CS61B Lecture #26

Today:

- Sorting algorithms: why?
- Insertion Sort.
- Inversions

Purposes of Sorting

- Sorting supports searching
- Binary search standard example
- Also supports other kinds of search:
  - Are there two equal items in this set?
  - Are there two items in this set that both have the same value for property X?
  - What are my nearest neighbors?
- Used in numerous unexpected algorithms, such as convex hull (smallest convex polygon enclosing set of points).

Some Definitions

- A sort is a *permutation* (re-arrangement) of a sequence of elements that brings them into order, according to some *total order.* A total order, $\preceq$, is:
  - **Total:** $x \preceq y$ or $y \preceq x$ for all x, y.
  - **Reflexive:** $x \preceq x$;
  - **Antisymmetric:** $x \preceq y$ and $y \preceq x$ iff $x = y$.
  - **Transitive:** $x \preceq y$ and $y \preceq z$ implies $x \preceq z$.
- However, our orderings may allow unequal items to be equivalent:
  - E.g., can be two dictionary definitions for the same word: if entries sorted only by word, then sorting could put either entry first.
  - A sort that does not change the relative order of equivalent entries is called *stable.*

Classifications

- *Internal sorts* keep all data in primary memory
- *External sorts* process large amounts of data in batches, keeping what won't fit in secondary storage (in the old days, tapes).
- *Comparison-based* sorting assumes only thing we know about keys is order
- *Radix sorting* uses more information about key structure.
- *Insertion sorting* works by repeatedly inserting items at their appropriate positions in the sorted sequence being constructed.
- *Selection sorting* works by repeatedly selecting the next larger (smaller) item in order and adding it one end of the sorted sequence being constructed.

Sorting by Insertion

- Simple idea:
  - starting with empty sequence of outputs.
  - add each item from input, *inserting* into output sequence at right point.
- Very simple, good for small sets of data.
- With vector or linked list, time for find + insert of one item is at worst $\Theta(k)$, where k is # of outputs so far.
- So gives us $O(N^2)$ algorithm. Can we say more?

Inversions

- Can run in $\Theta(N)$ comparisons if already sorted.
- Consider a typical implementation for arrays:

```
for (int i = 1; i < A.length; i += 1) {
  int j;
  Object x = A[i];
  for (j = i-1; j >= 0; j -= 1) {
    if (A[j].compareTo(x) <= 0)  /* (1) */
      break;
    A[j+1] = A[j];
  }
  A[j+1] = x;
}
```

- #times (1) executes $\approx$ how far `x` must move.
- If all items within K of proper places, then takes $O(KN)$ operations.
- Thus good for any amount of *nearly sorted* data.
- One measure of unsortedness: # of *inversions:* pairs that are out of order (= 0 when sorted, $N(N-1)/2$ when reversed).
- Each step of `j` decreases inversions by 1.

Shell's sort

Idea: Improve insertion sort by first sorting *distant* elements:

- First sort subsequences of elements $2^k - 1$ apart:
  - sort items #$0,\ 2^k - 1,\ 2(2^k - 1),\ 3(2^k - 1),\ \ldots$, then
  - sort items #$1,\ 1 + 2^k - 1,\ 1 + 2(2^k - 1),\ 1 + 3(2^k - 1),\ \ldots$, then
  - sort items #$2,\ 2 + 2^k - 1,\ 2 + 2(2^k - 1),\ 2 + 3(2^k - 1),\ \ldots$, then
  - *etc.*
  - sort items #$2^k - 2,\ 2(2^k - 1) - 1,\ 3(2^k - 1) - 1,\ \ldots$,
  - Each time an item moves, can reduce #inversions by as much as $2^k + 1$.
- Now sort subsequences of elements $2^{k-1} - 1$ apart:
  - sort items #$0,\ 2^{k-1} - 1,\ 2(2^{k-1} - 1),\ 3(2^{k-1} - 1),\ \ldots$, then
  - sort items #$1,\ 1 + 2^{k-1} - 1,\ 1 + 2(2^{k-1} - 1),\ 1 + 3(2^{k-1} - 1),\ \ldots$,
  - $\vdots$
- End at plain insertion sort ($2^0 = 1$ apart), but with most inversions gone.
- Sort is $\Theta(N^{1.5})$ (take CS170 for why!).