Logic Programming

Announcements

The Logic Language

The Logic Language

The *Logic* language was invented for Structure and Interpretation of Computer Programs

- Based on Prolog (1972)
- Expressions are facts or queries, which contain relations
- Expressions and relations are Scheme lists
- For example, **(likes john dogs)** is a relation

Simple Facts

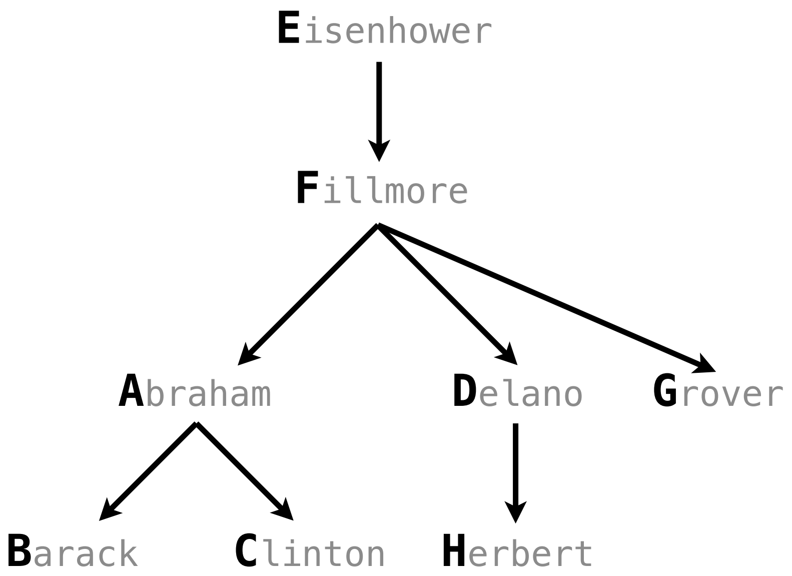
A simple fact expression in the Logic language declares a relation to be true

Let's say I want to track the heredity of a pack of dogs

Language Syntax:

- A relation is a Scheme list
- A fact expression is a Scheme list of relations

```
logic> (fact (parent delano herbert))
logic> (fact (parent abraham barack))
logic> (fact (parent abraham clinton))
logic> (fact (parent fillmore abraham))
logic> (fact (parent fillmore delano))
logic> (fact (parent fillmore grover))
logic> (fact (parent eisenhower fillmore))
```



Relations are Not Procedure Calls

In *Logic*, a relation is **not** a call expression.

- *Scheme*: the expression **(abs -3)** calls *abs* on -3. It returns 3.
- *Logic*: **(abs -3 3)** asserts that *abs* of -3 is 3.

To assert that 1 + 2 = 3, we use a relation: **(add 1 2 3)**

We can ask the Logic interpreter to complete relations based on known facts.

(add ? 2 3)	**1**
(add 1 ? 3)	**2**
(add 1 2 ?)	**3**
(? 1 2 3)	**add**

Queries

Queries

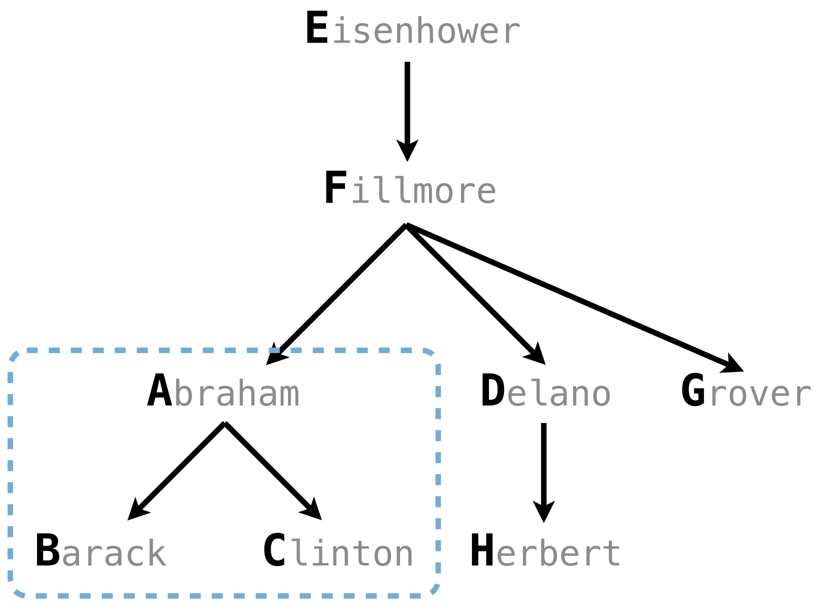
A *query* contains one or more relations that may contain variables.

Variables are symbols starting with **?**

```
logic> (fact (parent delano herbert))
logic> (fact (parent abraham barack))
logic> (fact (parent abraham clinton))
logic> (fact (parent fillmore abraham))
logic> (fact (parent fillmore delano))
logic> (fact (parent fillmore grover))
logic> (fact (parent eisenhower fillmore))

logic> (query (parent abraham ?puppy))
Success!
puppy: barack
puppy: clinton
```

A variable can have any name

Each line is an assignment of variables to values



(Demo)

Compound Facts and Queries

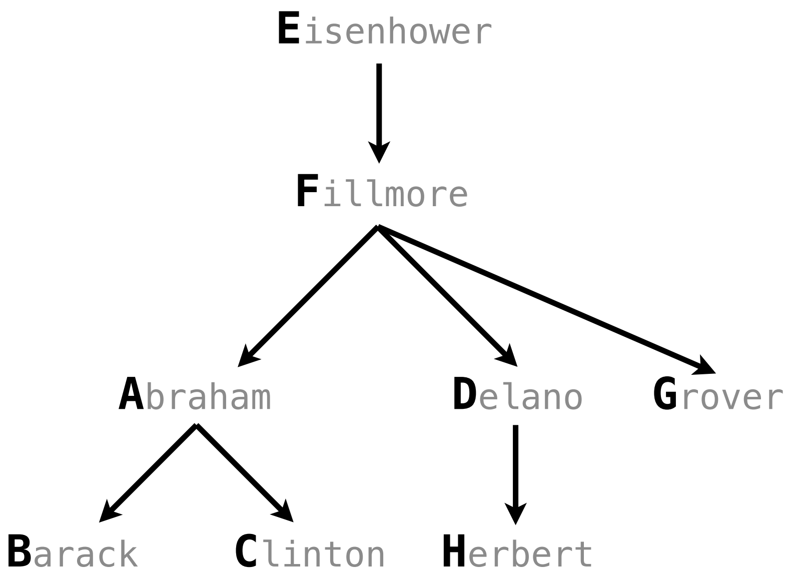
Compound Facts

A fact can include multiple relations and variables as well.

(fact <conclusion> <hypothesis$_0$> <hypothesis$_1$> ... <hypothesis$_N$>)

Means <conclusion> is true if all the <hypothesis$_K$> are true.

```
logic> (fact (child ?c ?p) (parent ?p ?c))

logic> (query (child herbert delano))
Success!

logic> (query (child eisenhower clinton))
Failure.

logic> (query (child ?kid fillmore))
Success!
kid: abraham
kid: delano
kid: grover
```



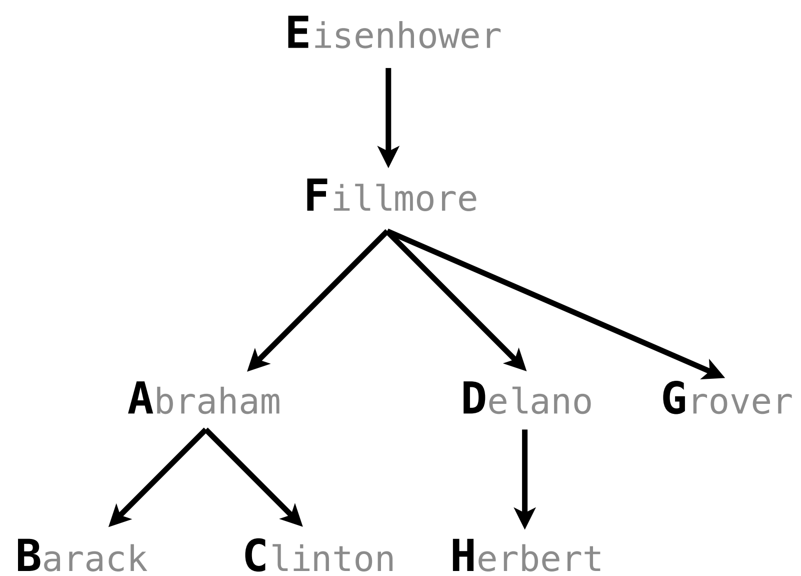
Compound Queries

An assignment must satisfy all relations in a query.

(query <relation$_0$> <relation$_1$> ... <relation$_N$>)

is satisfied if all the <relation$_K$> are true.

```
logic> (fact (child ?c ?p) (parent ?p ?c))

logic> (query (parent ?grampa ?kid)
              (child clinton ?kid))
Success!
grampa: fillmore    kid: abraham

logic> (query (child ?y ?x)
              (child ?x eisenhower))
Success!
y: abraham    x: fillmore
y: delano     x: fillmore
y: grover     x: fillmore
```

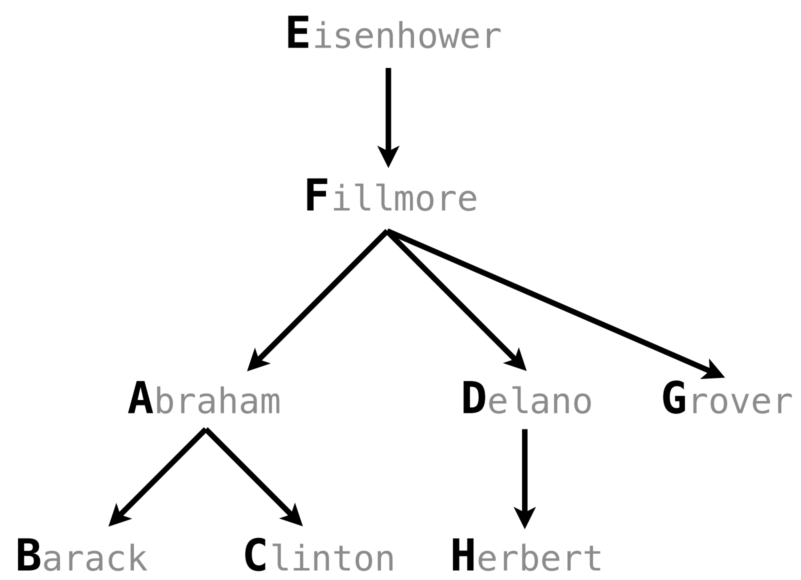


Recursive Facts

Recursive Facts

A fact is recursive if the same relation is mentioned in a hypothesis and the conclusion.

```
logic> (fact (ancestor ?a ?y) (parent ?a ?y))

logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))

logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
a: eisenhower

logic> (query (ancestor ?a barack)
              (ancestor ?a herbert))
Success!
a: fillmore
a: eisenhower
```



Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
a: eisenhower
```

```
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

```
(parent delano herbert)      ; (1), a simple fact
(ancestor delano herbert)    ; (2), from (1) and the 1st ancestor fact
(parent fillmore delano)     ; (3), a simple fact
(ancestor fillmore herbert) ; (4), from (2), (3), & the 2nd ancestor fact
```

Hierarchical Facts

Hierarchical Facts

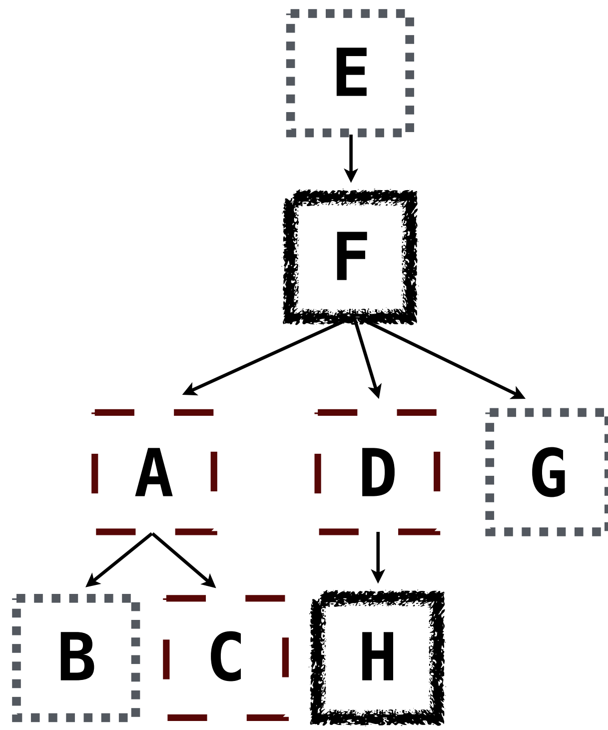
Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
Success!
type: long
```

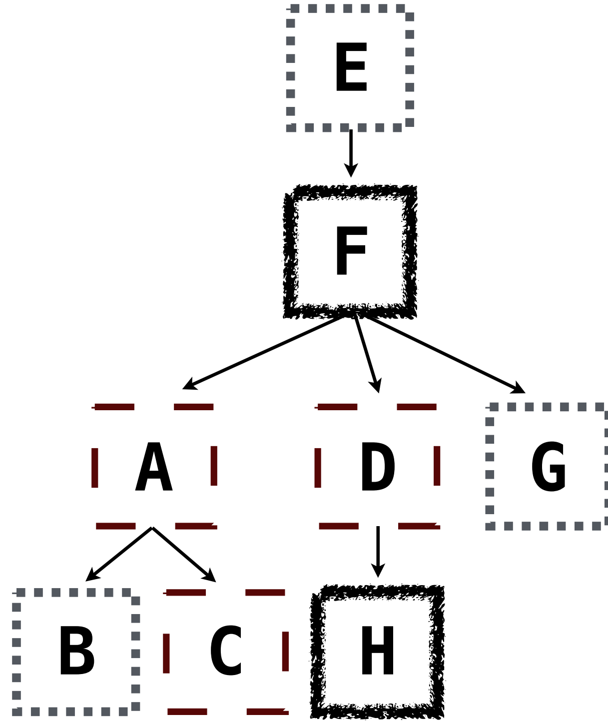
```
logic> (query (dog (name clinton) ?stats))
Success!
stats: (fur long)
```



Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack    fur: short   y: eisenhower
x: clinton   fur: long    y: abraham
x: grover    fur: short   y: eisenhower
x: herbert   fur: curly   y: fillmore
```



Appending Lists

(Demo)

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

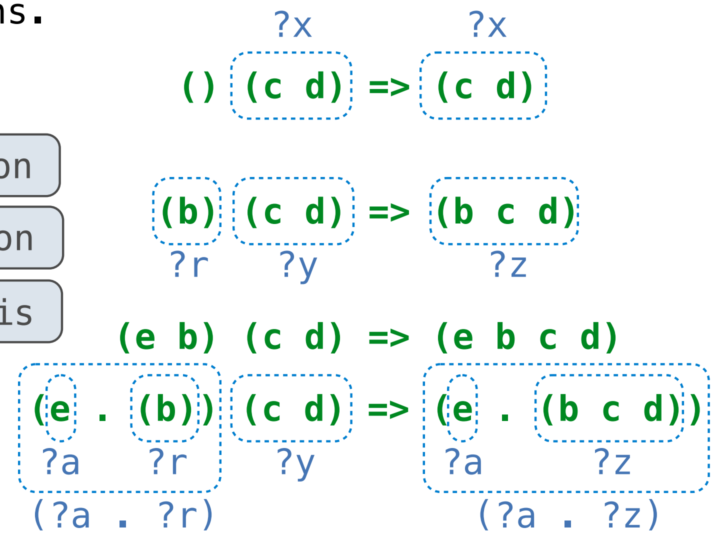
```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)



The interpreter lists all bindings that it can find to satisfy the query.

(Demo)

Which Hypotheses Complete append-3?

```
I can append (a b) and (1 2) and (x y) to form (a b 1 2 x y).

(fact (append-3 ?x ?y ?z ?xyz)

  1:   (append-to-form ?x ?y ?xy)
       (append-to-form ?y ?z ?yz) )

  2:   (= (append-to-form ?x ?y) ?xy)
       (= (append-to-form ?y ?z) ?yz) )

  3:   (append-to-form ?x ?y ?xy)
       (append-to-form ?xy ?z ?xyz) )

  4:   (= (append-to-form ?x ?y) ?xy)
       (= (append-to-form ?xy ?z) ?xyz) )

  5:  None of the above
```

Define Base Fact of append-to-form So That No Lists Can Be Empty

```
; (append-to-form () (1 2 3) (1 2 3))
; (append-to-form (1) (2 3) (1 2 3))
; (append-to-form (1 2) (3) (1 2 3))
; (append-to-form (1 2 3) () (1 2 3))
```

```
1: (fact (append-to-form () ?x ?x))

2: (fact (append-to-form ?a ?x (?a . ?x)))

3: (fact (append-to-form ?a (?b . ?x) (?a ?b . ?x)))

4: (fact (append-to-form (?a) ?x (?a . ?x)))

5: (fact (append-to-form (?a) (?b . ?x) (?a ?b . ?x)))
```

```
Recursive fact:  (fact (append-to-form (?a . ?r) ?y (?a . ?z))
                       (append-to-form      ?r  ?y       ?z))
```

Permuting Lists

Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element   List   List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Bigger list with ?a somewhere

```
(fact (insert ?a (?b . ?r) (?b . ?s))
      (insert ?a       ?r       ?s))
```

List with ?a somewhere

```
(fact (anagram () ()))

(fact (anagram (?a . ?r) ?b)
      (insert   ?a   ?s  ?b)
      (anagram  ?r   ?s))
```

```
a | r t

  r t
ar t
 rat
 r ta

  t r
at r
 tar
 t ra
```

(Demo)

Unification

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

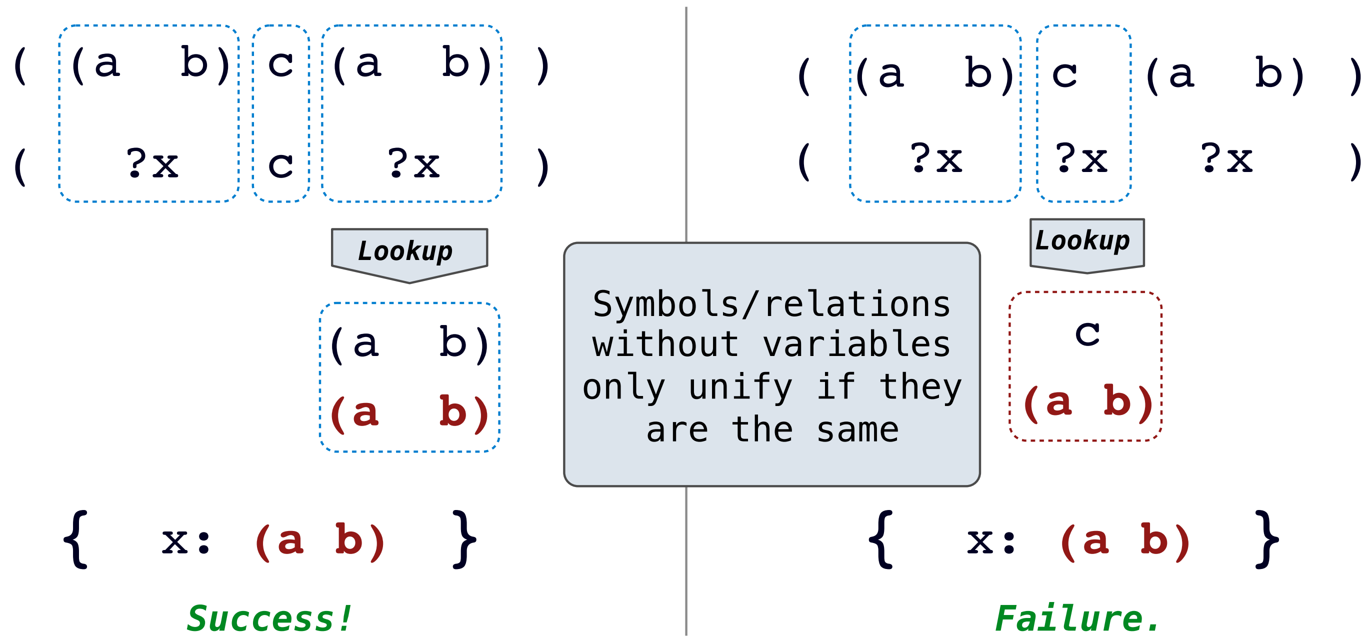
Unification is finding an assignment to variables that makes two relations the same.

```
( (a  b) c  (a  b) )
(   ?x   c    ?x   )      True, {x: (a b)}

( (a  b) c  (a  b) )
( (a ?y) ?z (a  b) )      True, {y: b, z: c}

( (a  b) c  (a  b) )
(   ?x  ?x    ?x   )      False
```

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.



Unifying Variables

Two relations that contain variables can be unified as well.

```
(    ?x           ?x      )
((a ?y c)    (a b ?z))
```

```
True, {x: (a ?y c),
       y: b,
       z: c}
```

Lookup

```
(a  ?y  c)
(a   b ?z)
```

Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

lookup('?x') ⇨ (a ?y c)   **lookup('?y')** ⇨ b   **ground('?x')** ⇨ (a b c)

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
( (a   b) c (a   b) )
(    ?x   c    ?x   )
```

Lookup

```
(a   b)
(a   b)
```

env: { x: (a b) }

Search

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

Variables are local to facts & queries

```
(app (e . ?r) (c d) (e b c d))

(app (b . ?r2) (c d) (b c d))

(app () (c d) (c d))
```

?left: (e . (b)) ⇨ (e b)

?r: (b . ()) ⇨ (b)

Depth-First Search

The space of facts is searched exhaustively, starting from the query and following a *depth-first* exploration order.

Depth-first search: Each proof approach is explored exhaustively before the next.

```
def search(clauses, env):
  for fact in facts:
    env_head = an environment extending env
    if unify(conclusion of fact, first clause, env_head):
      for env_rule in search(hypotheses of fact, env_head):
        for result in search(rest of clauses, env_rule):
          yield each successful result
```

Environment now contains new unifying bindings

- Limiting depth of the search avoids infinite loops.
- Each time a fact is used, its variables are renamed.
- Bindings are stored in separate frames to allow backtracking.

(Demo)

Addition

(Demo)