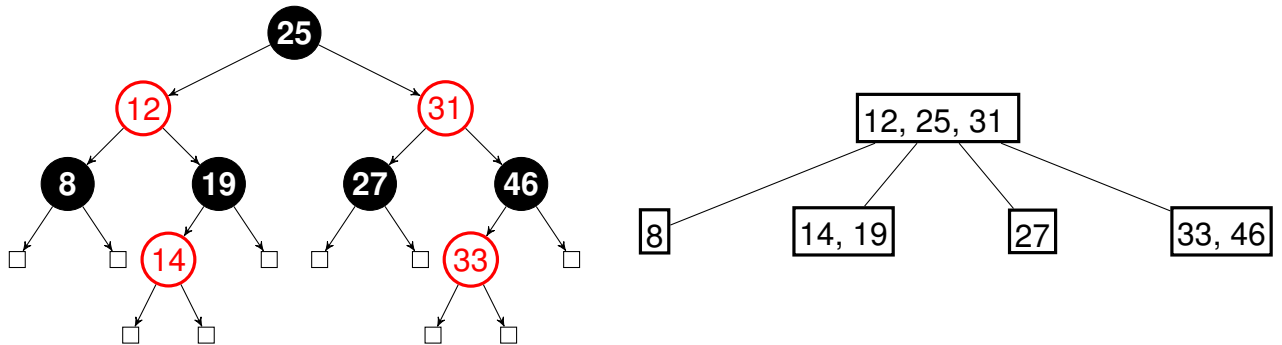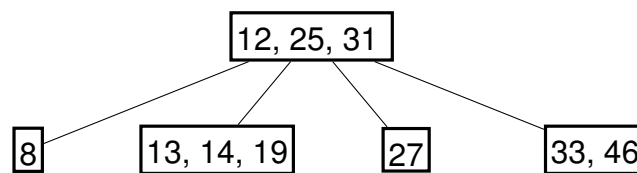## 1   Balanced Search Trees

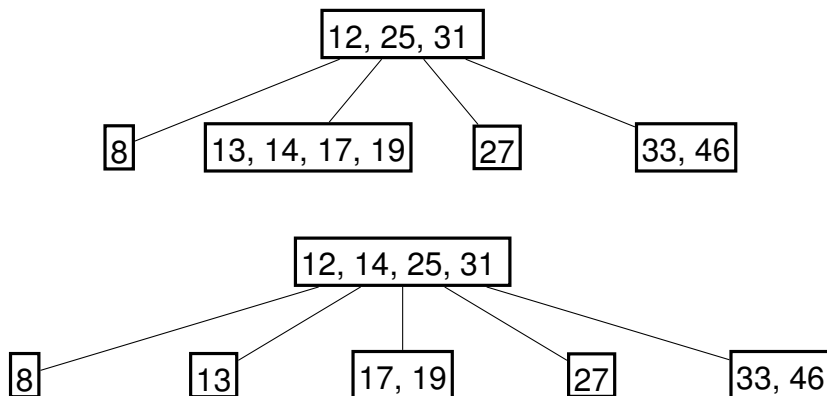(a) Convert the red-black tree into a 2-4 tree. Solid nodes are black.

(b) Insert the keys 13 and 17 into the resulting 2-4 tree. Assume that, if a node has 4 keys, we choose to push up the left of the 2 middle keys (so the 2nd key from the left).
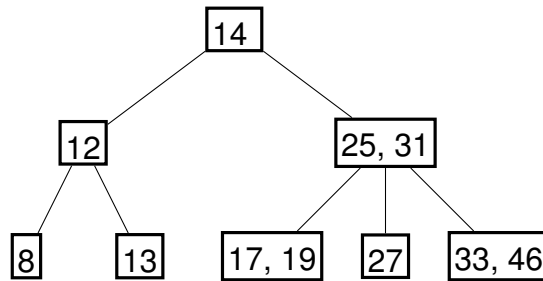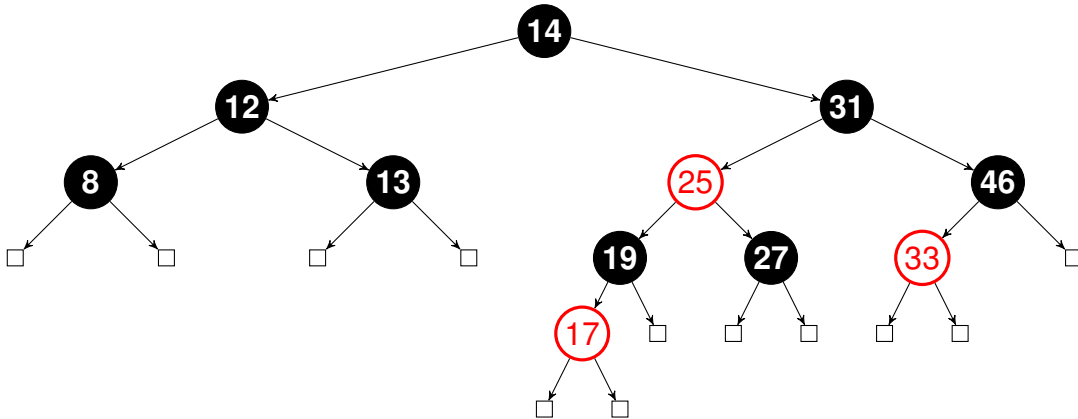
After adding 13:

Intermediate steps after adding 17:

(c) Convert the resulting 2-4 tree into a valid left-leaning red-black tree.

(d) Given a 2-4 tree containing N keys, describe how you can obtain the keys in sorted order in worst case $O(N)$ time.

Generalize an in-order traversal: traverse the left (first) child of the node, emit the first key of the node, traverse the second child of the node, emit the second key of the node, etc.

(e) If a 2-4 tree has depth H (that is, the leaves are at a distance of H from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?

$2H$ comparisons. The number of black nodes from root to leaf is the same for all nodes in a red-black tree and it is equal to the height of its equivalent 2-4 tree. The maximum number of comparisons occurs on the longest root to leaf path (i.e. the path with the most nodes). We can also have at most one red node after each black node (we can't have more than two red nodes in a row in a red-black tree), resulting in a longest possible path that is $2H$ nodes long. As a result, we will make at most $2H$ comparisons.
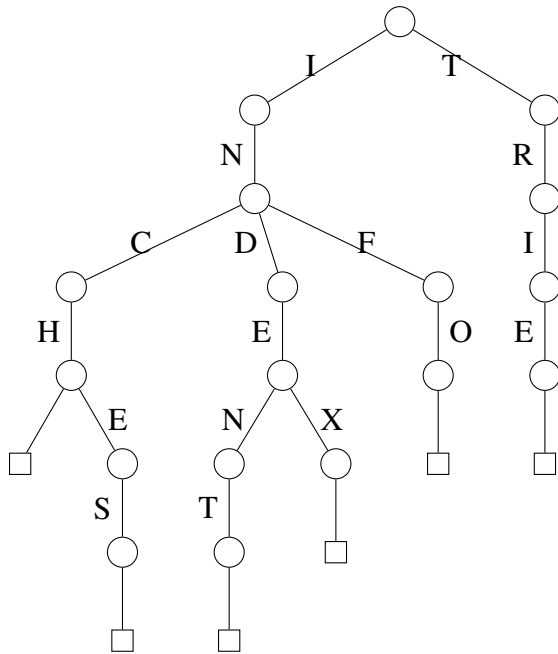
2 Tries

List the words encoded by the following trie, then draw the resulting trie after inserting the words *indent*, *inches*, and *trie*.
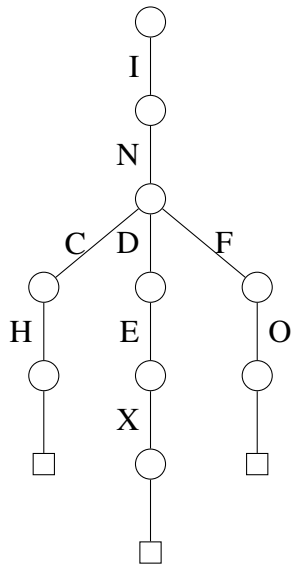
The encoded words are *index*, *info*, and *inch*.

After inserting the words *indent*, *inches*, and *trie*, we get the following structure:

3 Skip Lists

Draw the resulting skip list after adding the following numbers at the specified random heights. Highlight the links traversed to find 148.

Number	40	41	43	48	54	59	77	128	131	139	148	161	170	179	189
Height	2	1	3	1	3	1	4	2	2	1	1	3	2	1	1

The links used to traverse to the item are in blue, and the links that overshoot are in red.