

## Lazy Evaluation

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## Announcements

Promises

## Delay Creates a Promise

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From the **Revised<sup>5</sup> Report on the Algorithmic Language Scheme**

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(**delay** *<expression>*)

## Delay Creates a Promise

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**(delay <expression>)**

The **delay** construct is used together with the procedure **force** to implement *lazy evaluation* or *call by need*. **(delay <expression>)** returns an object called a *promise* which at some point in the future may be asked (by the **force** procedure) to evaluate *<expression>*, and deliver the resulting value...

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**(force** *<promise>*)

Forces the value of promise...

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**(force** *<promise>*)

Forces the value of promise...

`(force (delay (+ 1 2))) ⇒ 3`

`(let ((p (delay (+ 1 2)))) (list (force p) (force p))) ⇒ (3 3)`

## A Promise Can Be Represented as Function

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(force (delay (+ 1 2))) ⇒ 3
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(let ((p (delay (+ 1 2)))) (list (force p) (force p))) ⇒ (3 3)
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## A Promise Can Be Represented as Function

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A delayed expression can be captured along with the current environment using a lambda

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A delayed expression can be captured along with the current environment using a lambda

E.g., `(let ((p (lambda () (+ 1 2)))) (list (p) (p)))`

`(force (delay (+ 1 2))) ⇒ 3`

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E.g., `(let ((p (lambda () (+ 1 2)))) (list (p) (p)))`

(Demo)

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## Assignment and Caching

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scm> (define x 2)
x
scm> (set! x 3)
okay
scm> x
3
```

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Local, non-local, and global assignment all use `set!`

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scm> x
3
```

Local, non-local, and global assignment all use `set!`

```
(define (sum a b)
  (let ((total 0))
    (define (iter x)
      (if (< x b)
          (begin
             (set! total (+ total x))
             (iter (+ x 1))))))
    (iter a)
    total))
```

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```

Local, non-local, and global assignment all use `set!`

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    (define (iter x)
      (if (< x b)
          (begin
             (set! total (+ total x))
             (iter (+ x 1))))))
    (iter a)
    total))
```

```
def sum(a, b):
    total = 0
    def iter(x):
        nonlocal total
        if x < b:
            total = total + x
            iter(x + 1)
    iter(a)
    return total
```

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Forces the value of promise. If no value has been computed for the promise, then a value is computed and returned. The value of the promise is cached (or "memoized") so that if it is forced a second time, the previously computed value is returned.

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```
scm> (define x 2)
x
scm> (let ((p (delay (set! x (+ x 1)))) (begin (force p) (force p)))
okay
scm> x
3
scm
```

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```
(define make-promise  
  (lambda (proc)  
    (let ((result-ready? #f)
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Evaluates proc and gives it a local name

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Assignment is required in order to cache the value of a promise (from R5RS)

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Did (proc) get cached while evaluating (proc)?



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```
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```
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# Meta-Circular Evaluator

## A Scheme Evaluator in Scheme

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Lots of different programming languages can be expressed using nested combinations

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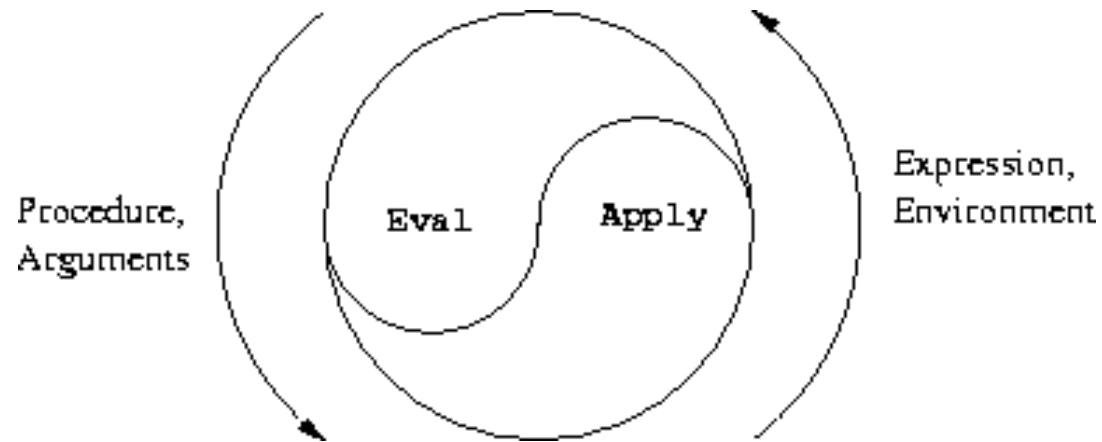
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## Lazy Evaluator



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(Demo)