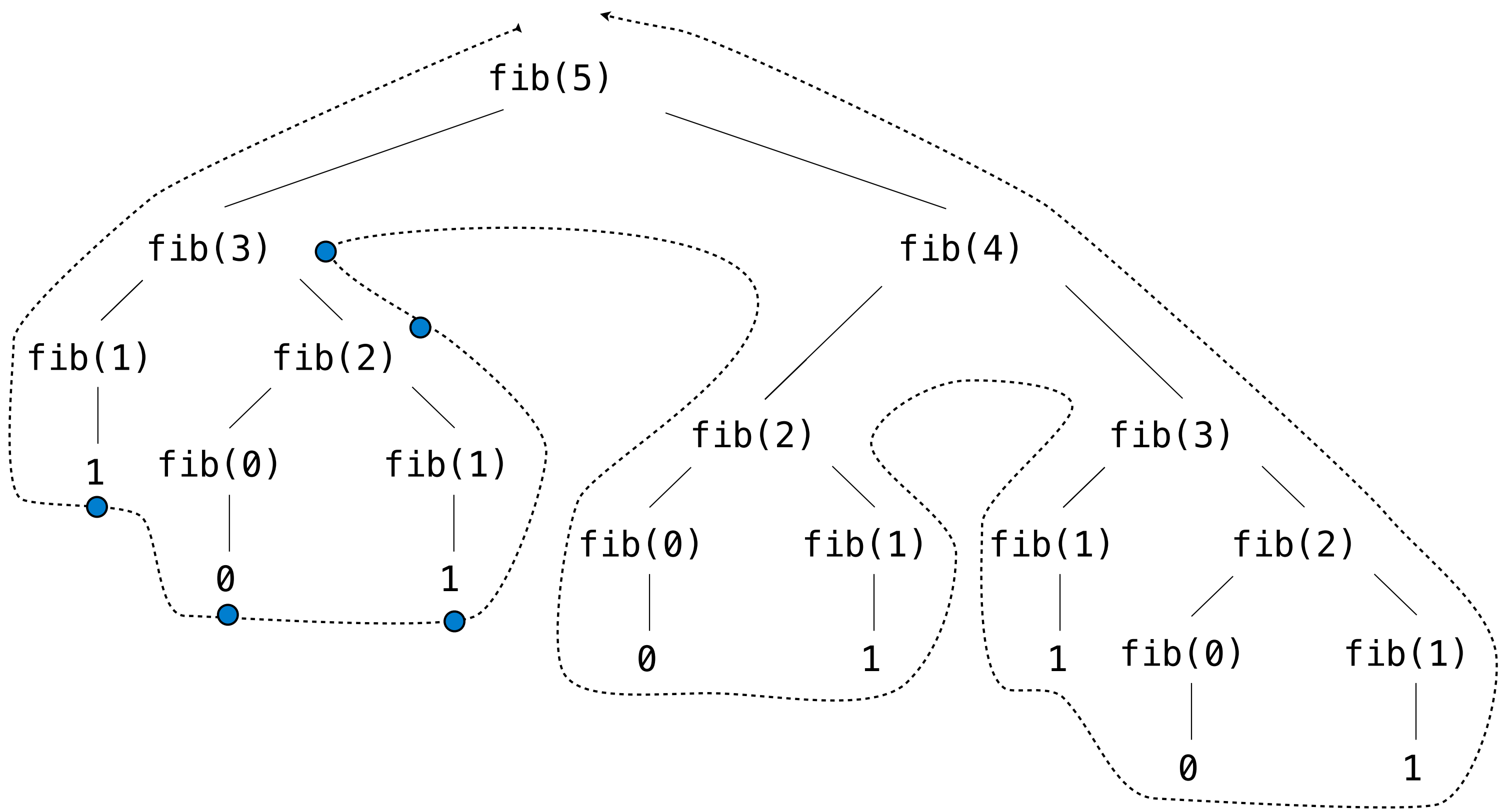
A Tree-Recursive Process

```
The computational process of fib evolves into a tree structure
```



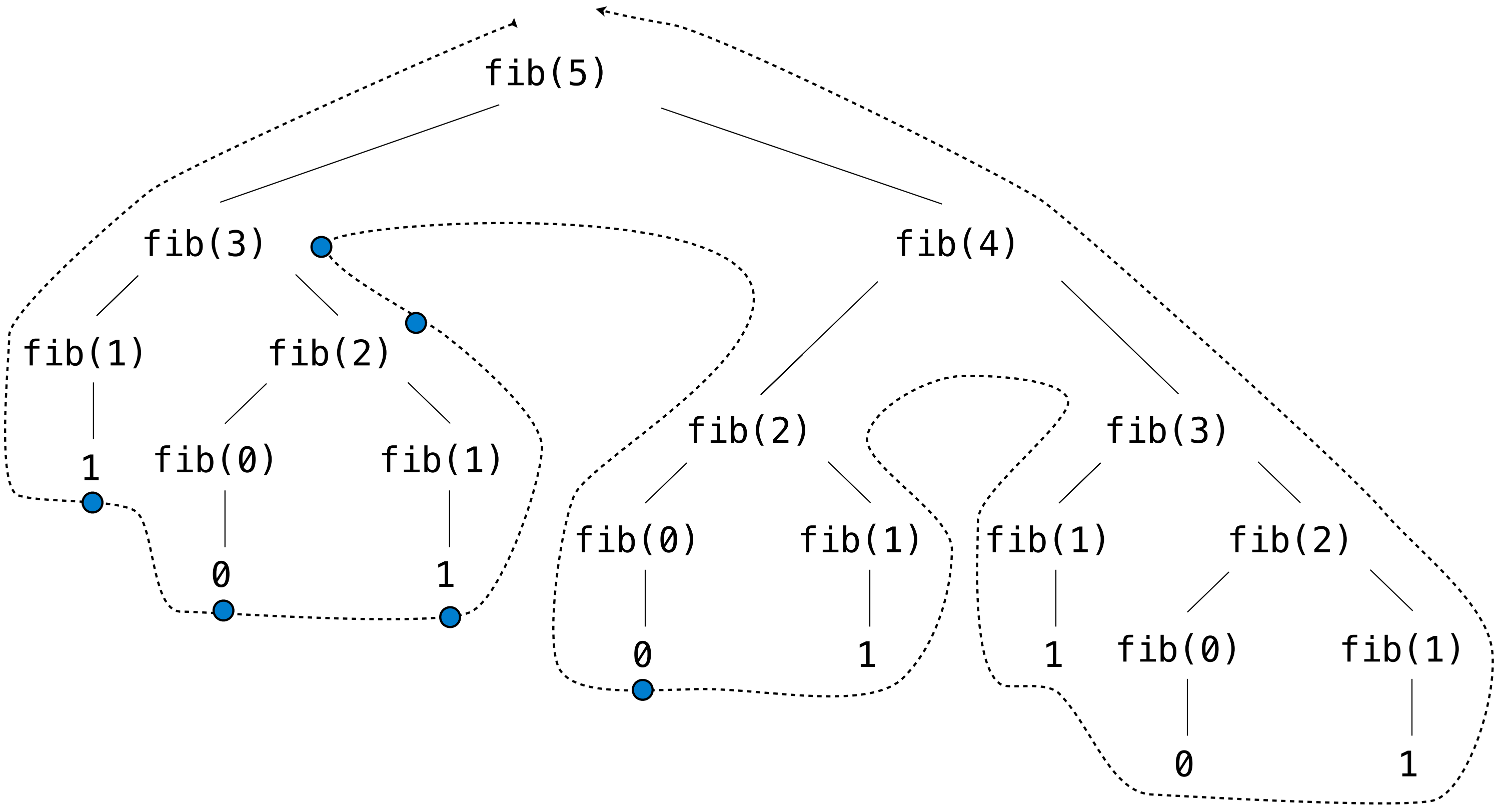
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



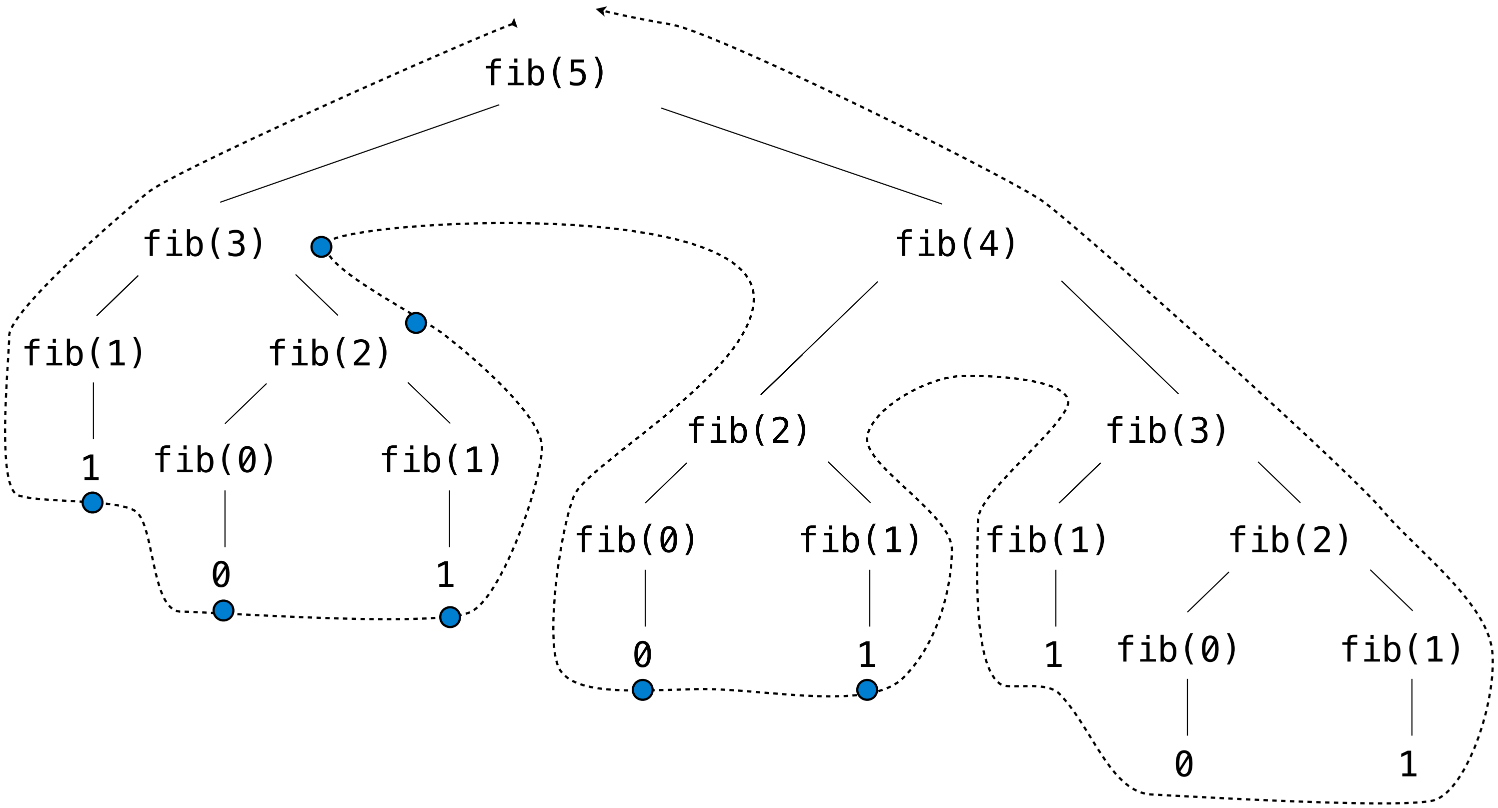
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



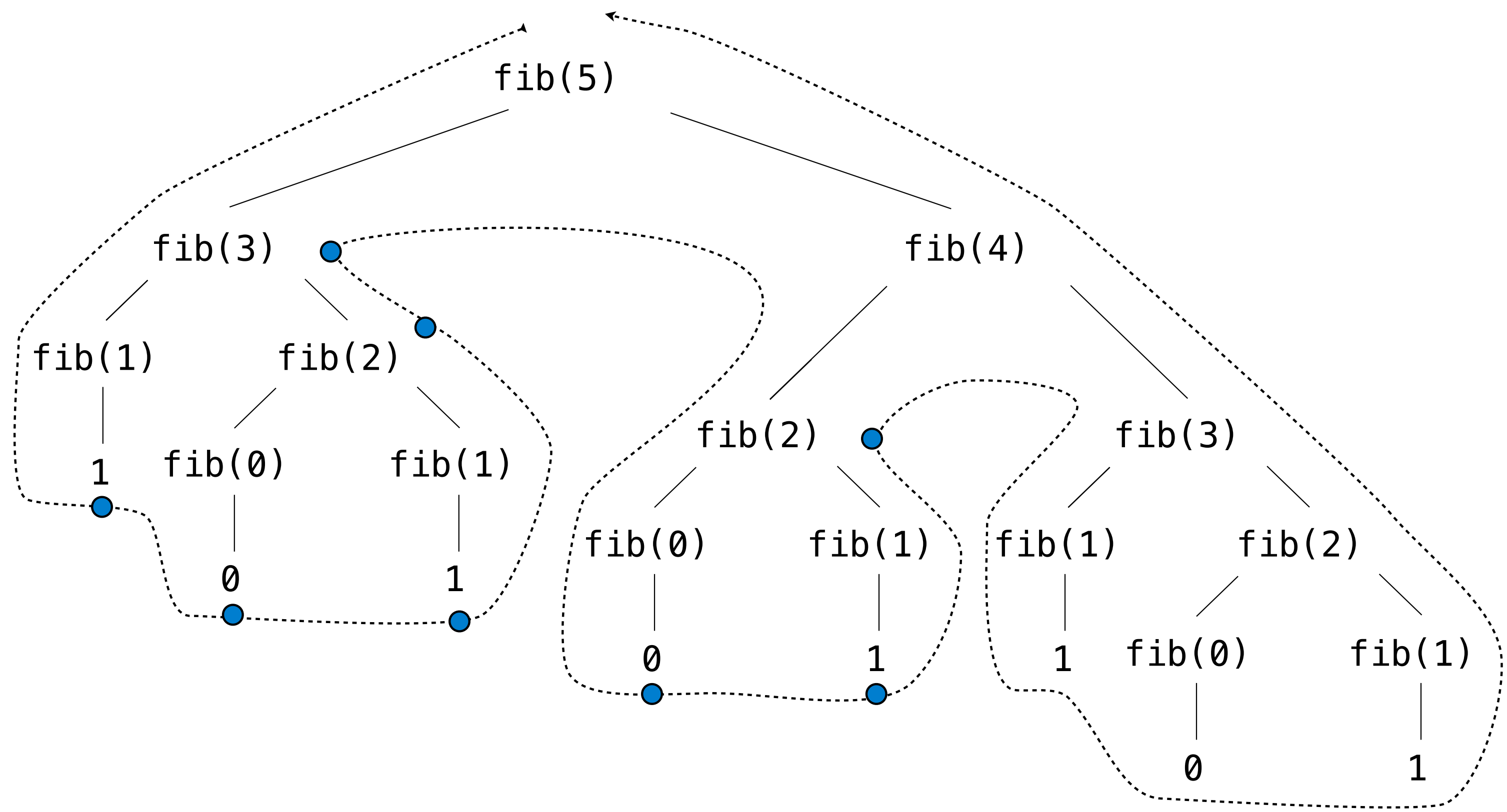
A Tree-Recursive Process

```
The computational process of fib evolves into a tree structure
```



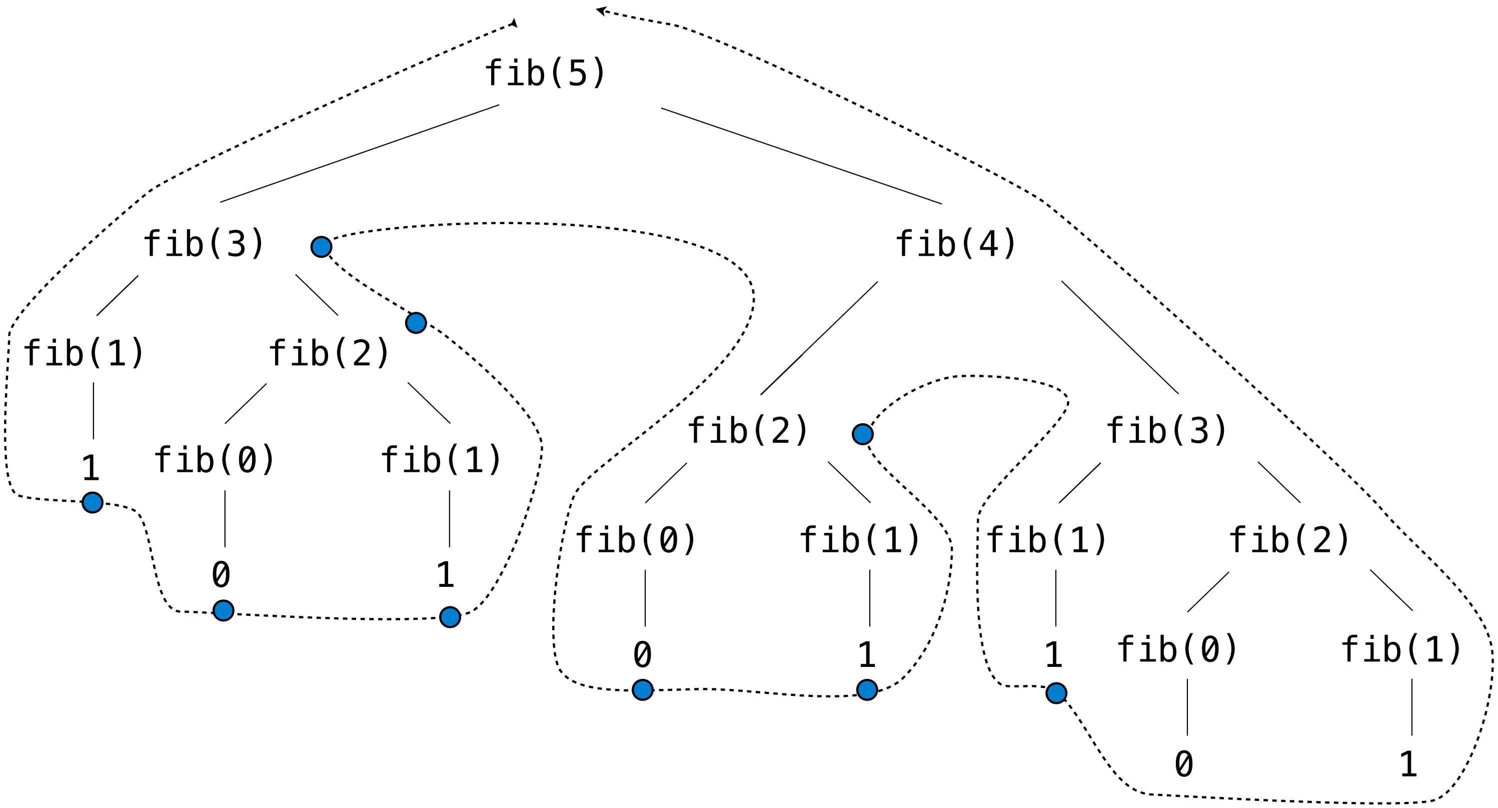
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



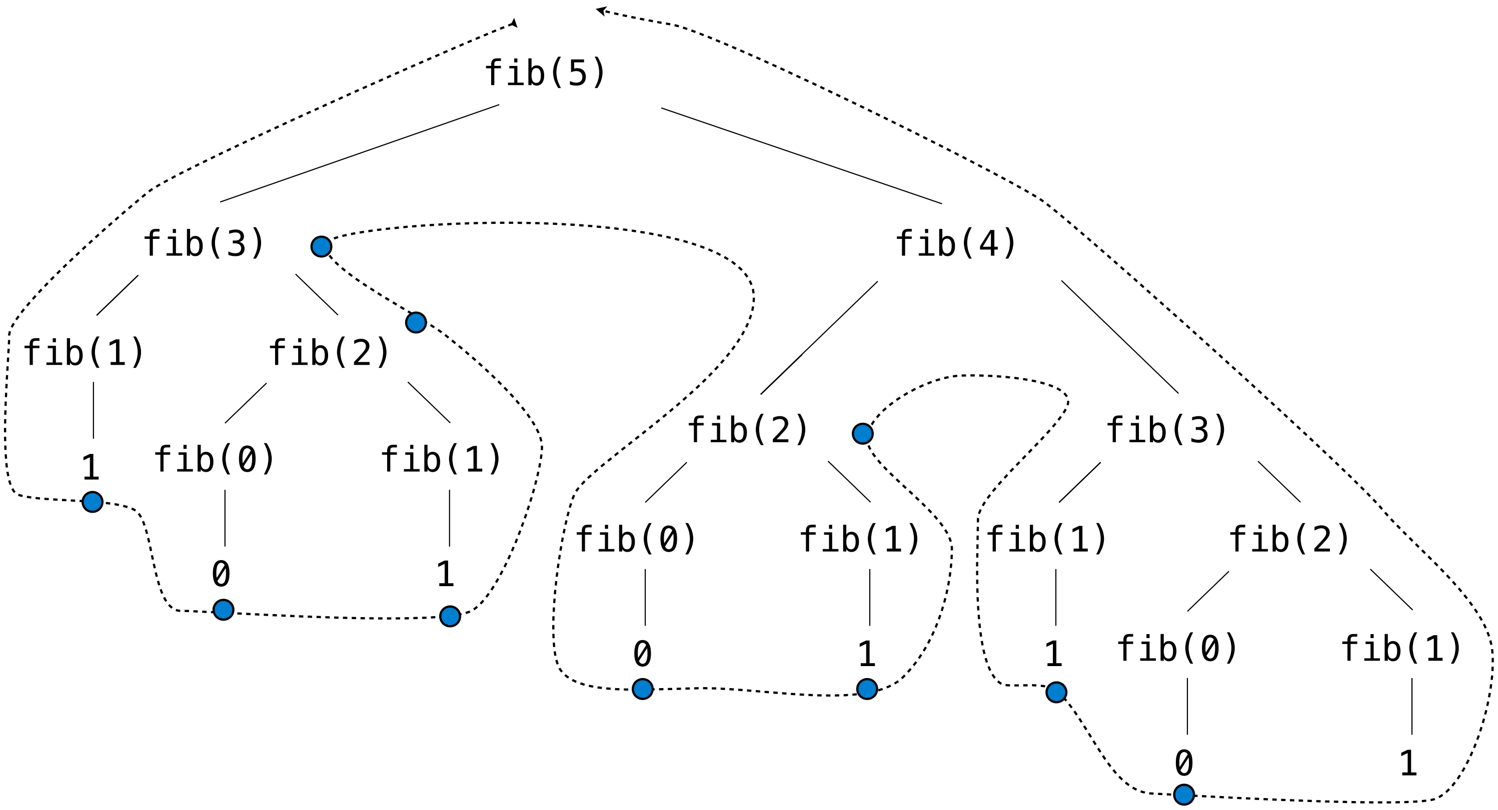
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



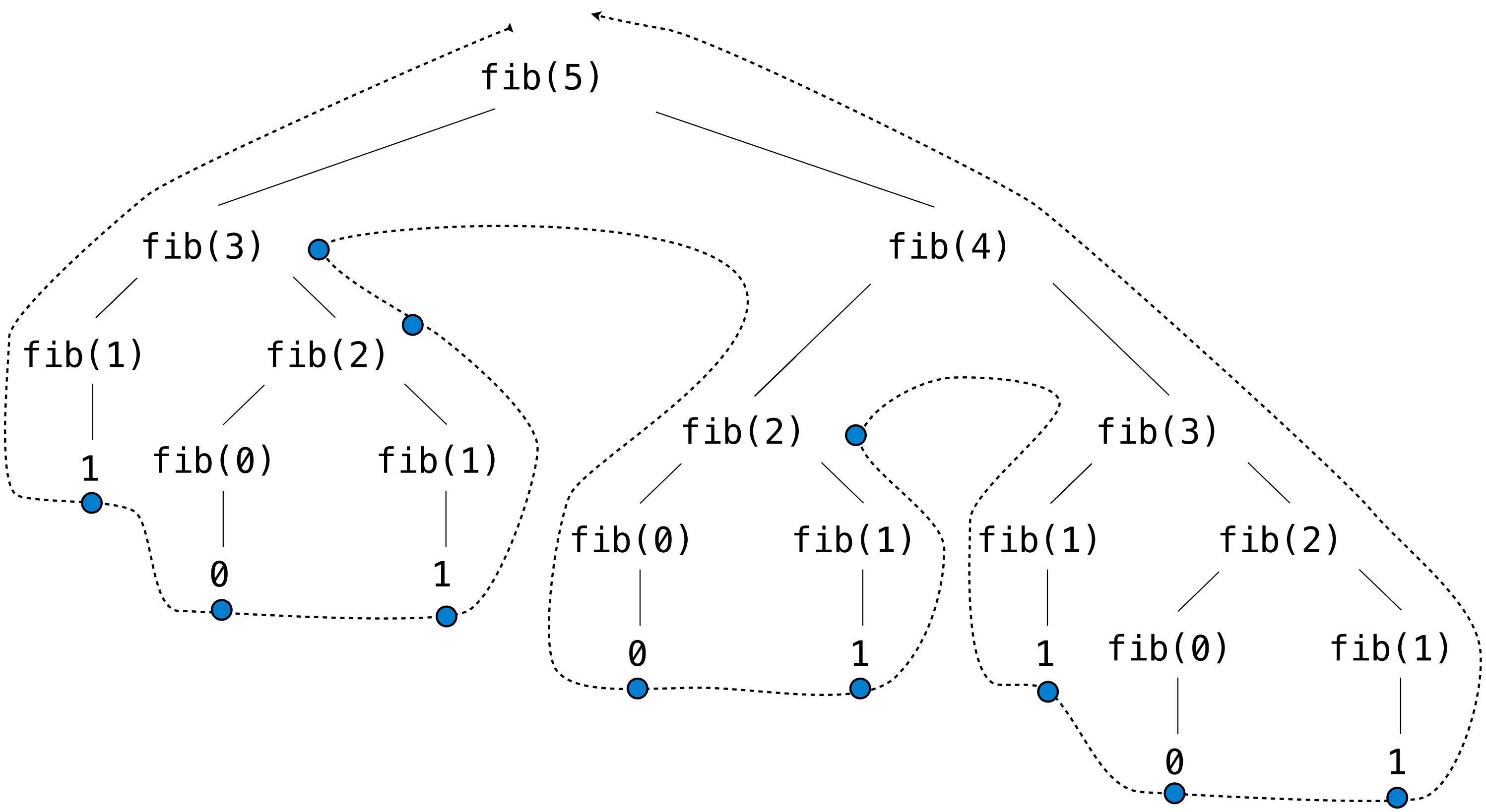
A Tree-Recursive Process

`The computational process of fib evolves into a tree structure`



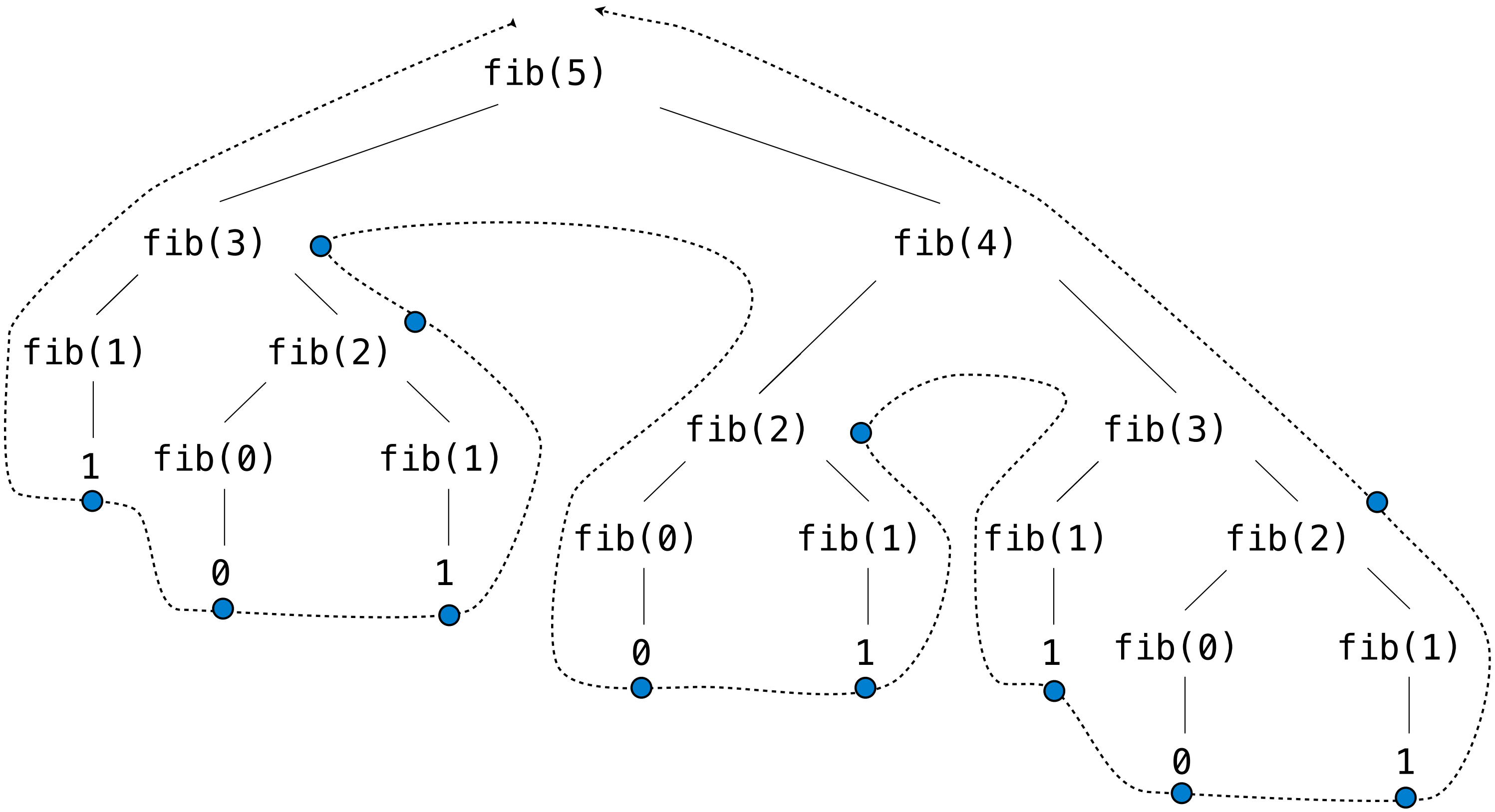
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



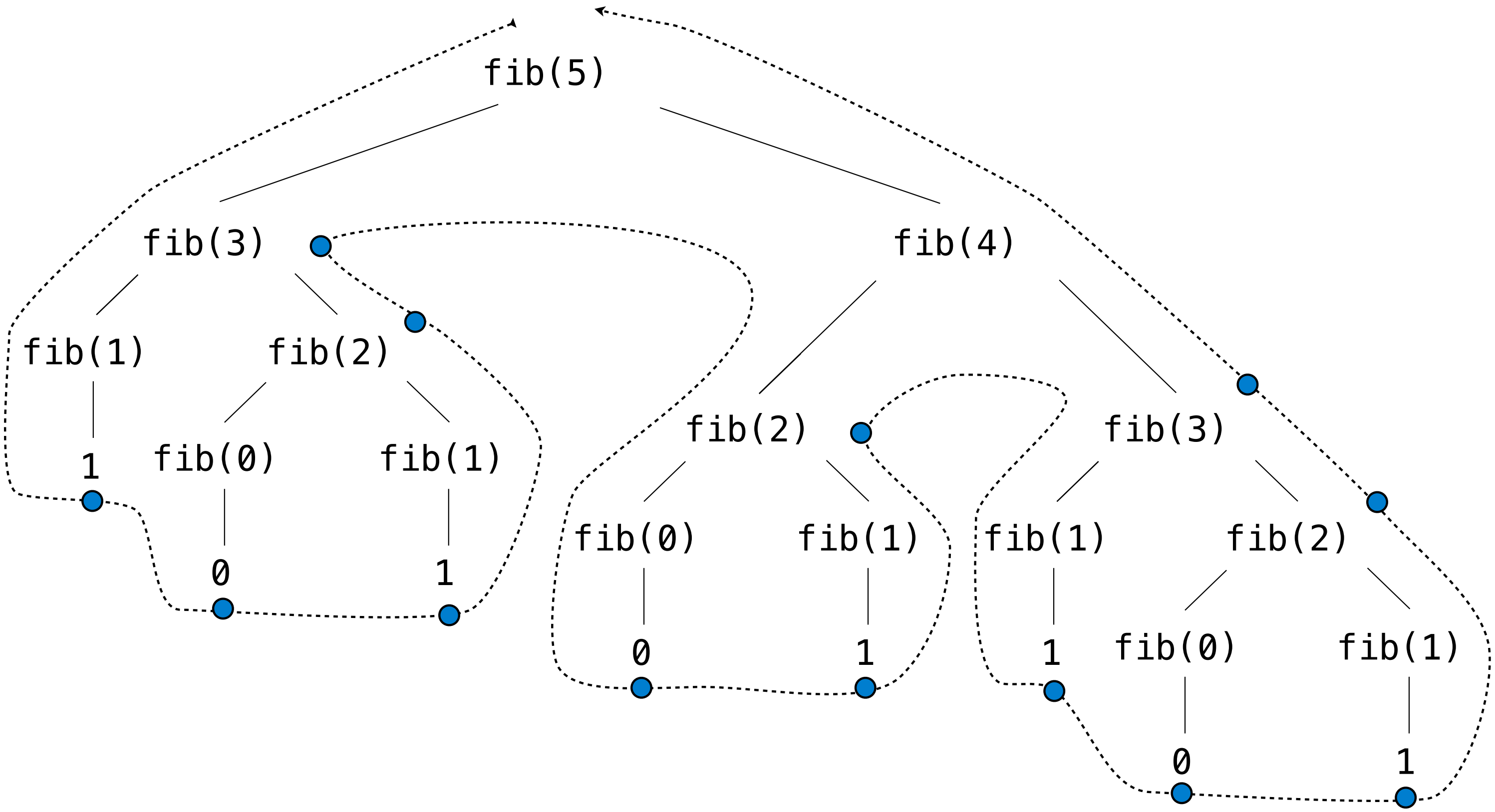
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



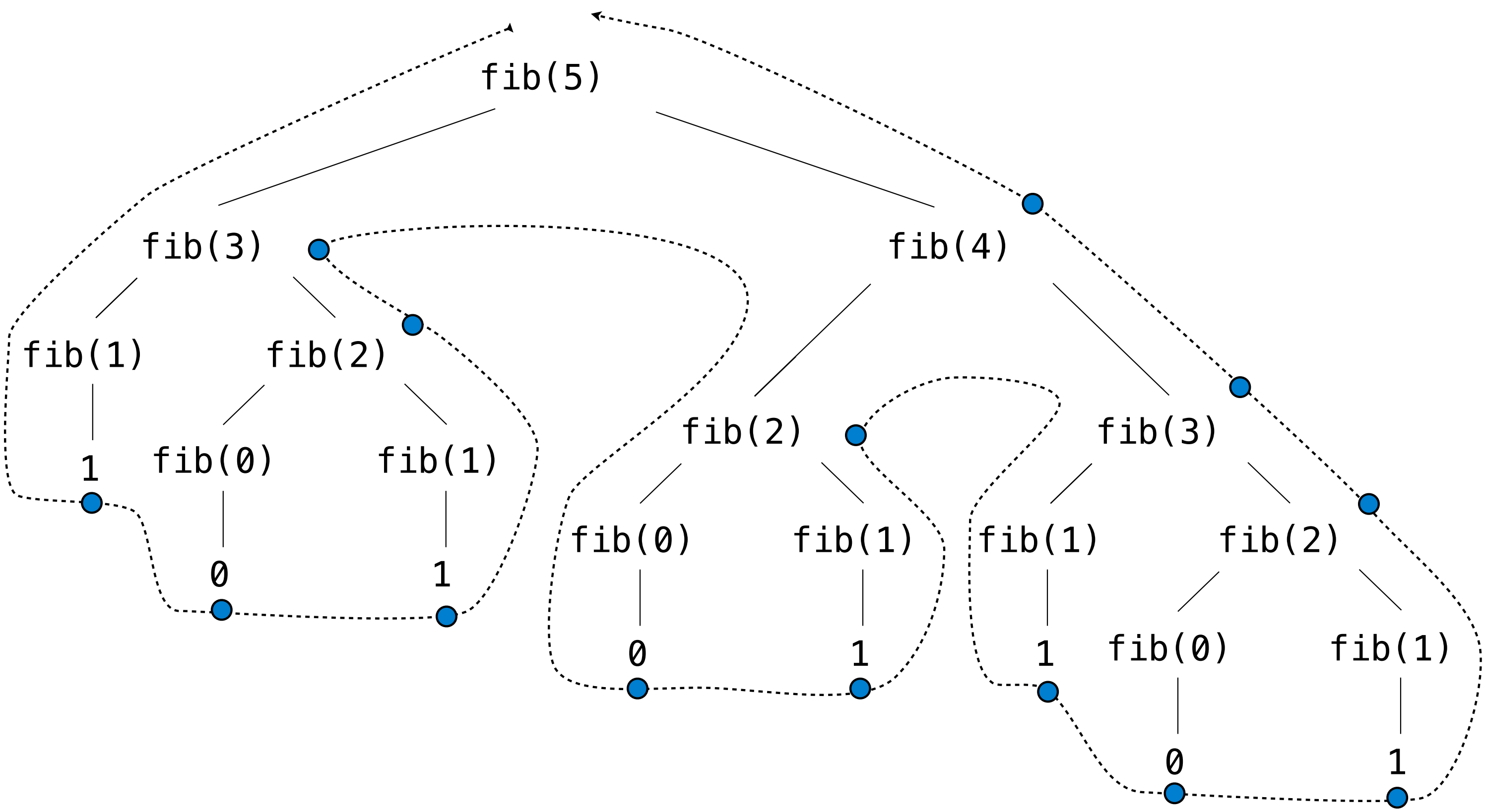
A Tree-Recursive Process

```
The computational process of fib evolves into a tree structure
```



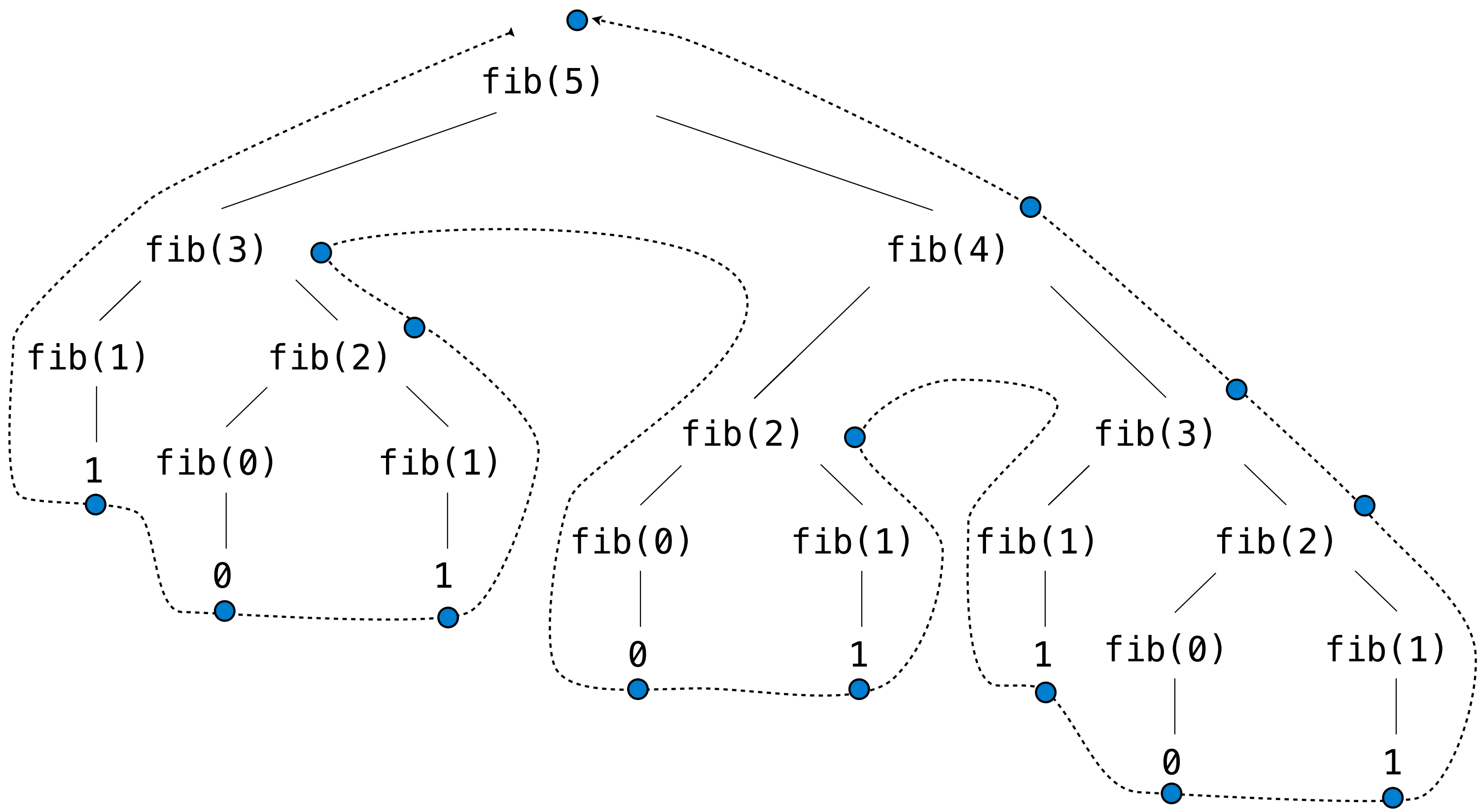
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



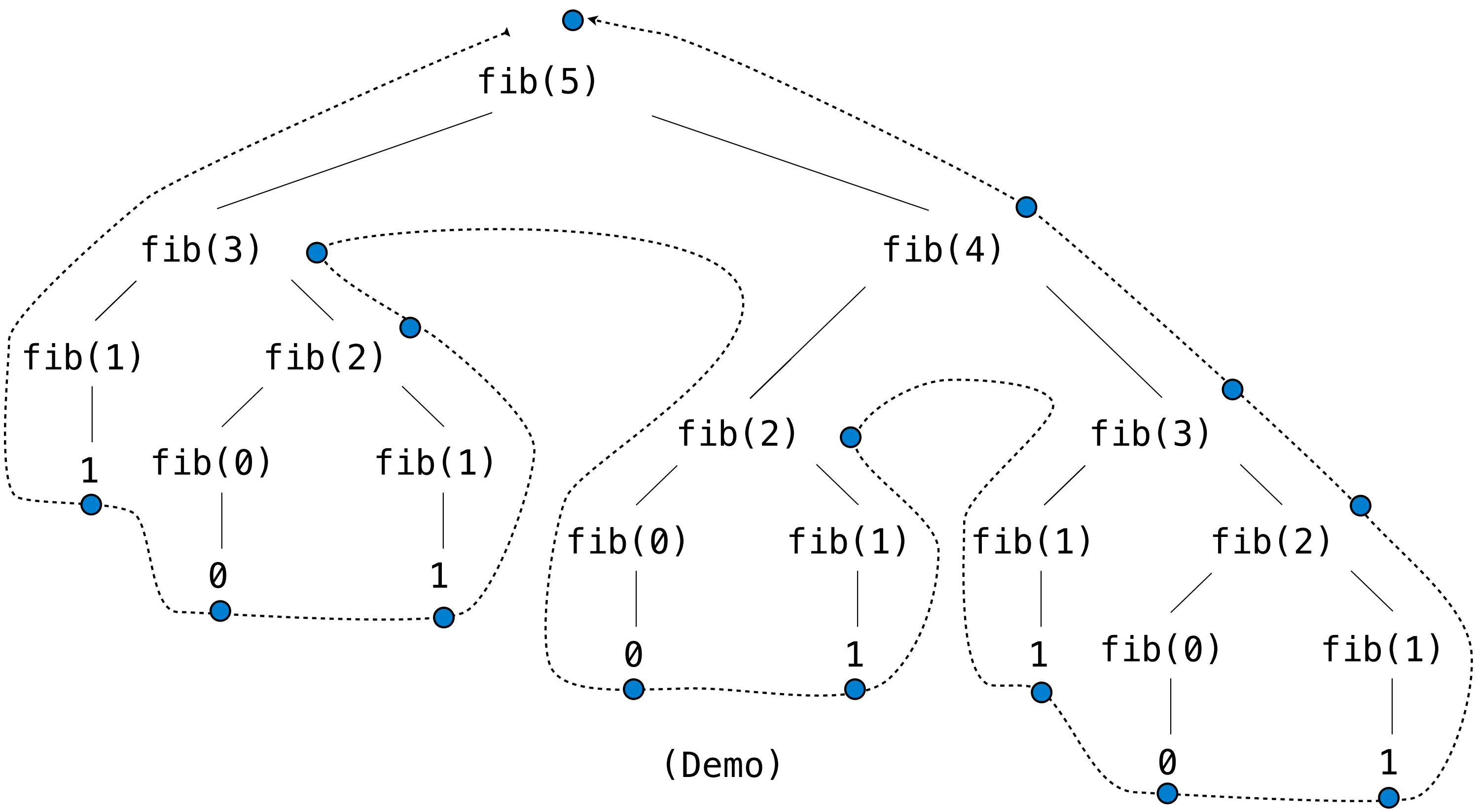
A Tree-Recursive Process

The computational process of fib evolves into a tree structure



A Tree-Recursive Process

The computational process of fib evolves into a tree structure



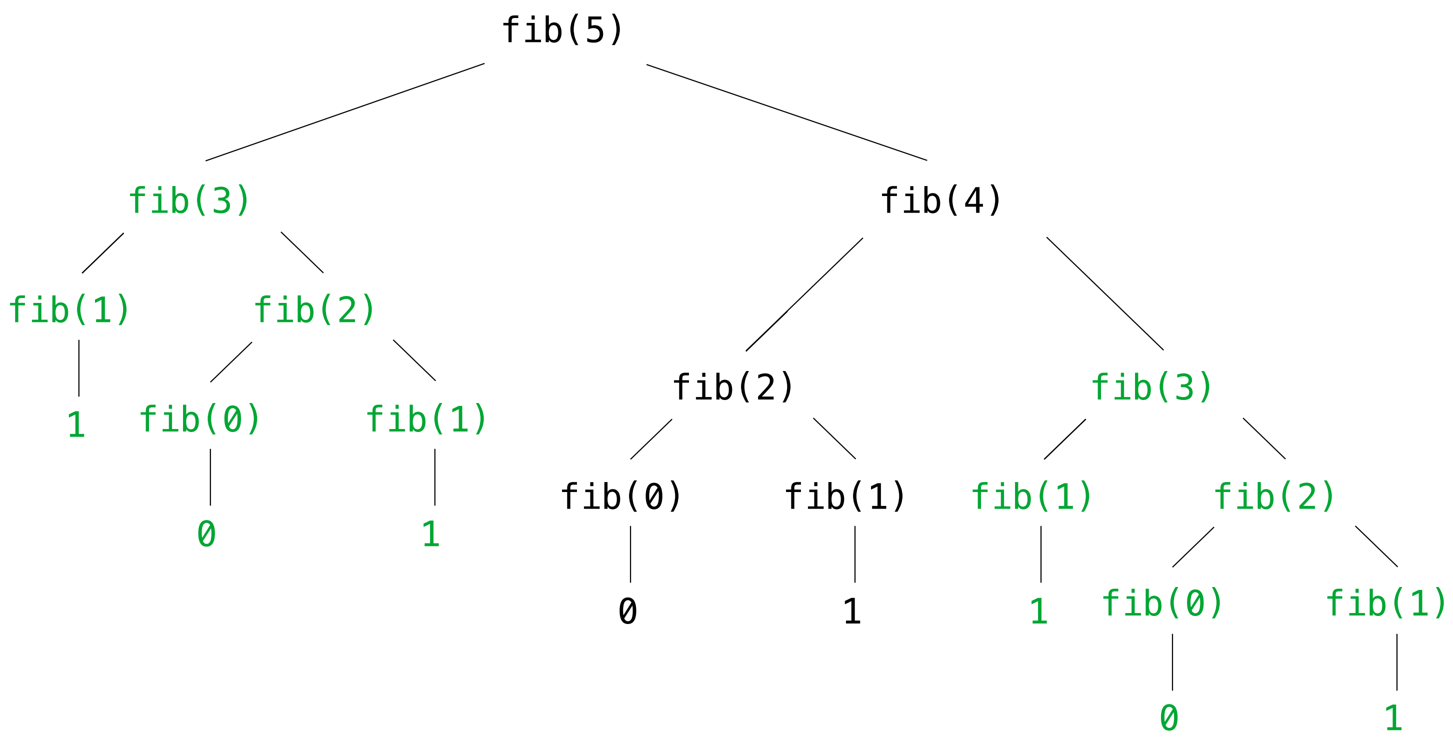
Repetition in Tree-Recursive Computation

Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times

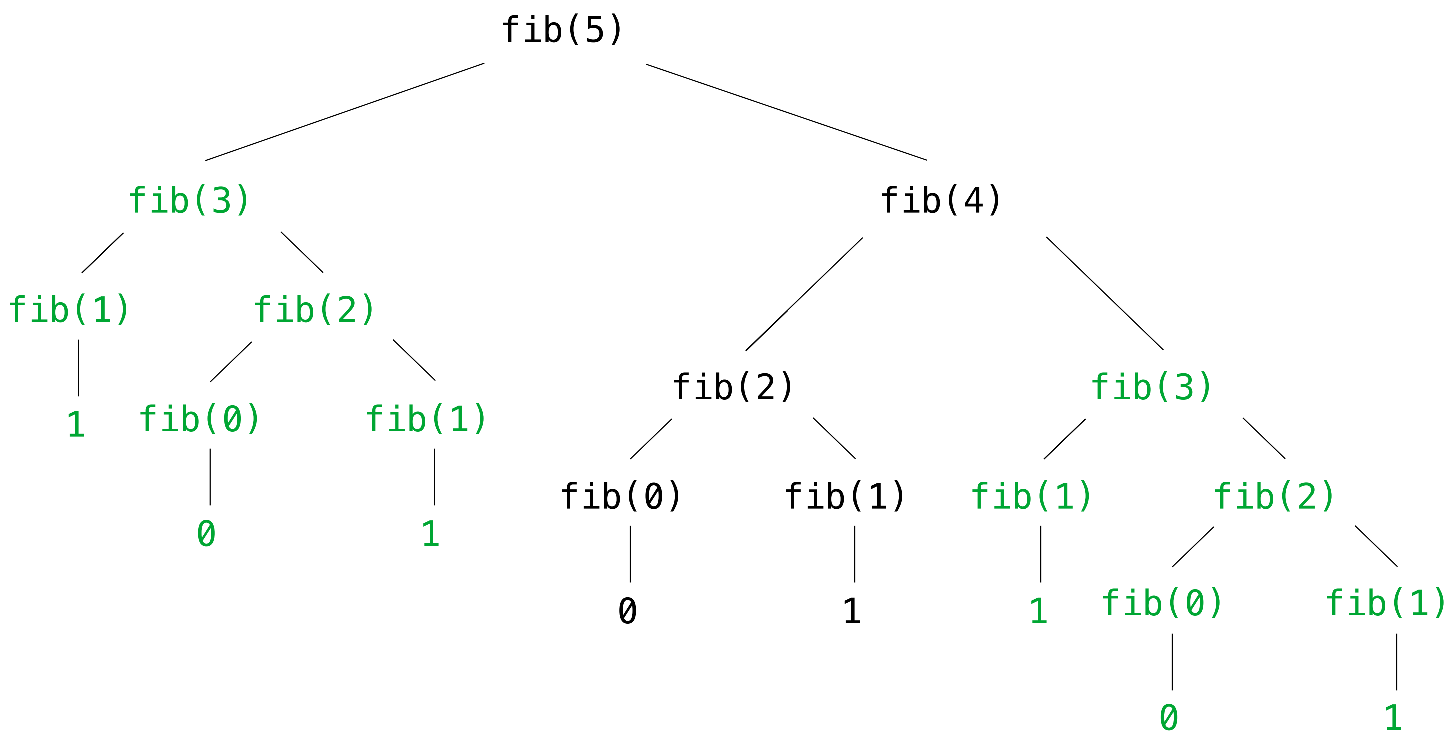
Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times



Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times



(We will speed up this computation dramatically in a few weeks by remembering results)

Example: Counting Partitions

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

```
count_partitions(6, 4)
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

```
count_partitions(6, 4)
```

```
2 + 4 = 6
1 + 1 + 4 = 6
3 + 3 = 6
1 + 2 + 3 = 6
1 + 1 + 1 + 3 = 6
2 + 2 + 2 = 6
1 + 1 + 2 + 2 = 6
1 + 1 + 1 + 1 + 2 = 6
1 + 1 + 1 + 1 + 1 + 1 = 6
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

```
count_partitions(6, 4)
```

2 + 4 = 6

1 + 1 + 4 = 6

3 + 3 = 6

1 + 2 + 3 = 6

1 + 1 + 1 + 3 = 6

2 + 2 + 2 = 6

1 + 1 + 2 + 2 = 6

1 + 1 + 1 + 1 + 2 = 6

1 + 1 + 1 + 1 + 1 + 1 = 6

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

2 + 4 = 6

1 + 1 + 4 = 6

3 + 3 = 6

1 + 2 + 3 = 6

1 + 1 + 1 + 3 = 6

2 + 2 + 2 = 6

1 + 1 + 2 + 2 = 6

1 + 1 + 1 + 1 + 2 = 6

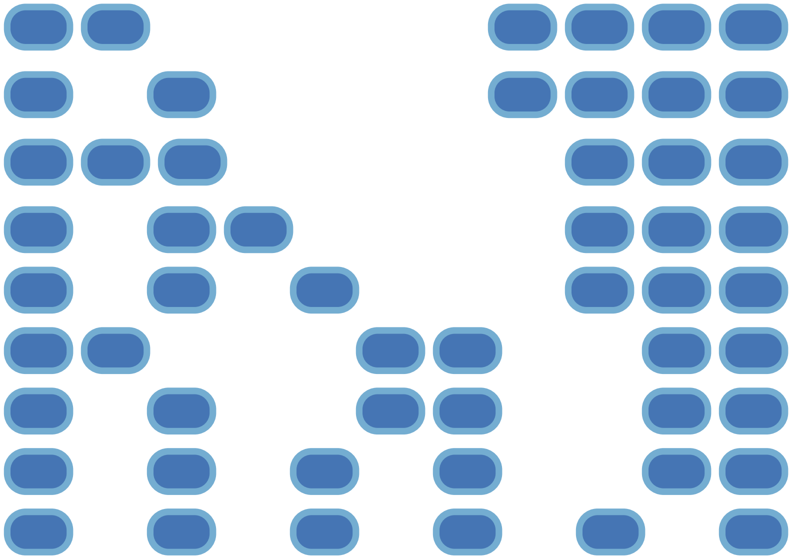
1 + 1 + 1 + 1 + 1 + 1 = 6

Counting Partitions

```
The number of partitions of a positive integer n, using parts up to size m, is the number
of ways in which n can be expressed as the sum of positive integer parts up to m in
increasing order.
```

```
count_partitions(6, 4)
```

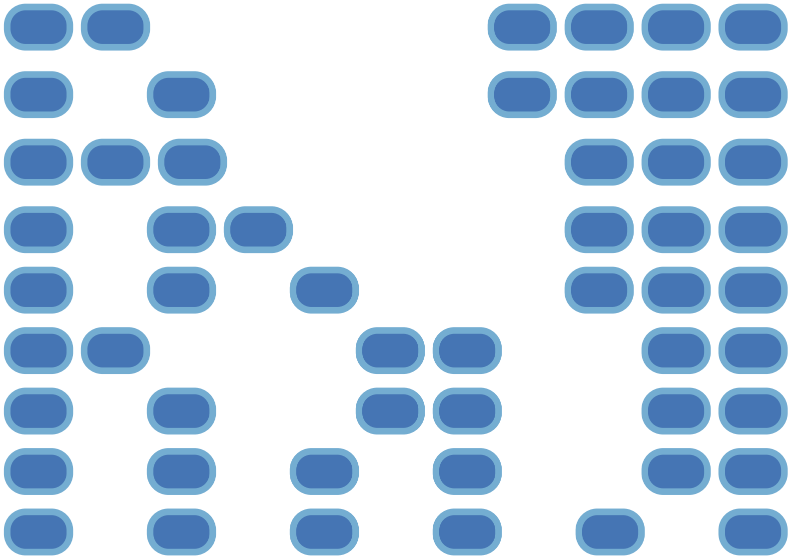
```
2 + 4 = 6
1 + 1 + 4 = 6
3 + 3 = 6
1 + 2 + 3 = 6
1 + 1 + 1 + 3 = 6
2 + 2 + 2 = 6
1 + 1 + 2 + 2 = 6
1 + 1 + 1 + 1 + 2 = 6
1 + 1 + 1 + 1 + 1 + 1 = 6
```



Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

```
count_partitions(6, 4)
```

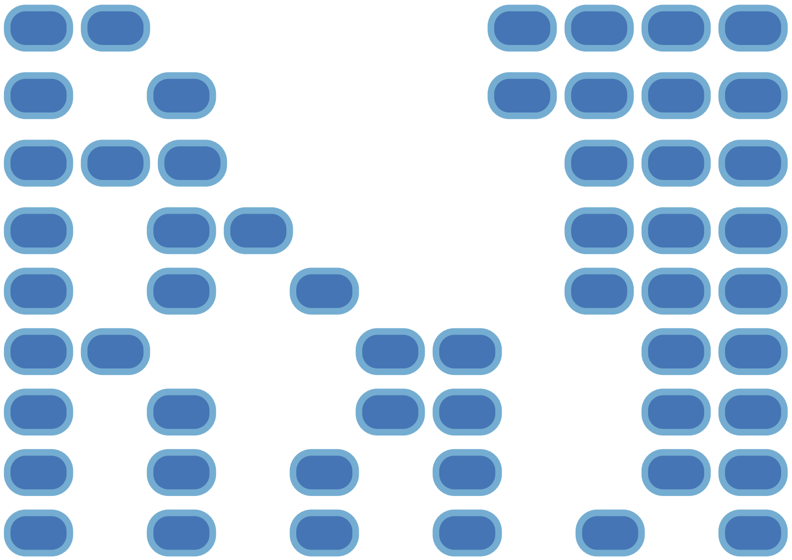


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.

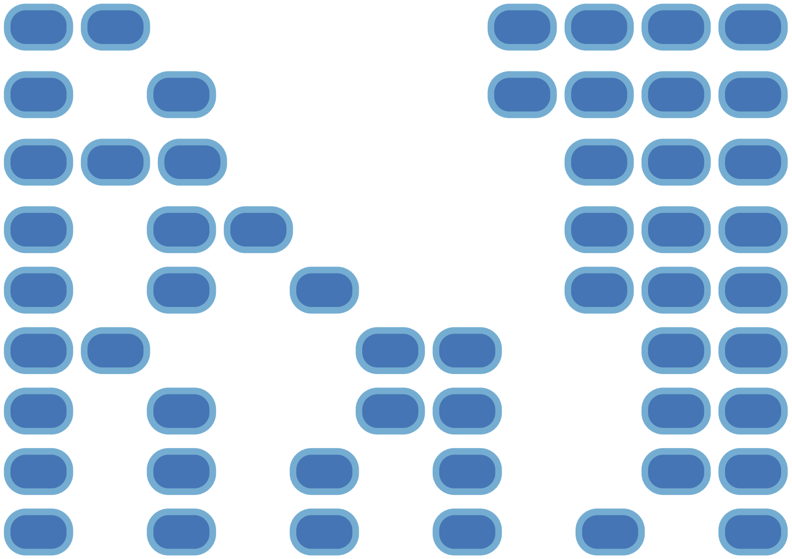


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:

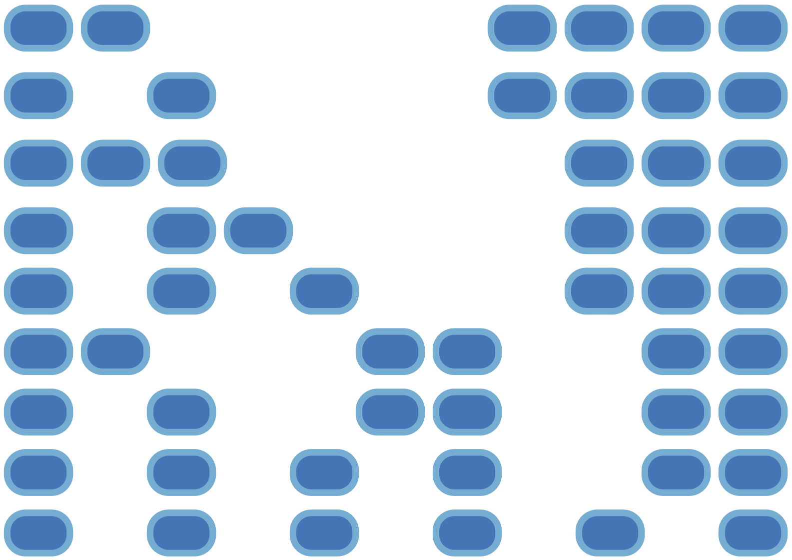


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4

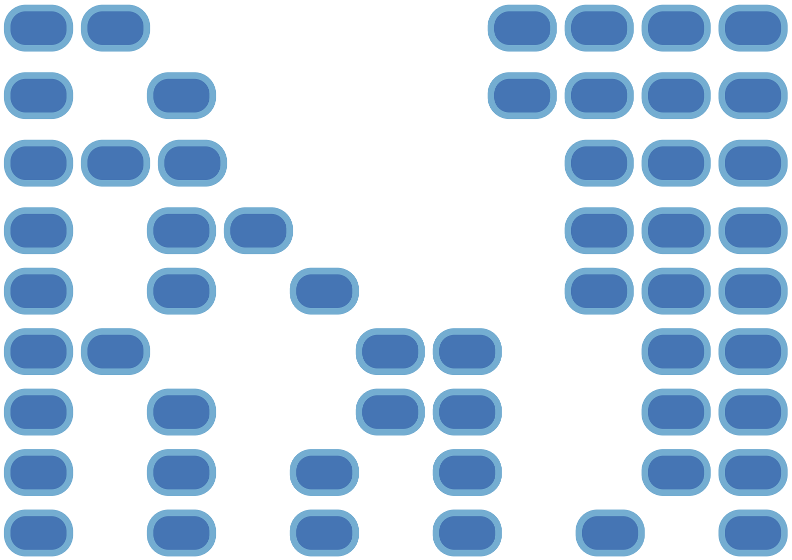


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

`count_partitions(6, 4)`

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4

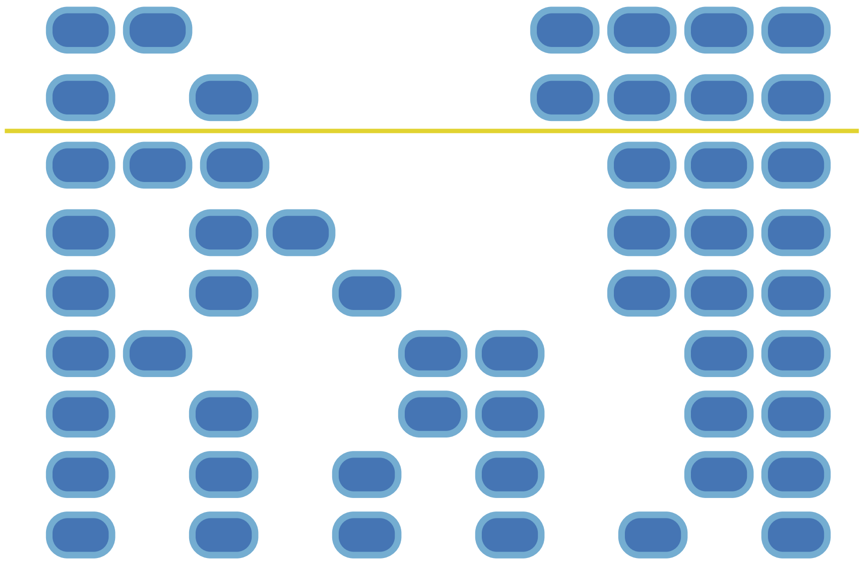


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4

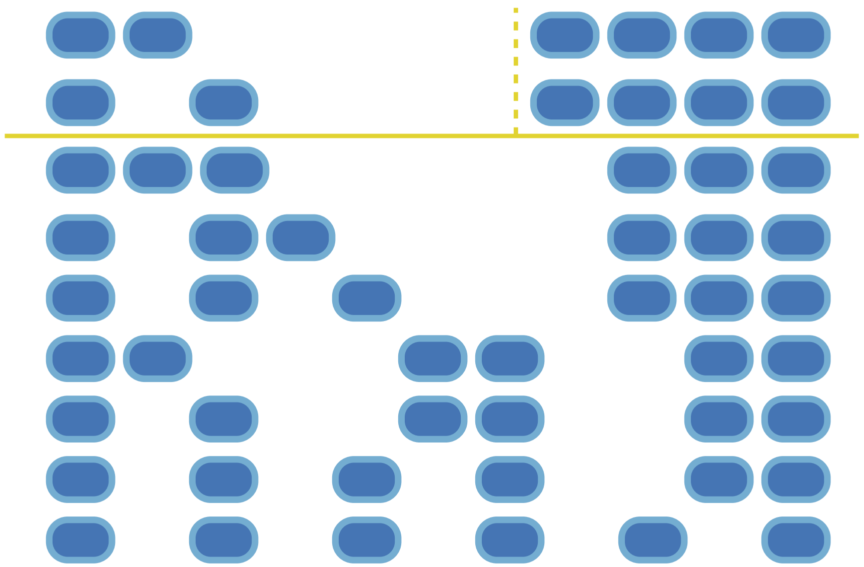


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4

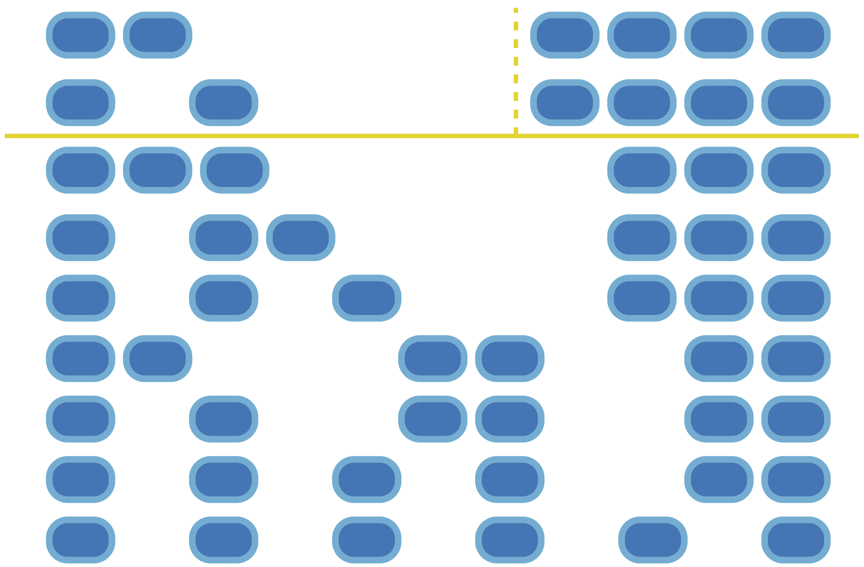


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:

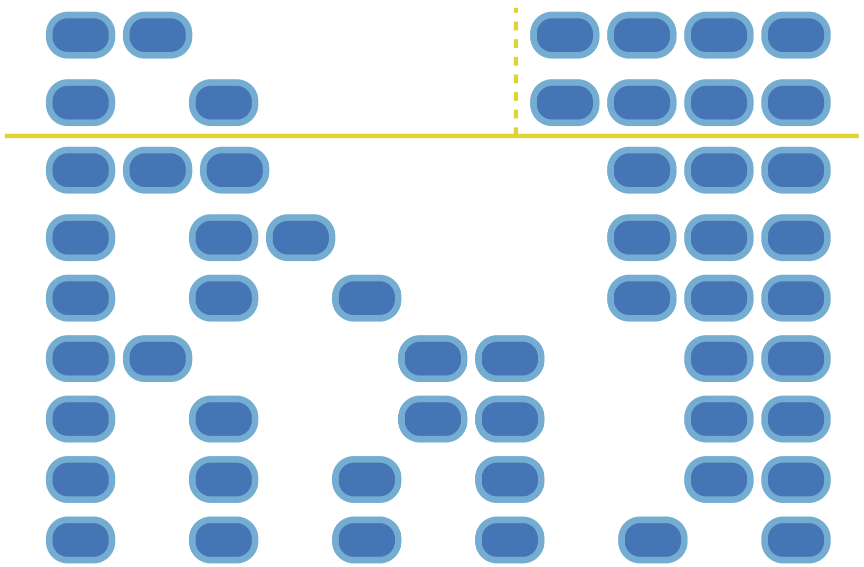


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)

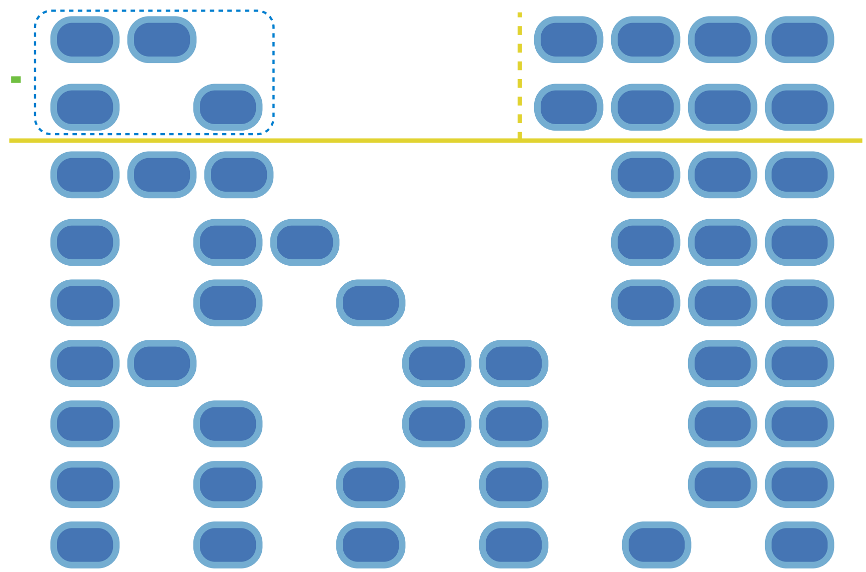


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)

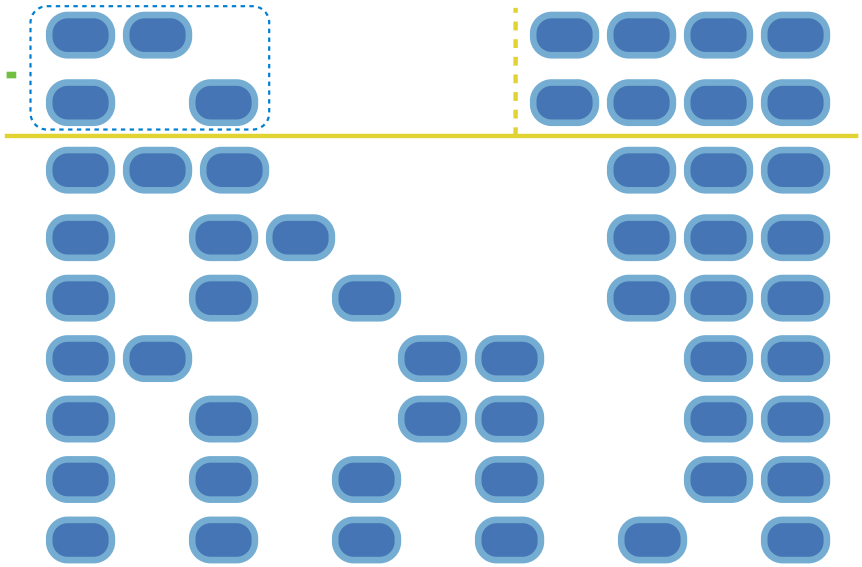


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)

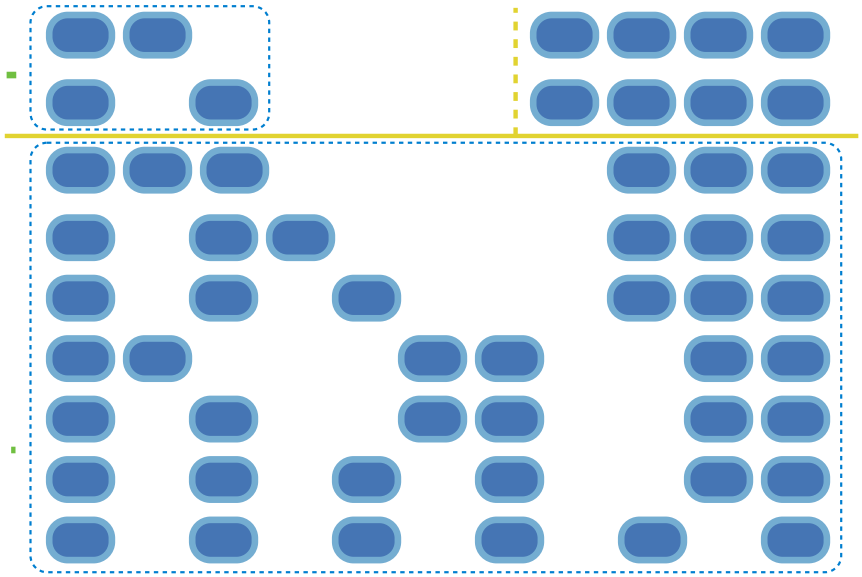


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)

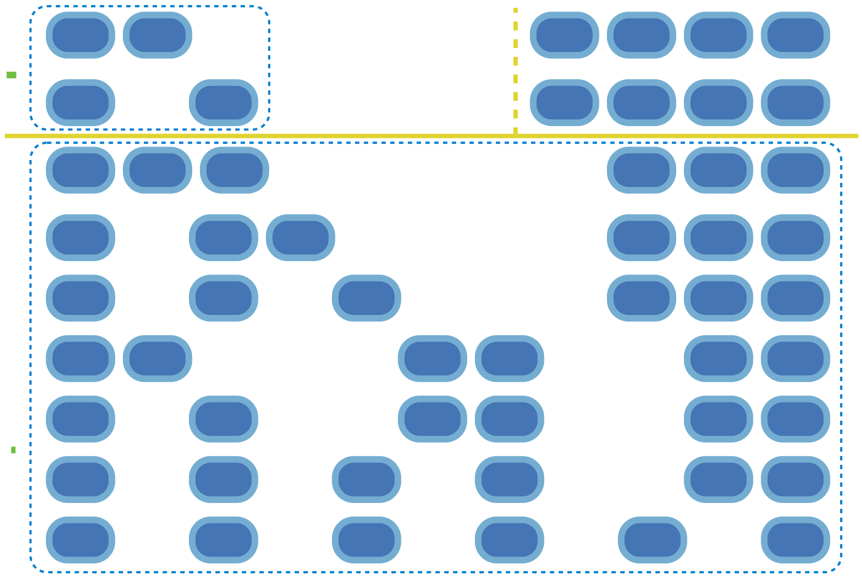


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

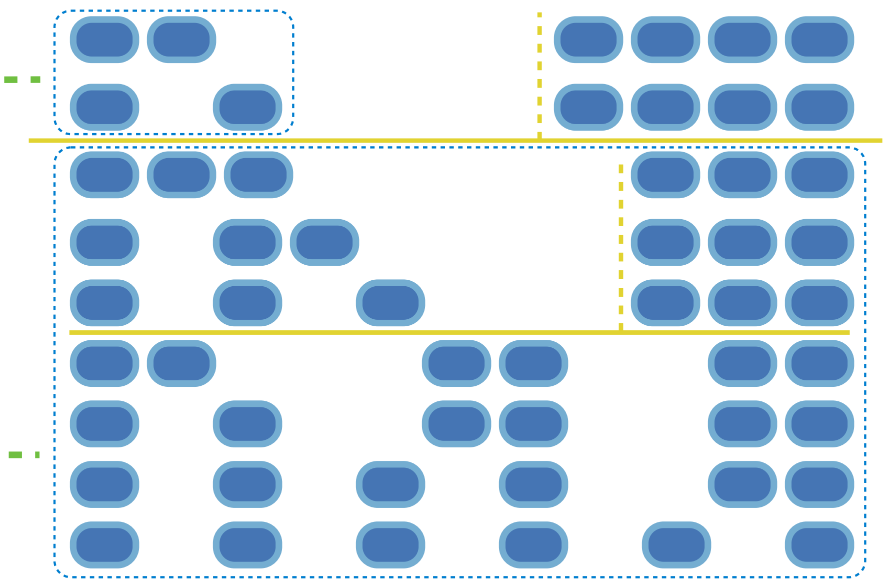


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

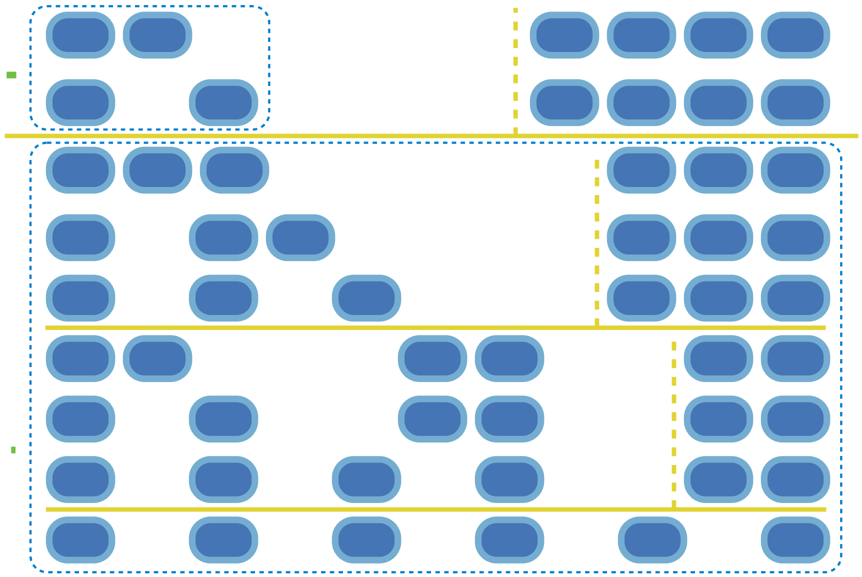


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

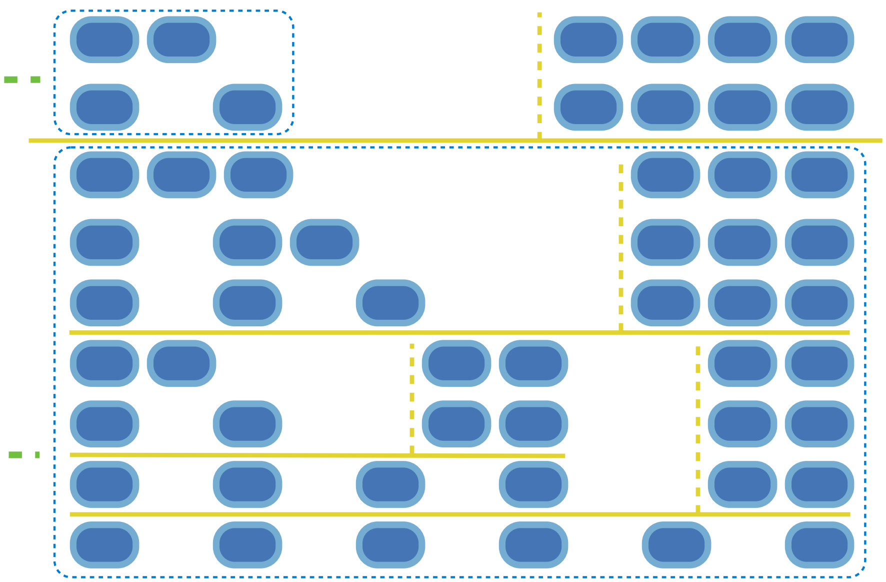


Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.



Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):



    else:
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):




    else:
        with_m = count_partitions(n-m, m)
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):



    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):



    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):


    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Counting Partitions

```
The number of partitions of a positive integer n, using parts up to size m, is the number
of ways in which n can be expressed as the sum of positive integer parts up to m in
increasing order.
```

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):



    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:



    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1



    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:


    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0


    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:

    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

(Demo)

Interactive Diagram