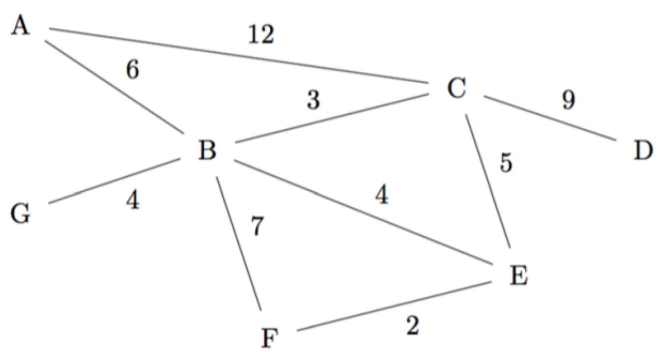
CS 61B	More Shortest Paths and MSTs
Fall 2021	Exam Prep Discussion 14: November 29, 2021

1 Introduction to MSTs



(a) For the graph above, list the edges in the order they're added to the MST by Kruskal's and Prim's algorithm. Assume Prim's algorithm starts at vertex A. Assume ties are broken in alphabetical order. Denote each edge as a pair of vertices (e.g. AB is the edge from A to B)

Prim's algorithm order:
Kruskal's algorithm order:

(b) Is there any vertex for which the shortest paths tree from that vertex is the same as your Prim MST? If there are multiple viable vertices, list all.

(c) True/False: Adding 1 to the smallest edge of a graph G with unique edge weights must change the total weight of its MST

(d) True/False: The shortest path from vertex A to vertex B in a graph G is the same as the shortest path from A to B using only edges in T, where T is the MST of G.

(e) True/False: Given any cut, the maximum-weight crossing edge is in the maximum spanning tree.

2 Multiple MSTs

Recall a graph can have multiple MSTs if there are multiple spanning trees of minimum weight.

(a) For each subpart below, select the correct option and justify your answer. If you select "never" or "always," provide a short explanation. If you select "sometimes", provide two graphs that fulfill the given properties — one with multiple MSTs and one without. Assume G is an undirected, connected graph.

1. If **none** the edge weights are **identical**, there will

   ◯ never be multiple MSTs in G.

   ◯ sometimes be multiple MSTs in G.

   ◯ always be multiple MSTs in G.

   Justification:

2. If **some** of the edge weights are **identical**, there will

   ◯ never be multiple MSTs in G.

   ◯ sometimes be multiple MSTs in G.

   ◯ always be multiple MSTs in G.

   Justification:

3. If **all** of the edge weights are **identical**, there will

   ◯ never be multiple MSTs in G.

   ◯ sometimes be multiple MSTs in G.

   ◯ always be multiple MSTs in G.

   Justification:

(b) Suppose we have a connected, undirected graph G with N vertices and N edges, where all the **edge weights are identical**. Find the maximum and minimum number of MSTs in G and explain your reasoning.

Minimum: _________
Maximum: _________

Justification:

(c) It is possible that Prim's and Kruskal's find **different** MSTs on the same graph `G` (as an added exercise, construct a graph where this is the case!). Given any graph `G` with integer edge weights, modify G to **ensure** that Prim's and Kruskal's will always find the same MST. You may not modify Prim's or Kruskal's.
Hint: Look at subpart 1 of part a.

3 Graph Algorithm Design

Given a **undirected**, **weighted** graph G with **positive**, **integer** edge weights, we want to find a path from **u** to **v** that minimizes the total cost. For each "catch" below, find the path of optimal cost no slower than $O(ElogV)$.

(a) Excluding the start and end vertex, we partition the vertices into 5 subsets, and we must visit vertices in order of their subset. That is, if we are in subset **k**, the next vertex we visit must be in subset $\mathbf{k + 1}$.

(b) We must visit two designated vertices s and k on our path.

(c) If two paths from u to v are of the same cost, we will choose the path with fewer edges.

(d) Instead of starting from u and ending at v, we can start from any vertex in a subset of vertices and end at any vertex in a subset of vertices. Each subset is of size k.