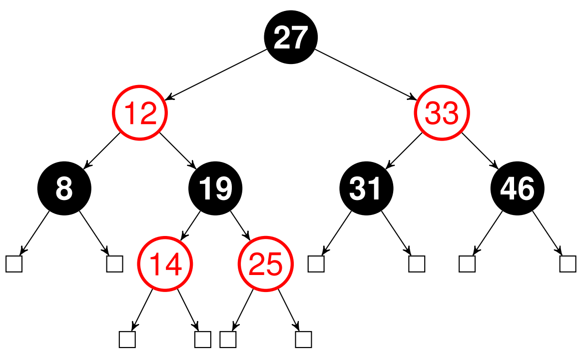
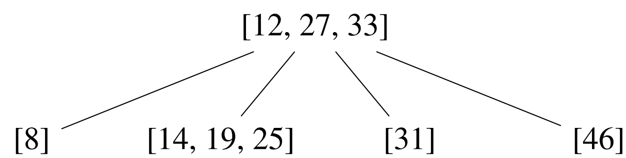
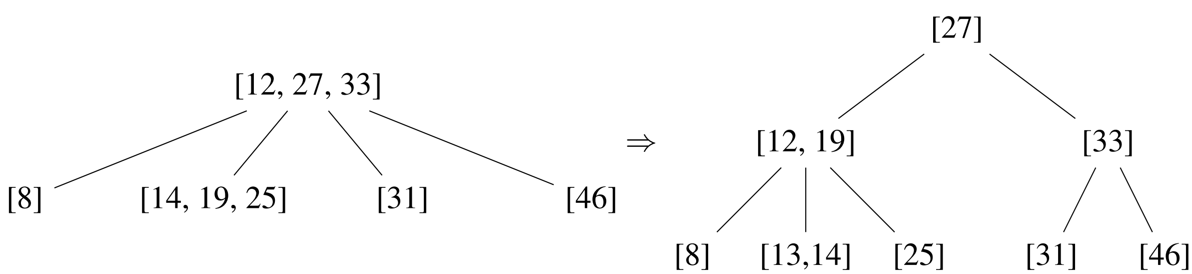
1 Balanced Search Trees

(a) Convert the red-black tree into a 2-4 tree.

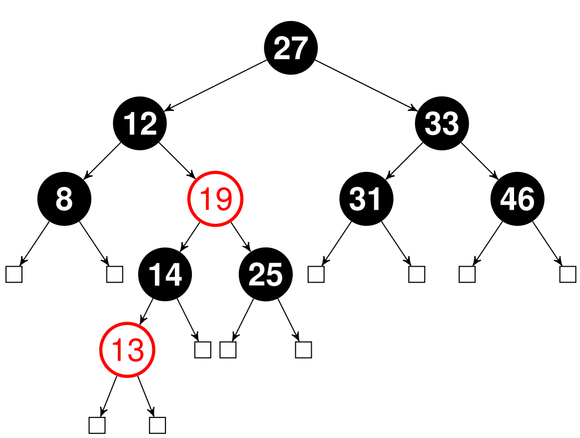




(b) Insert 13 into the 2-4 tree.



(c) Convert the resulting 2-4 tree into a valid red-black tree.

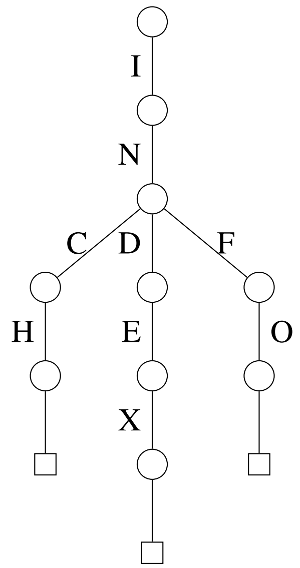


(d) Given a (2, 4) tree containing N keys, how would you obtain the keys in sorted order in worst case O(N) time? We don't need actual code—pseudo code or an unambiguous description will do (Final Fall '13). Simply generalize an inorder traversal: traverse the left (first) child of the node, emit the first key, traverse the second child of the node, emit the second key, etc

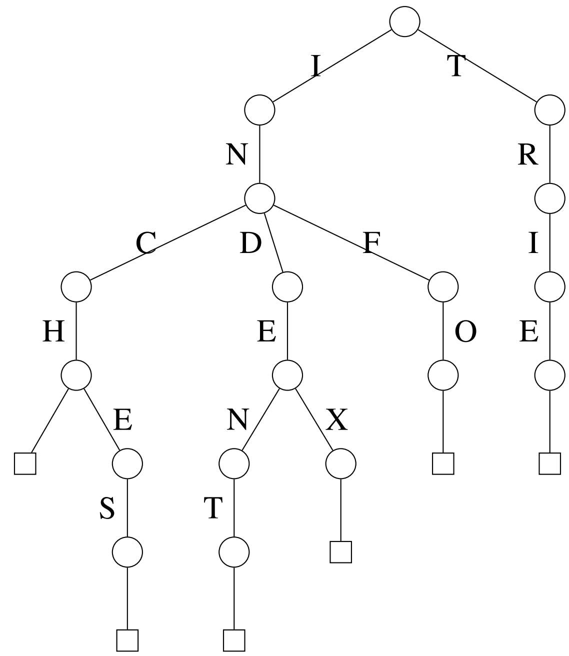
(e) f a (2,4) tree has depth h (that is, the (empty) leaves are at distance h from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree? (Final Spring '06) 2h comparisons.

2 Tries

First, list the words encoded by the trie. Then draw the trie after inserting the words *indent*, *inches*, and *trie*.



Encoded words: index, info, inch



3 Runtime Analysis

(a) Give the best and worst case runtimes for method A in $\Theta(\cdot)$ in terms of N.

```
public boolean A(int[] arr, int x) {
    //Assume arr is sorted; N is arr.length
    return A(arr, x, 0 , arr.length-1);
}

public boolean A(int[] arr, int x, int low, int high) {
    if (low > high) return false;
    int mid = (low + high) / 2;
    if (arr[mid] == x) return true;
    return A(arr, x, low, mid-1) || A(arr, x, mid+1, high);
}
```

This is almost binary search, except that both halves are recursed on.
Best case: $\Theta(1)$. Worst case: $\Theta(N)$.

(b) Give the best and worst case runtimes for method B in $\Theta(\cdot)$ in terms of N.

```
public int B(int[] arr) {
    //N is arr.length
    int count = arr.length - 1;
    while(count > 50) {
        count = count - arr.length / 50;
    }
    return count;
}
```

No matter how big the input array is, the loop will only execute about 50 times.
Best case: $\Theta(1)$. Worst case: $\Theta(1)$.