61A Extra Lecture 3

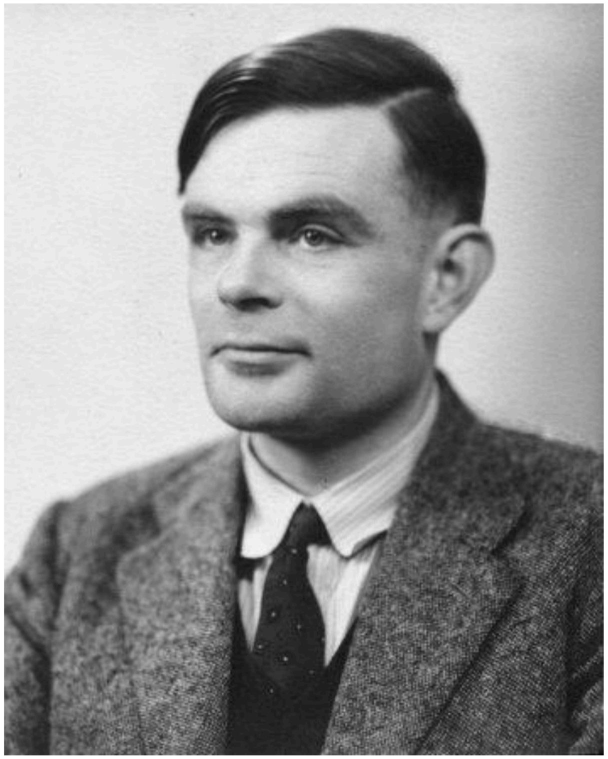
Announcements

cs61a.org/extra.html

Church-Turing Thesis

The Church-Turing Thesis

A function on the natural numbers is computable by a human following an algorithm, ignoring resource limitations, if and only if it is computable by a Turing machine.



Representation

Functions Can Represent Boolean Values

If all we have to work with are functions and call expressions, is there any way to represent other primitive values?

```
t = lambda a: lambda b: a
f = lambda a: lambda b: b

def py_pred(p):
    return p(True)(False)

def f_not(p):
    """Define Not.

    >>> py_pred(f_not(t))
    False
    >>> py_pred(f_not(f))
    True
    """
    return lambda a: lambda b: p(b)(a)
```

Exercise:

```
def f_and(p, q):
    """Define And.

    >>> py_pred(f_and(t, t))
    True
    >>> py_pred(f_and(t, f))
    False
    >>> py_pred(f_and(f, t))
    False
    >>> py_pred(f_and(f, f))
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    """
    return ____________
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    """
    return __________
```

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    """
    return __p(t)(q)__
```

Functions Can Represent Natural Numbers

If all we have to work with are functions and call expressions, is there any way to represent other primitive values?

```
def zero(s):
    return lambda z: z

def one(s):
    return lambda z: s(z)

def two(s):
    return lambda z: s(s(z))

def successor(n):
    return lambda s: lambda z: s(n(s)(z))

three = successor(two)
```

```
def add_church(m, n):
    return lambda s: lambda x: m(s)(n(s)(z))

def mul_church(m, n):
    return lambda s: m(n(s))

def pow_church(m, n):
    return n(m)
```

Note: **lambda x: f(x)** is the same as **f**

Lambda Calculus Notation

Lambda Calculus

Variables: single letters, such as **x**

Functions: Instead of **lambda x: x** , write **λx.x** ; Instead of **lambda x, y: x** , write **λxy.x**

Assignment: Write **var f = ...**

Application: Instead of **f(x)** , write **(f x)** ; **f(x)(y)** and **f(x, y)** are both written **(f x y)**

Follow along! http://chenyang.co/lambda/

**To type λ, just type **

var I = λx.x	Are (I I) and I the same?	Are (K I I) and (K I K) the same?
var K = λr.λs.r	Are (K I) and I the same?	What's ((K K) (K K)) the same as?
	Are (K K I) and K the same?	Can you construct a 4-argument function by just calling K & I?

Boolean Values

Variables: single letters, such as **x**

Functions: Instead of **lambda x: x** , write **λx.x** ; Instead of **lambda x, y: x** , write **λxy.x**

Assignment: Write **var f = ...**

Application: Instead of **f(x)** , write **(f x)** ; **f(x)(y)** and **f(x, y)** are both written **(f x y)**

Follow along! http://chenyang.co/lambda/

**To type λ, just type **

var T = λab.a

var F = λab.b

Define **and, or,** and **not**!

Define exclusive or:

```
xor(False, False) -> False
xor(False, True)  -> True
xor(True,  False) -> True
xor(True,  True)  -> False
```