61A Extra Lecture 13

Announcements

Prediction

Regression

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(Demo)

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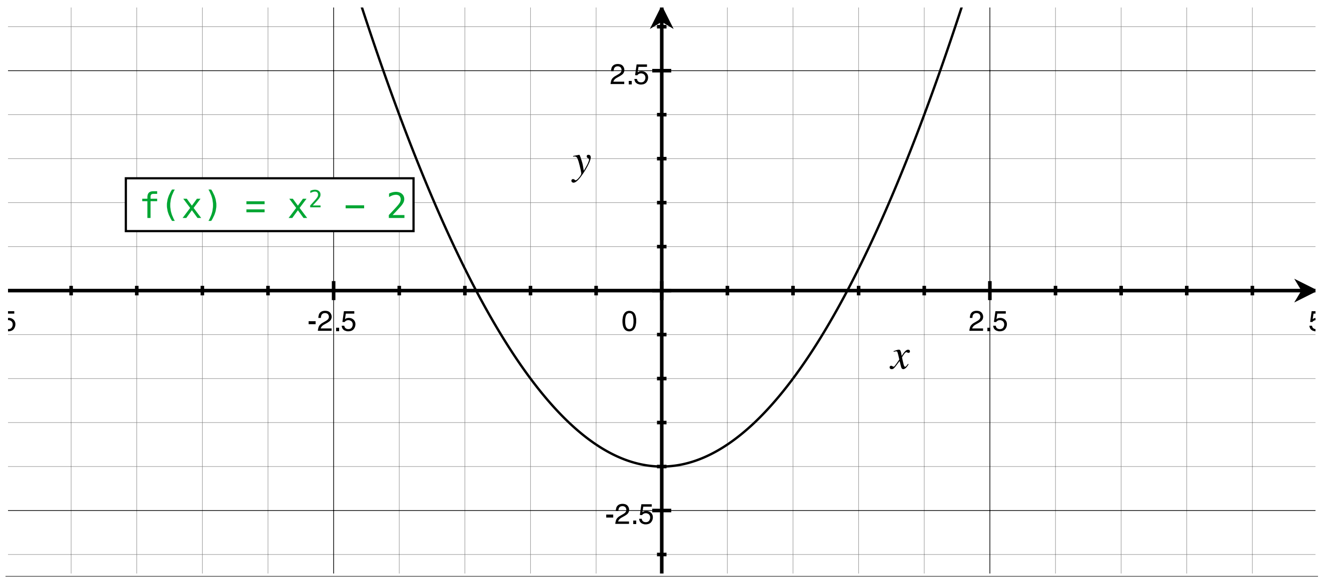

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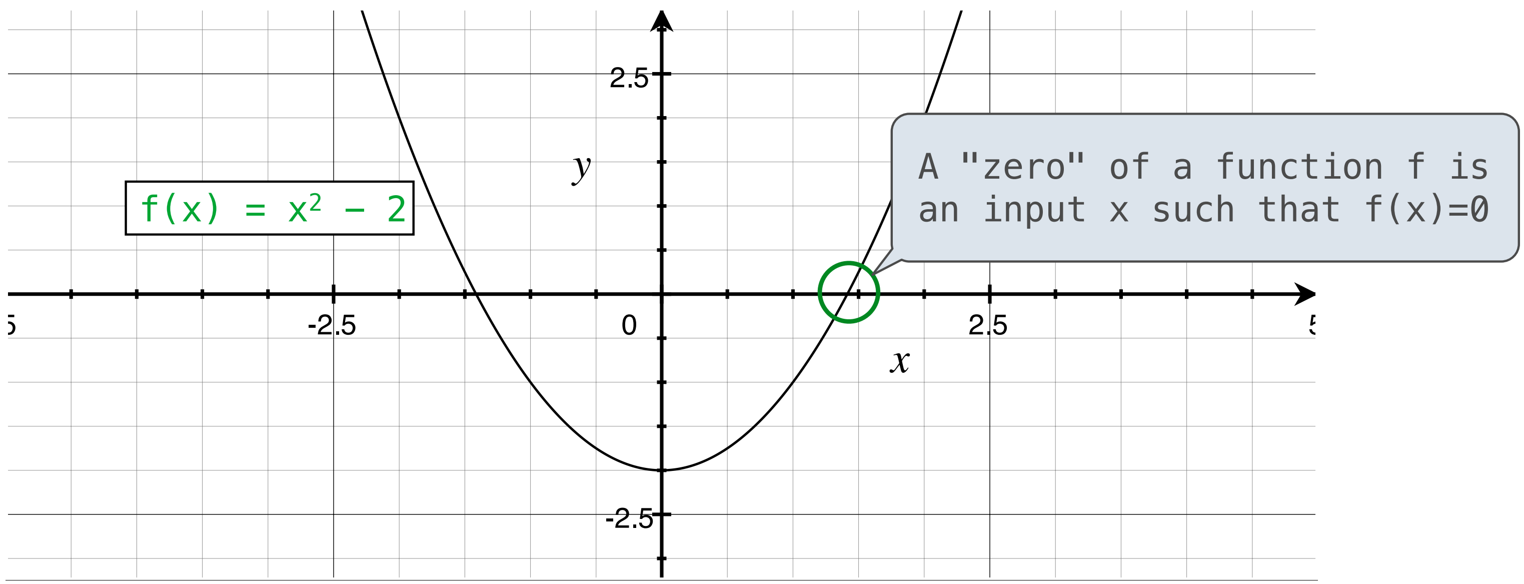

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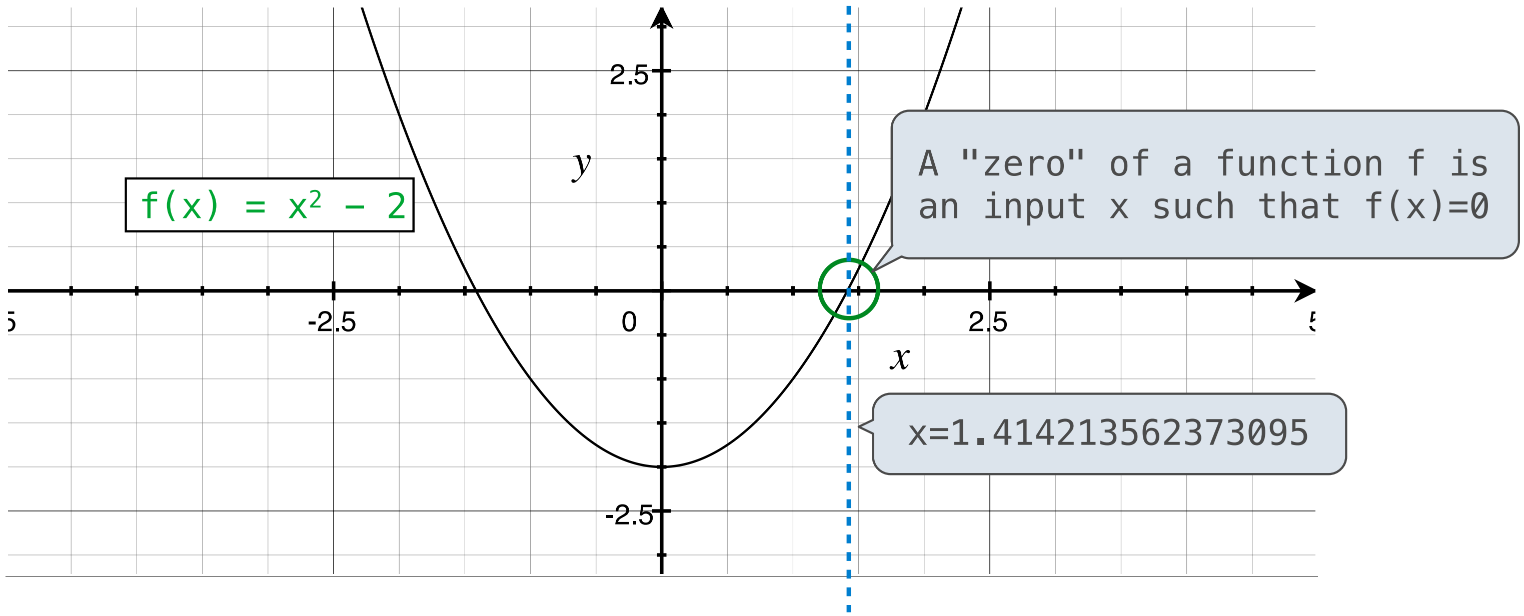

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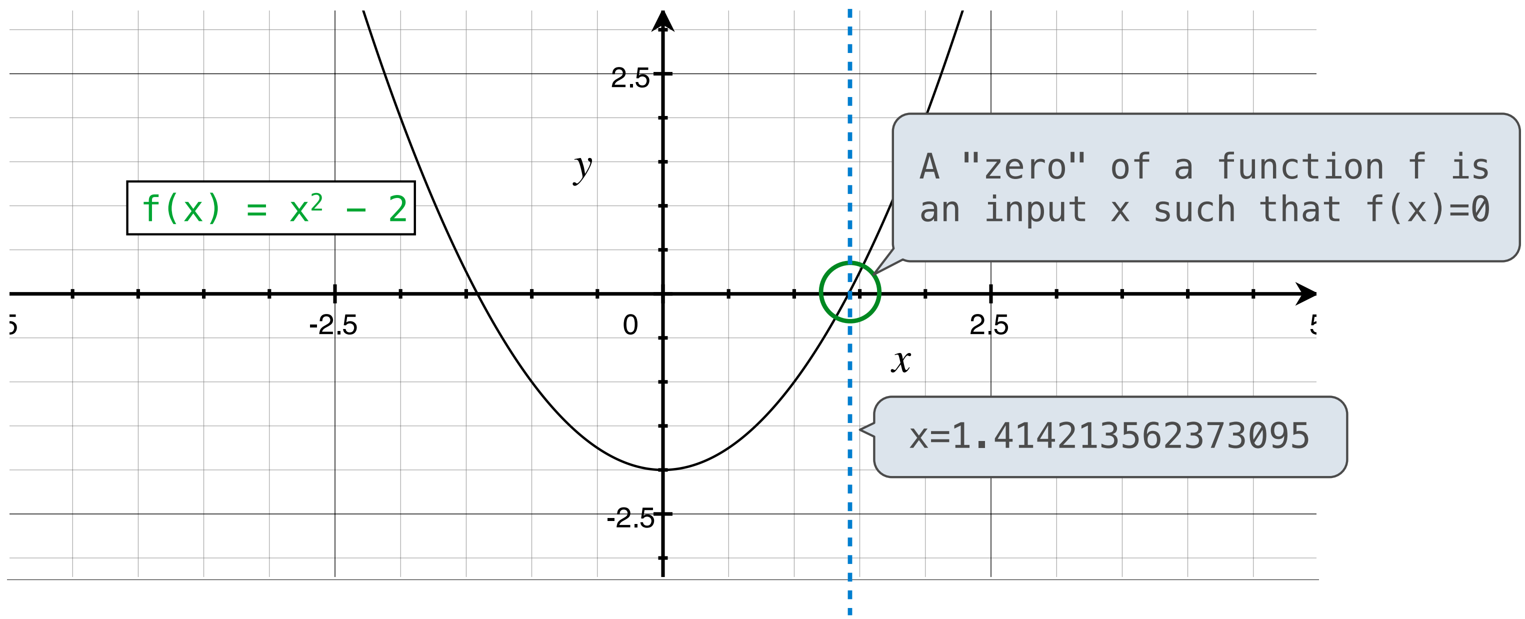

Application: Find the minimum of a function by finding the zero of its derivative

Approximate Differentiation

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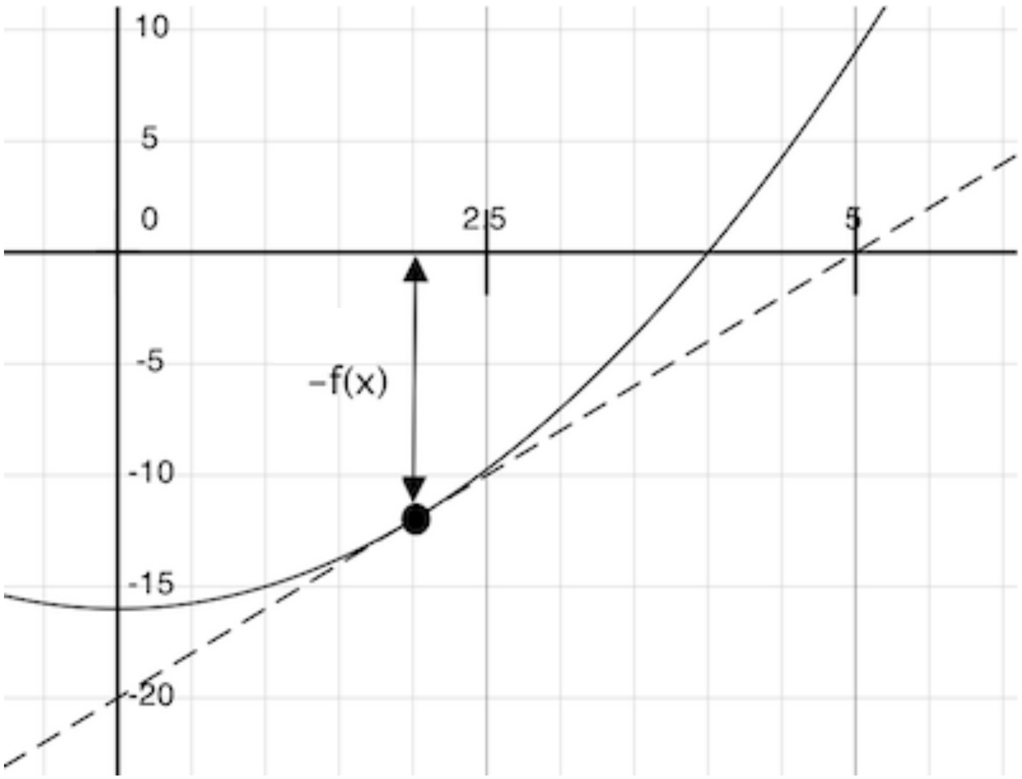

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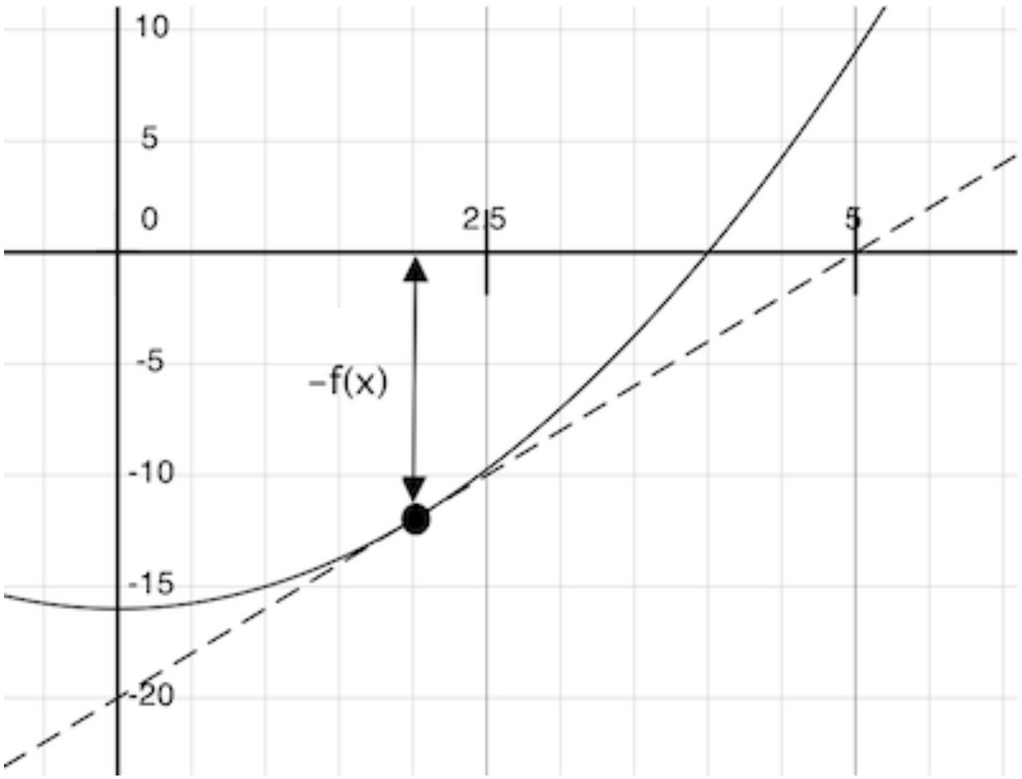
Differentiation can be performed symbolically or numerically



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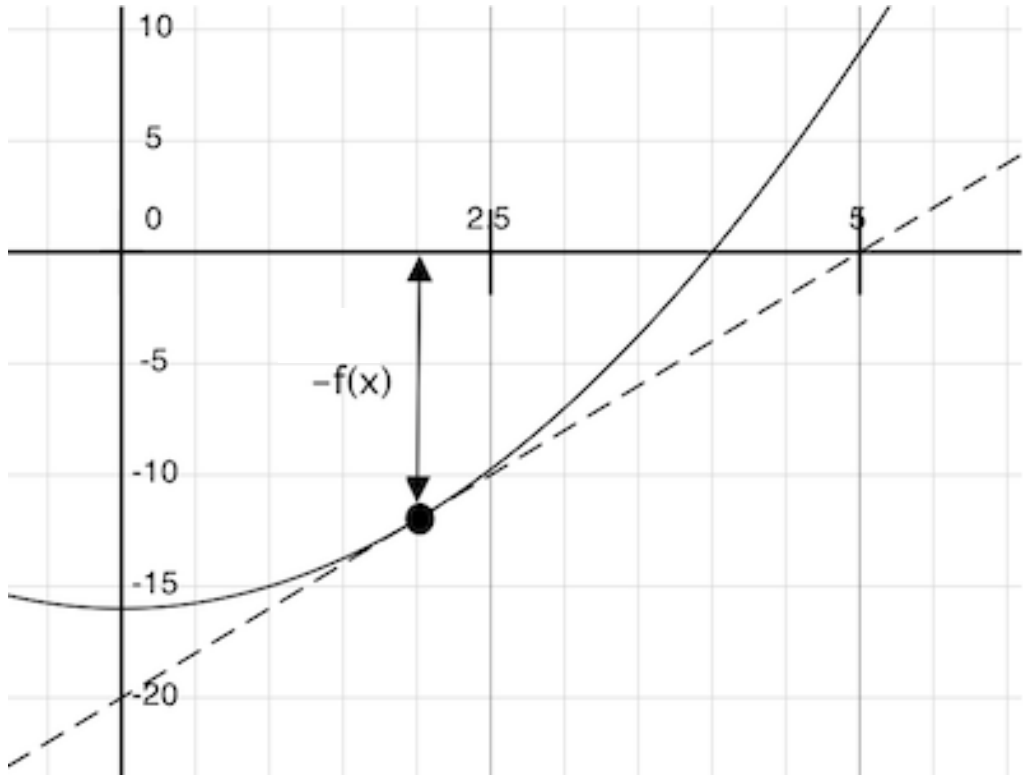

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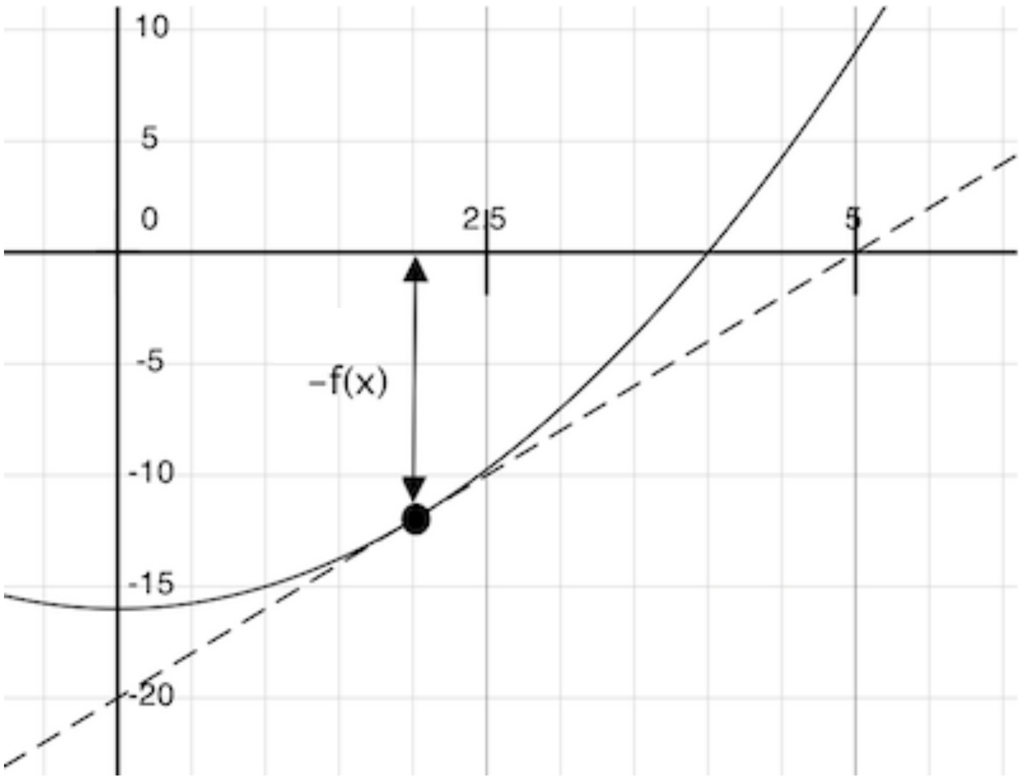

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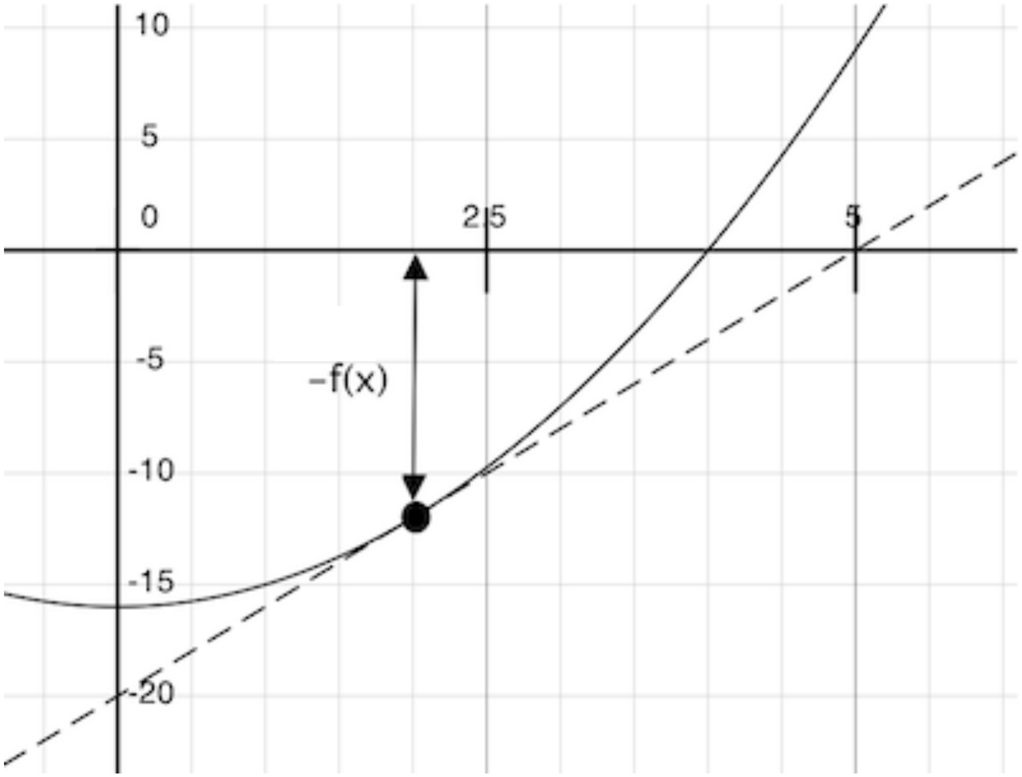

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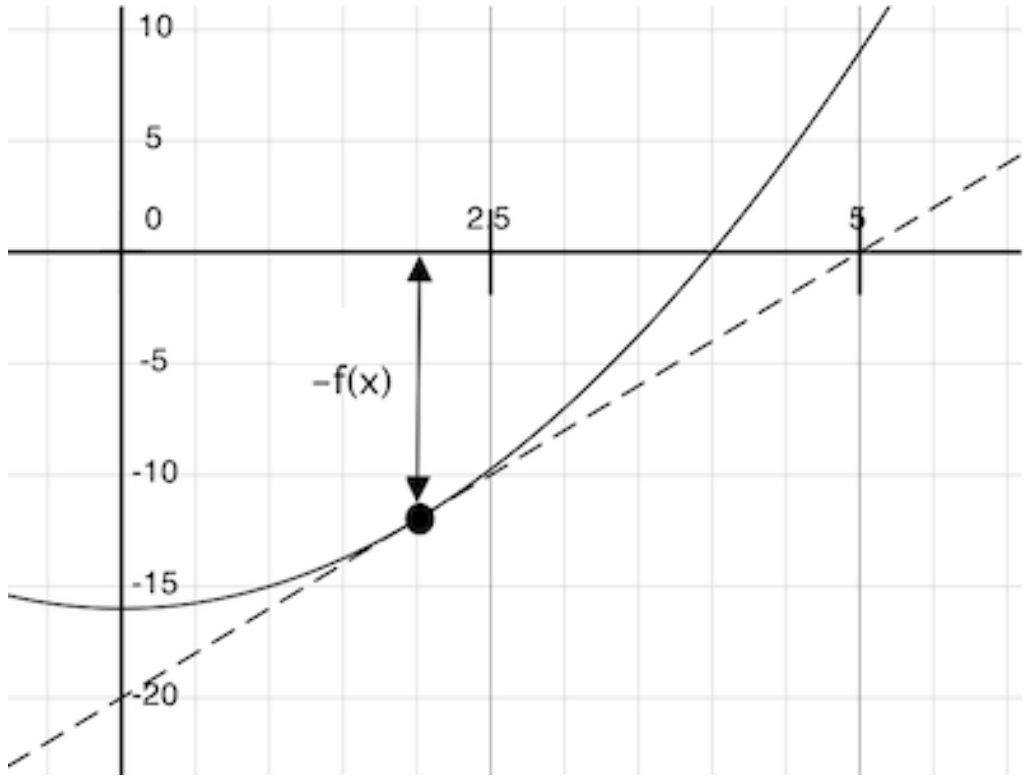

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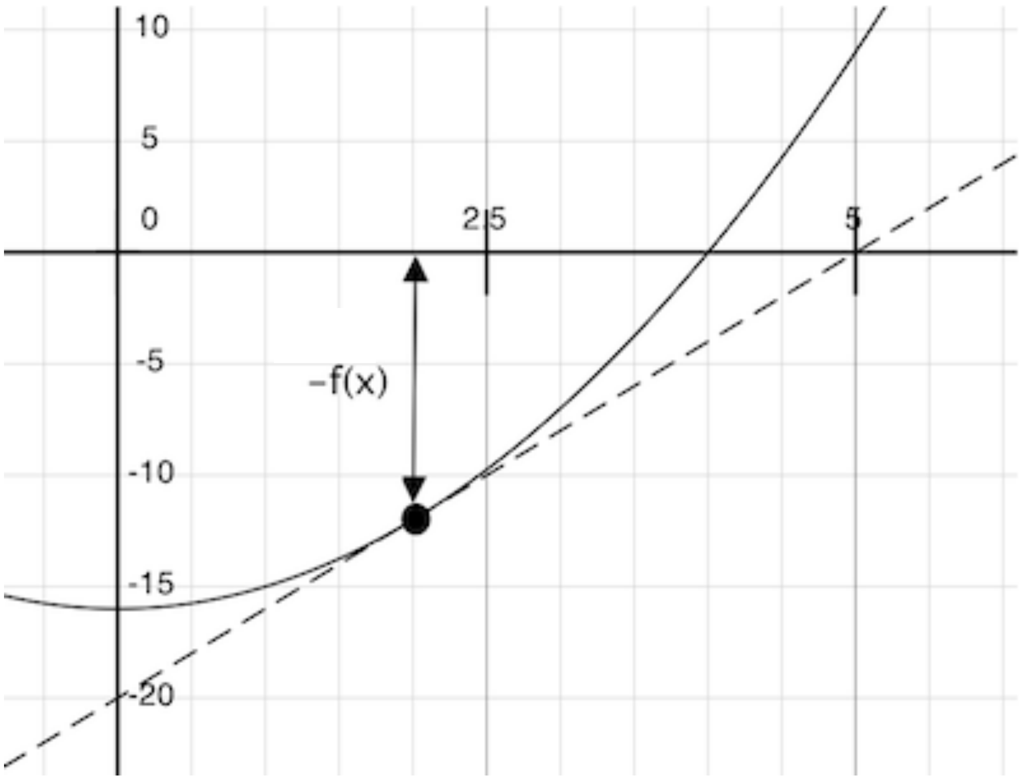

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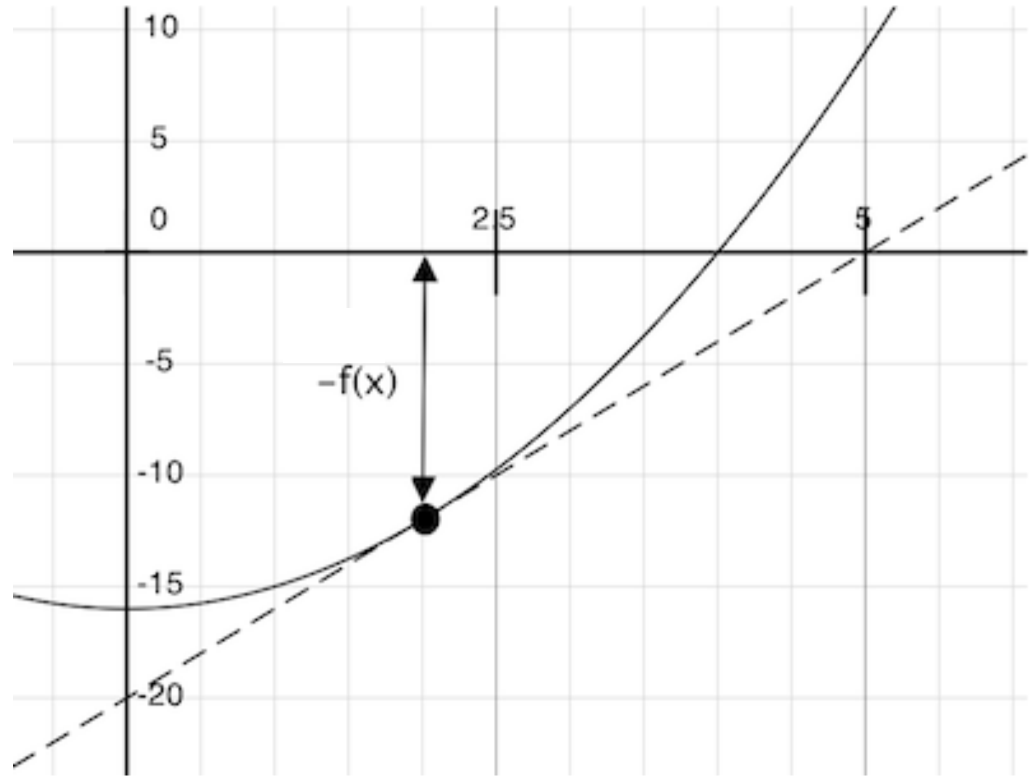

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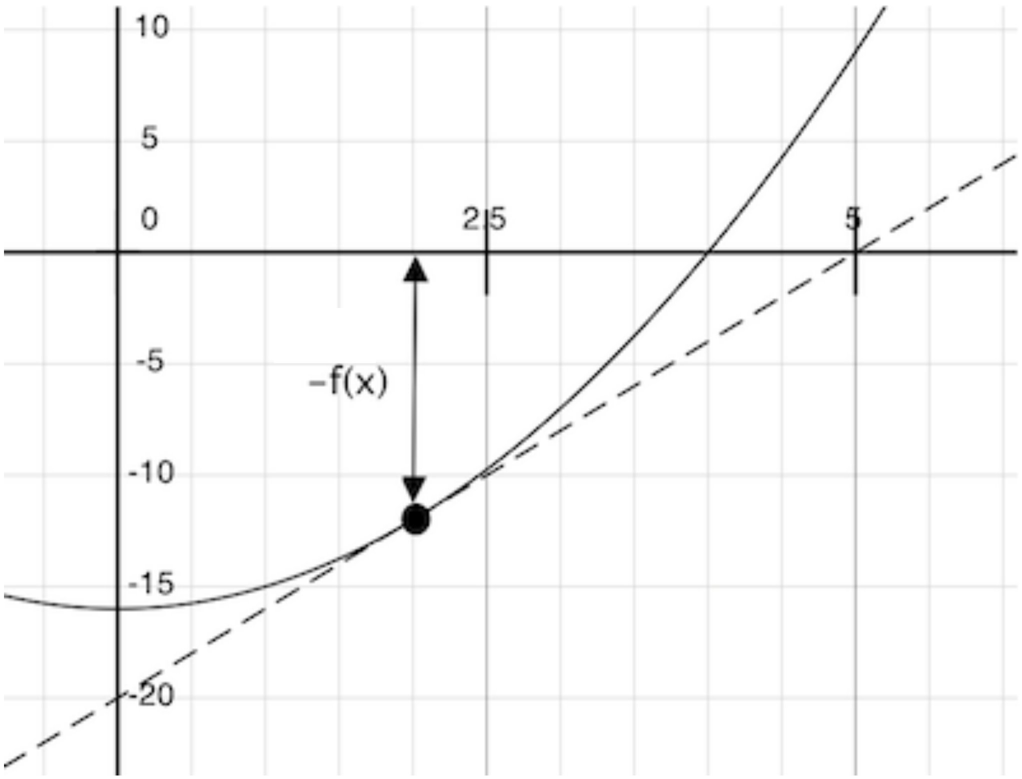

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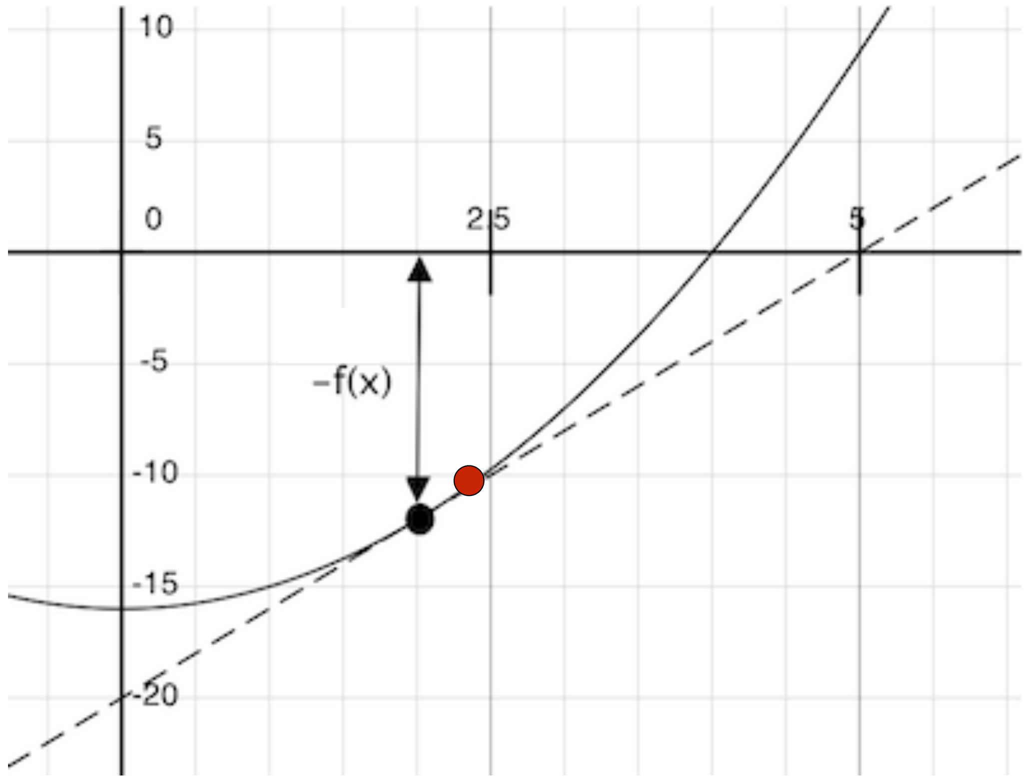

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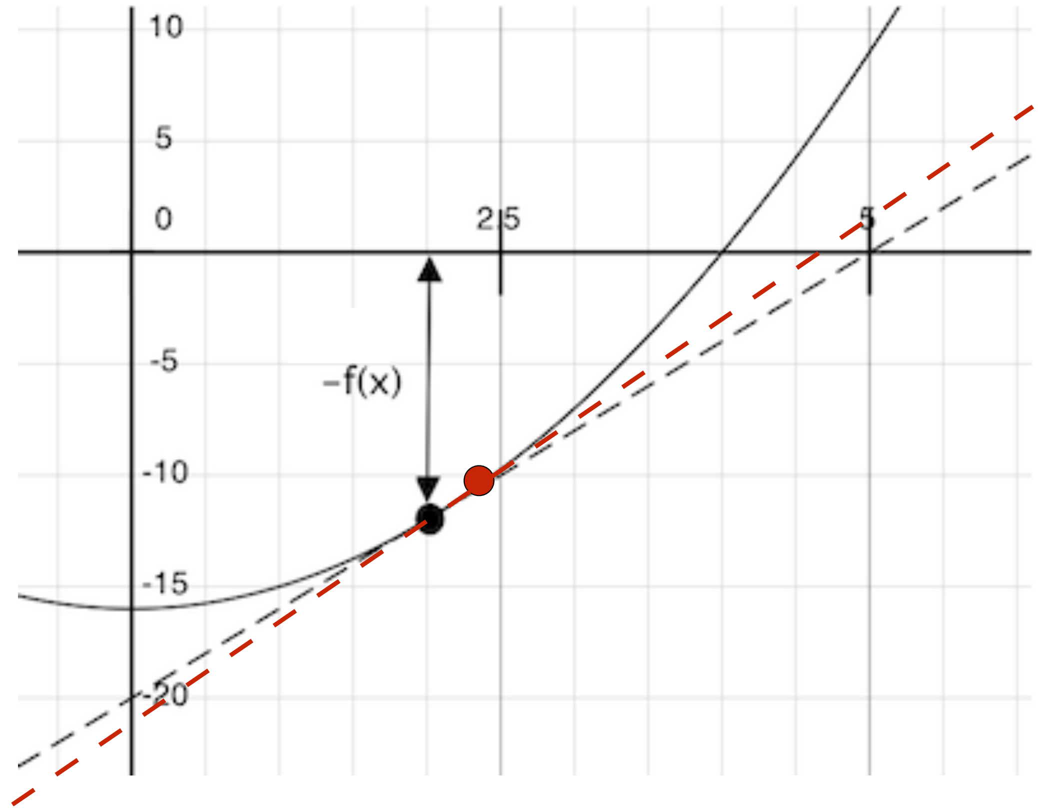

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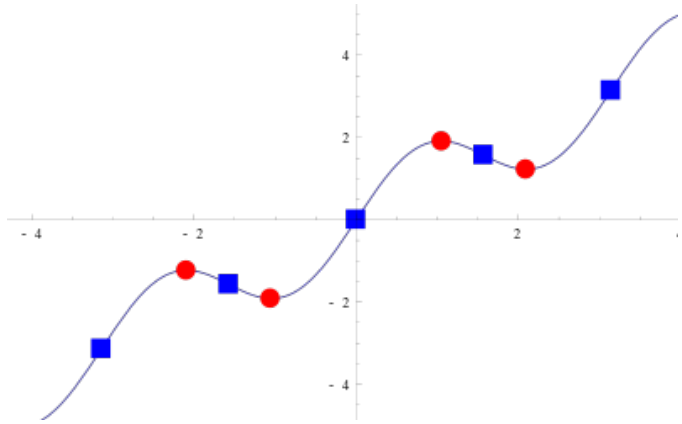

http://upload.wikimedia.org/wikipedia/commons/f/fd/Stationary_vs_inflection_pts.svg

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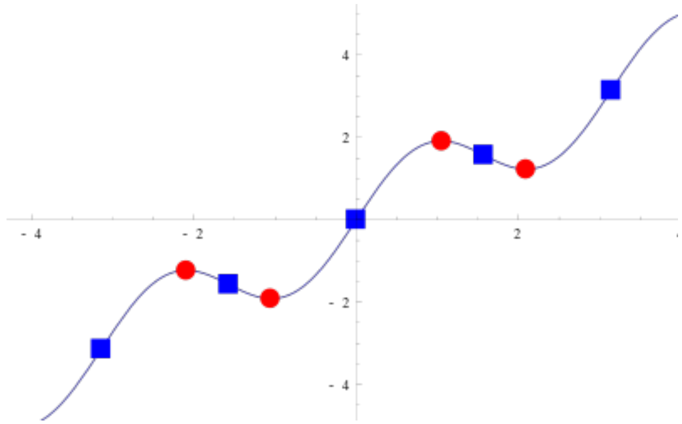

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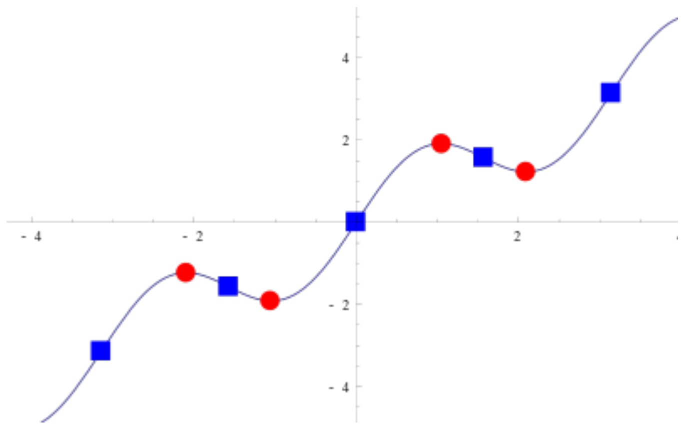

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Note: Root mean squared error can be optimized through linear algebra alone, but numerical optimization works for a much larger class of related error measures