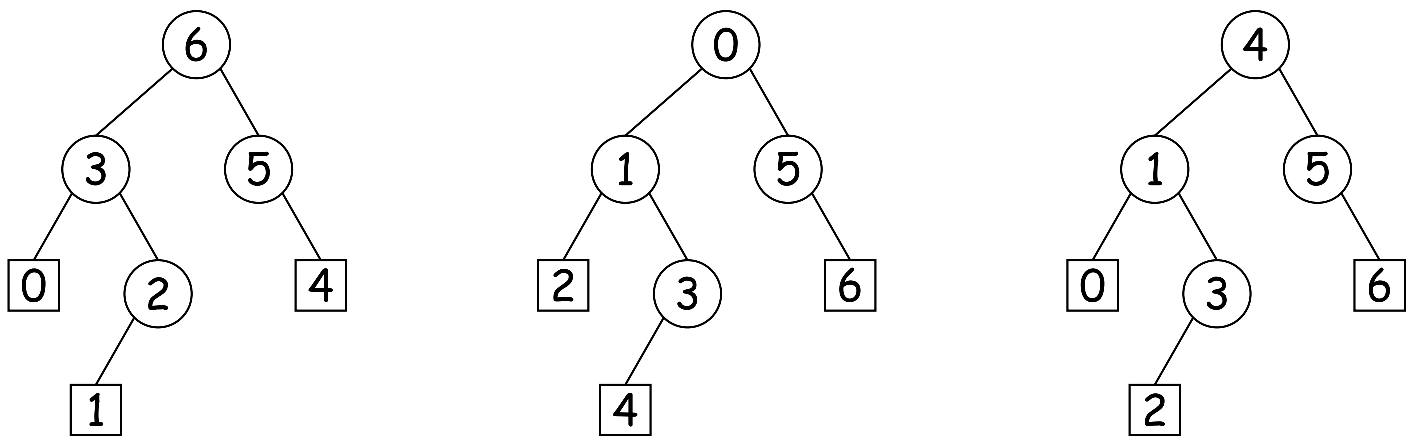
CS61B Lecture #20: Trees

A Recursive Structure

- Trees naturally represent recursively defined, hierarchical objects with more than one recursive subpart for each instance.
- Common examples: expressions, sentences.
  - Expressions have definitions such as "an expression consists of a literal or two expressions separated by an operator."
- Also describe structures in which we recursively divide a set into multiple subsets.

Fundamental Operation: Traversal

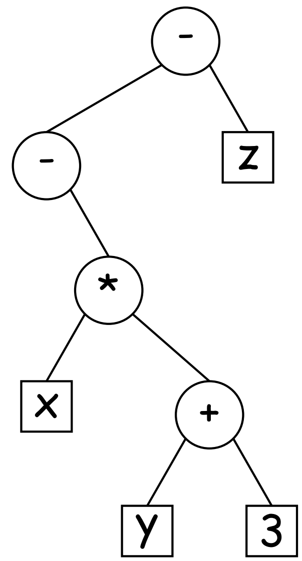
- *Traversing a tree* means enumerating (some subset of) its nodes.
- Typically done recursively, because that is natural description.
- As nodes are enumerated, we say they are *visited.*
- Three basic orders for enumeration (+ variations):
  - **Preorder:** visit node, traverse its children.
  - **Postorder:** traverse children, visit node.
  - **Inorder:** traverse first child, visit node, traverse second child (binary trees only).



Postorder *Preorder* *inorder*

Preorder Traversal and Prefix Expressions

Problem: Convert



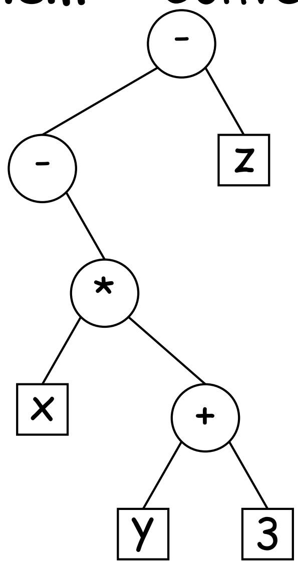
⇒ `(- (- (* x (+ y 3))) z)`

(Assume `Tree`<*Label*> is means "Tree whose labels have type *Label*.)

```
static String toLisp(Tree<String> T) {
  if (T == null) return "";
  else if (T.degree() == 0) return T.label();
  else {
    String R;  R = "";
    for (int i = 0; i < T.numChildren(); i += 1)
      R += " " + toLisp(T.child(i));
    return String.format("(%s%s)", T.label(), R);
  }
}
```

Inorder Traversal and Infix Expressions

Problem: Convert



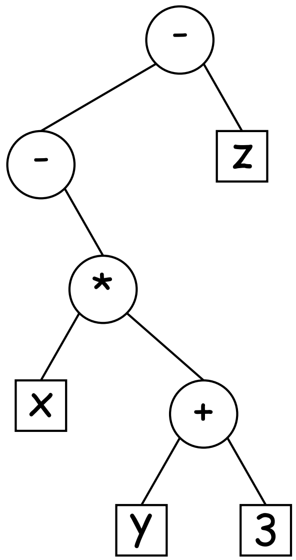
$\Rightarrow$ `((-(x*(y+3)))-z)`

To think about: how to get rid of all those parentheses.

```
static String toInfix(Tree<String> T) {
  if (T == null)
    return "";
  if (T.degree() == 0)
    return T.label();
  else {
    String left = toInfix(T.left()), right = toInfix(T.right());
    return String.format("(%s%s%s)", left, T.label(), right);
  }
}
```

Postorder Traversal and Postfix Expressions

Problem: Convert



$\Rightarrow$ `x y 3 +:2 *:2 -:1 z -:2`

```
static String toPolish(Tree<String> T) {
  if (T == null)
    return "";
  else {
    String R;  R = "";
    for (int i = 0; i < T.numChildren(); i += 1)
      R += toPolish(T.child(i)) + " ";
    return String.format("%s%s:%d", R, T.label(), T.degree());
  }
}
```

A General Traversal: The Visitor Pattern

```
void preorderTraverse(Tree<Label> T, Action<Label> whatToDo)
{
  if (T != null) {
    whatToDo.action(T);
    for (int i = 0; i < T.numChildren(); i += 1)
      preorderTraverse(T.child(i), whatToDo);
  }
}
```

- What is `Action`?

```
interface Action<Label> {
  void action(Tree<Label> T);
}
```

Now, using Java 8 lambda syntax, I can print all labels in the tree in preorder with:

```
preorderTraverse(myTree,
                 (Tree<String> T) -> System.out.print(T.label()));
```

Iterative Depth-First Traversals

- Tree recursion conceals data: a *stack* of nodes (all the `T` arguments) and a little extra information. Can make the data explicit:

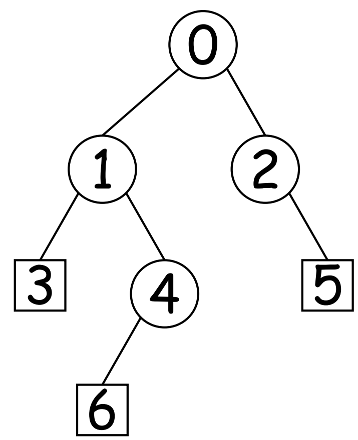
```
void preorderTraverse2(Tree<Label> T, Action whatToDo) {
  Stack<Tree<Label>> s = new Stack<>();
  s.push(T);
  while (!s.isEmpty()) {
    Tree<Label> node = s.pop();
    if (node != null) {
        whatToDo.action(node);
        for (int i = node.numChildren()-1; i >= 0; i -= 1)
           s.push(node.child(i));  // Why backward?
    }
  }
}
```

- To do a breadth-first traversal, use a queue instead of a stack, replace `push` with `add`, and `pop` with `removeFirst`.

- Makes breadth-first traversal worst-case linear time in all cases, but also linear *space* for "bushy" trees.

Level-Order (Breadth-First) Traversal

Problem: Traverse all nodes at depth 0, then depth 1, etc:



Breadth-First Traversal Implemented

A simple modification to iterative depth-first traversal gives breadth-first traversal. Just change the (LIFO) stack to a (FIFO) queue:

```
void preorderTraverse2(Tree<Label> T, Action whatToDo) {
  ArrayDeque<Tree<Label>> s = new ArrayDeque<>();  // (Changed)
  s.push(T);
  while (!s.isEmpty()) {
    Tree<Label> node = s.remove();    // (Changed)
    if (node != null) {
        whatToDo.action(node);
        for (int i = 0; i < node.numChildren(); i += 1) // (Changed)
           s.push(node.child(i));
    }
  }
}
```

Times

- The traversal algorithms have roughly the form of the `boom` example in §1.3.3 of *Data Structures*—an exponential algorithm.
- *However,* the role of M in that algorithm is played by the *height* of the tree, not the number of nodes.
- In fact, easy to see that tree traversal is *linear:* $\Theta(N)$, where N is the # of nodes: Form of the algorithm implies that there is one visit at the root, and then one visit for every *edge* in the tree. Since every node but the root has exactly one parent, and the root has none, must be $N-1$ edges in any non-empty tree.
- In positional tree, is also one recursive call for each empty tree, but # of empty trees can be no greater than kN, where k is arity.
- For k-ary tree (max # children is k), $h+1 \le N \le \frac{k^{h+1}-1}{k-1}$, where h is height.
- So $h \in \Omega(\log_k N) = \Omega(\lg N)$ and $h \in O(N)$.
- Many tree algorithms look at one child only. For them, time is proportional to the *height* of the tree, and this is $\Theta(\lg N)$, assuming that tree is *bushy*—each level has about as many nodes as possible.

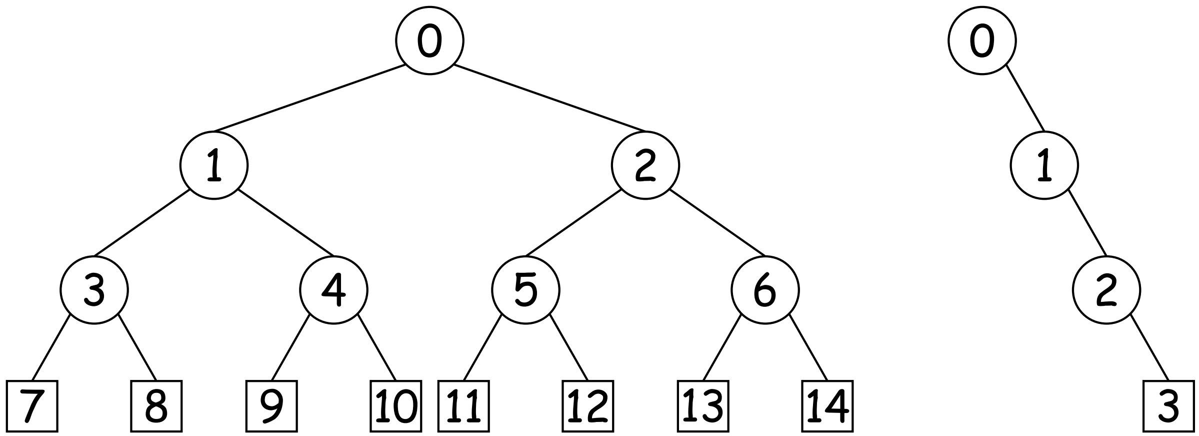
Recursive Breadth-First Traversal: Iterative Deepening

- For each level, k, of the tree from 0 to h, call `doLevel(T,k)`:

```
void doLevel(Tree T, int lev) {
  if (lev == 0)
    visit T
  else
    for each non-null child, C, of T {
      doLevel(C, lev-1);
    }
}
```

- We do breadth-first traversal by repeated (truncated) depth-first traversals.
- In `doLevel(T, k)`, we skip (i.e., traverse but don't visit) the nodes before level k, and then visit at level k, but not their children.

Iterative Deepening Time?



- Let h be height, N be # of nodes.
- Count # edges traversed (i.e, # of calls, not counting null nodes).
- First (full) tree: 1 for level 0, 3 for level 1, 7 for level 2, 15 for level 3.
- Or in general $(2^1 - 1) + (2^2 - 1) + \ldots + (2^{h+1} - 1) = 2^{h+2} - h \in \Theta(N)$, since $N = 2^{h+1} - 1$ for this tree.
- Second (*right leaning*) tree: 1 for level 0, 2 for level 2, 3 for level 3.
- Or in general $(h+1)(h+2)/2 = N(N+1)/2 \in \Theta(N^2)$, since $N = h+1$ for this kind of tree.

Iterators for Trees

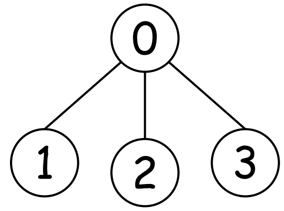
- Frankly, iterators are not terribly convenient on trees.
- But can use ideas from iterative methods.

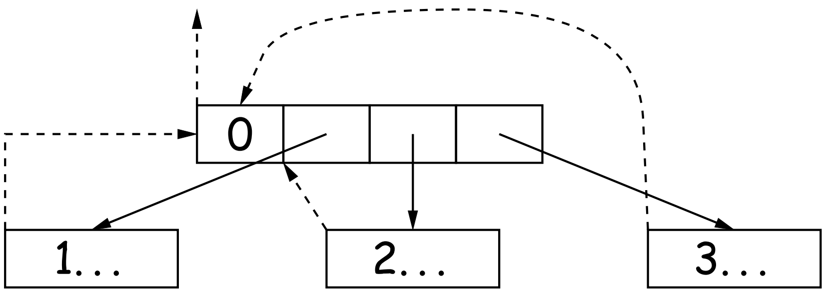
```
class PreorderTreeIterator<Label> implements Iterator<Label> {
  private Stack<Tree<Label>> s = new Stack<Tree<Label>>();

  public PreorderTreeIterator(Tree<Label> T) { s.push(T);  }

  public boolean hasNext() { return !s.isEmpty(); }
  public T next() {
    Tree<Label> result = s.pop();
    for (int i = result.numChildren()-1; i >= 0; i -= 1)
      s.push(result.child(i));
    return result.label();
  }
  void remove() { throw new UnsupportedOperationException(); }
}
```

Example: (what do I have to add to class `Tree` first?)

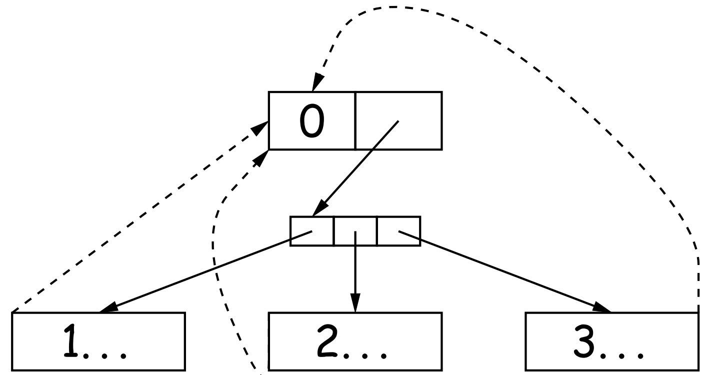
```
for (String label : aTree) System.out.print(label + " ");
```

Tree Representation

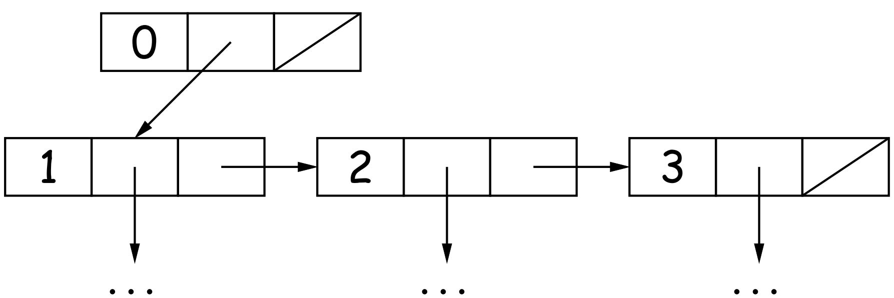




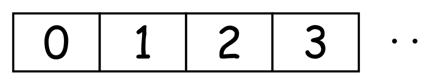
(a) Embedded child pointers
(+ optional parent pointers)



(b) Array of child pointers
(+ optional parent pointers)



(c) child/sibling pointers



(d) breadth-first array
(complete trees)