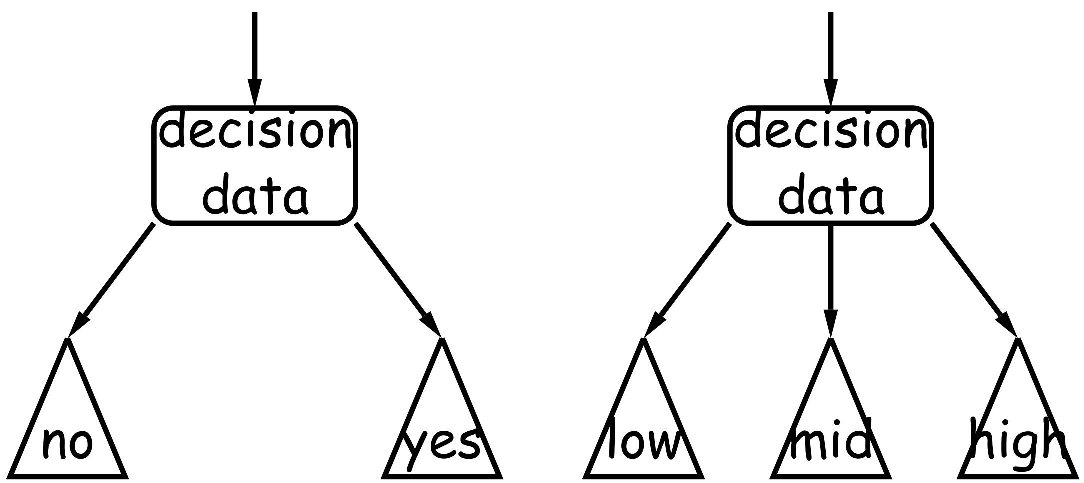
CS61B Lecture #21: Tree Searching

Divide and Conquer

- Much (most?) computation is devoted to finding things in response to various forms of query.
- Linear search for response can be expensive, especially when data set is too large for primary memory.
- Preferable to have criteria for *dividing* data to be searched into pieces recursively
- We saw the figure for $\lg N$ algorithms: at 1 μsec per comparison, could process 10^{300000} items in 1 sec.
- Tree is a natural framework for the representation:



Binary Search Trees

Binary Search Property:

- Tree nodes contain *keys,* and possibly other data.
- All nodes in left subtree of node have *smaller* keys.
- All nodes in right subtree of node have *larger* keys.
- "Smaller" means any complete transitive, anti-symmetric ordering on keys:
  - exactly one of $x \prec y$ and $y \prec x$ true.
  - $x \prec y$ and $y \prec z$ imply $x \prec z$.
  - (To simplify, won't allow duplicate keys this semester).
- E.g., in dictionary database, node label would be (*word, definition*): *word* is the key.
- For concreteness here, we'll just use the standard Java convention of calling `.compareTo`.

A Binary Search Type

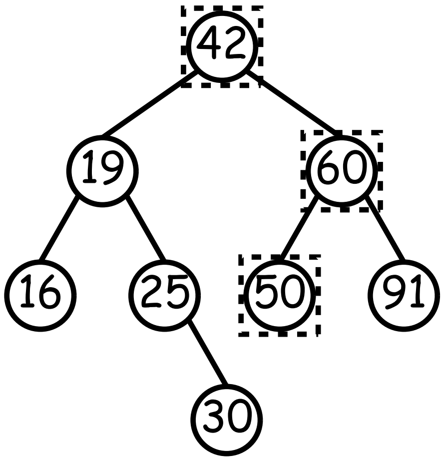
- Here, we'll use the following simple binary search tree type. Ignore all the style violations, please.

```
/** A node of a binary search tree associating a value of type VALUE
 *  with a key of type KEY.  (Thus, the labels in this tree are
 *  key/value pairs.) */
class BST<Key extends Comparable<Key>, Value> {
    Key key;
    Value value;
    BST<Key, Value> left, right;

    BST(Key key0, Value value0,
        BST<Key, Value> left0, BST<Key, Value> right0) {
        Body left to the reader.
    }
    BST(Key key0, Value value0) {
        this(key0, value0, null, null);
    }
}
```

- (Ignore the `Key extends Comparable<Key>` stuff for now. It just says that keys (of type `Key`) can be compared to each other.)

Finding

- Searching for 50 and 49:

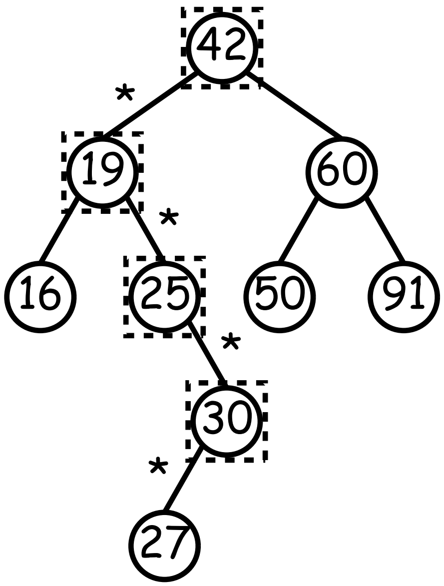



```
/** Return node in T containing L.  Null if none. */
static <Key extends Comparable<Key>, Value>
  BST<Key, Value> find(BST<Key, Value> T, Key L) {
    if (T == null)
        return T;
    if (L.compareTo(T.key) == 0)
        return T;
    else if (L.compareTo(T.key) < 0)
        return find(T.left, L);
    else
        return find(T.right, L);
}
```

- Dashed boxes show which node labels we look at.
- Number of nodes examined is proportional to height of tree.

Inserting

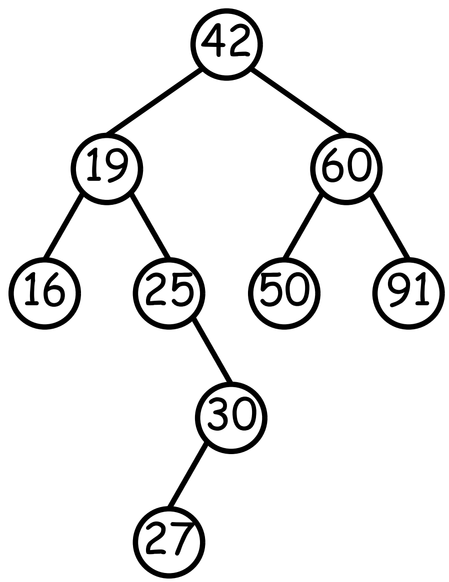
- Inserting 27



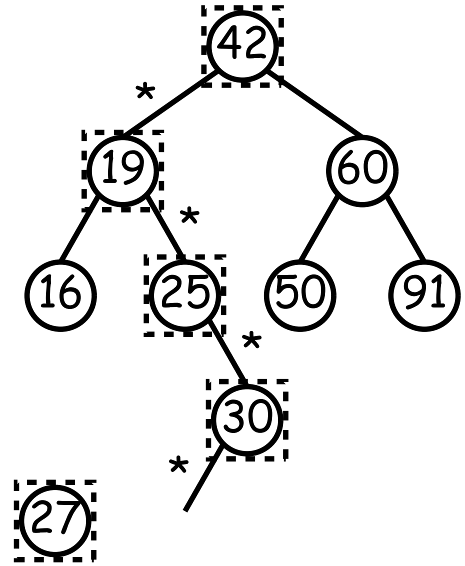

```
/** Insert V in T with key K, replacing existing
 *  value if present. Return the modified tree. */
static <Key extends Comparable<Key>, Value>
  BST<Key, Value> insert(BST<Key, Value> T,
                         Key K, Value V) {
    if (T == null)
        return new BST(K, V);
    if (K.compareTo(T.key) == 0)
        T.value = V;
    else if (K.compareTo(T.key) < 0)
        T.left = insert(T.left, K, V);
    else
        T.right = insert(T.right, K, V);
    return T;
}
```

- Starred edges are set (to themselves, unless initially null).
- Again, time proportional to height.

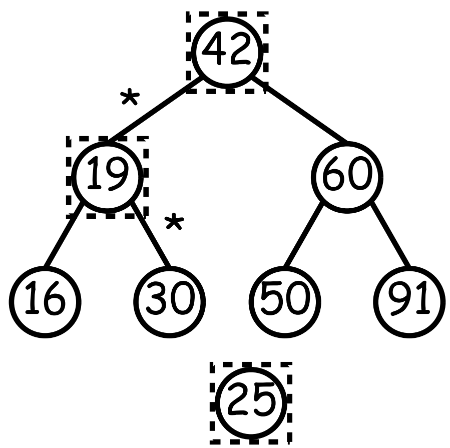
Deletion



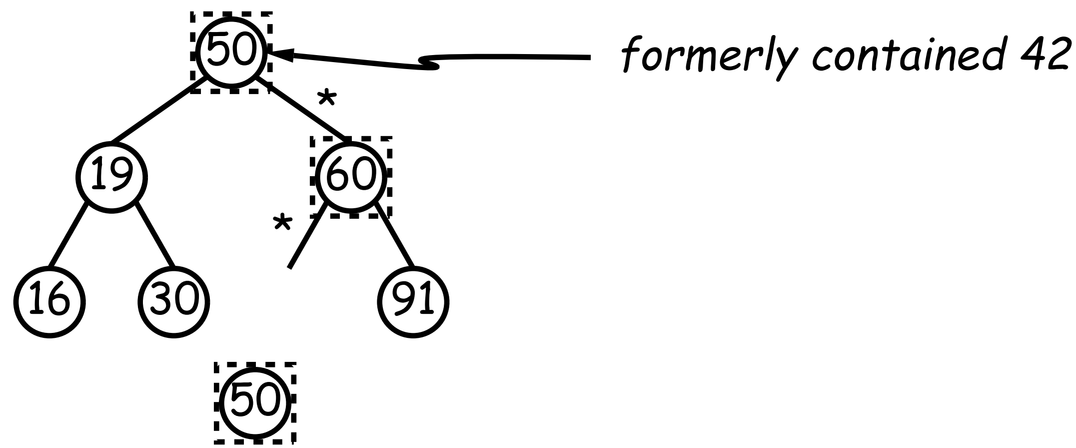
Initial



Remove 27



Remove 25



Remove 42

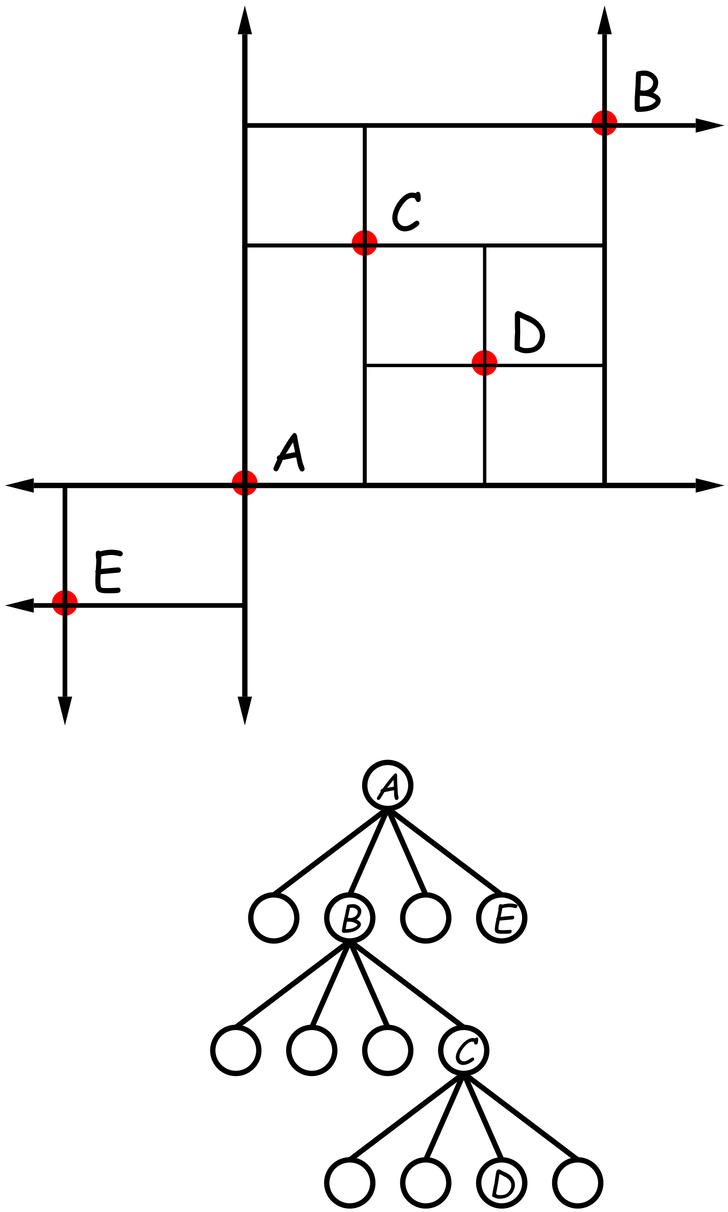
Deletion Algorithm

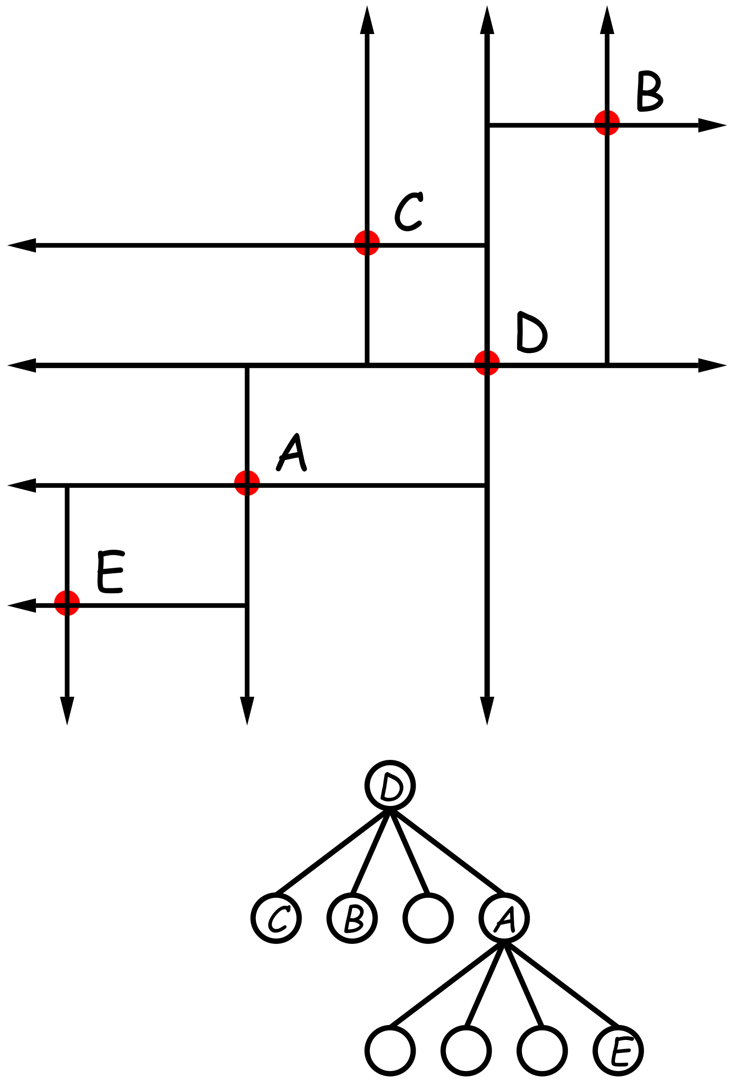
```
/** Remove K from T, and return the new tree. */
static <Key extends Comparable<Key>, Value>
  BST<Key, Value> remove(BST T, Key K) {

    if (T == null)
        return null;
    if (K.compareTo(T.key) == 0) {
        if (T.left == null)
            return T.right;
        else if (T.right == null)
            return T.left;
        else {
            BST<Key, Value> smallest = minNode(T.right);  // ??
            T.value = smallest.value;
            T.key = smallest.key;
            T.right = remove(T.right, smallest.key);
        }
    }
    else if (K.compareTo(T.key) < 0)
        T.left = remove(T.left, K);
    else
        T.right = remove(T.right, K);
    return T;
}
```

More Than Two Choices: Quadtrees

- Want to *index* information about 2D locations so that items can be retrieved by position. But how to compare positions "binarily?"
- Quadtrees do so using the same standard data-structuring trick as BSTs—*Divide and Conquer*—but with more subtrees.
- Idea: divide (2D) space into four *quadrants,* and store items in the appropriate quadrant. Repeat this recursively with each quadrant that contains more than one item.
- Original definition: a quadtree is either
  - Empty, or
  - An item at some position (x, y), called the root, plus
  - four quadtrees, each containing only items that are northwest, northeast, southwest, and southeast of (x, y).
- Big idea is that if you are looking for point (x', y') and the root is not the point you are looking for, you can narrow down which of the four subtrees of the root to look in by comparing coordinates (x, y) with (x', y').

Classical Quadtree: Example

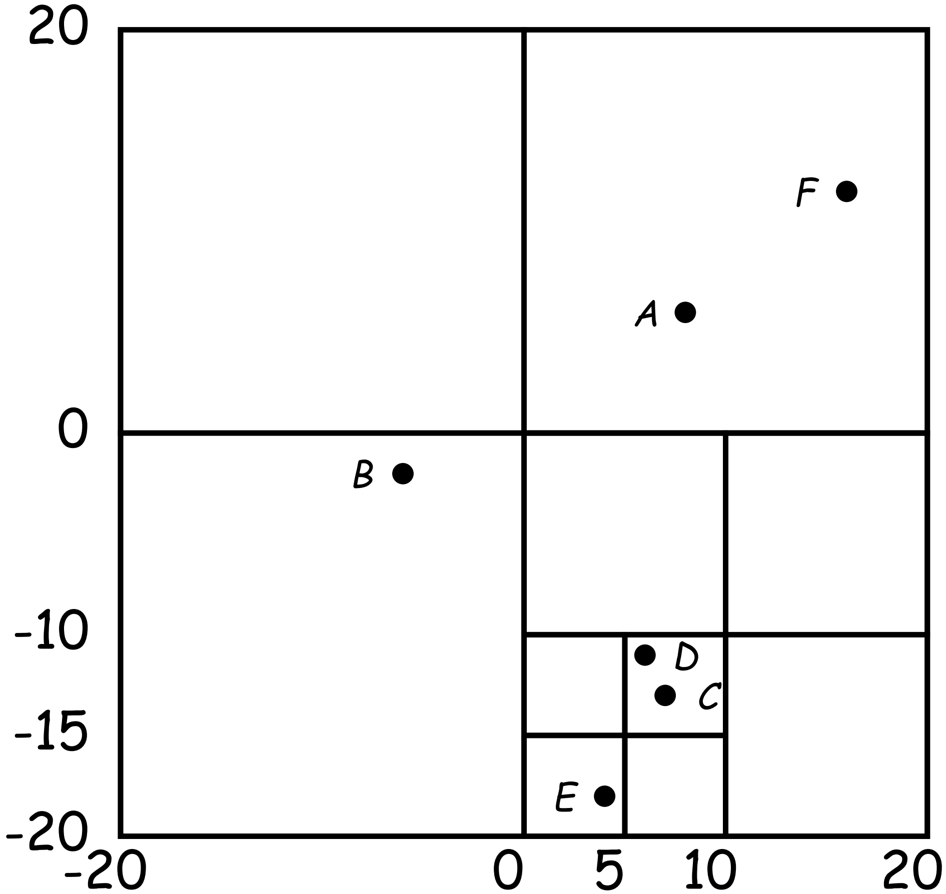




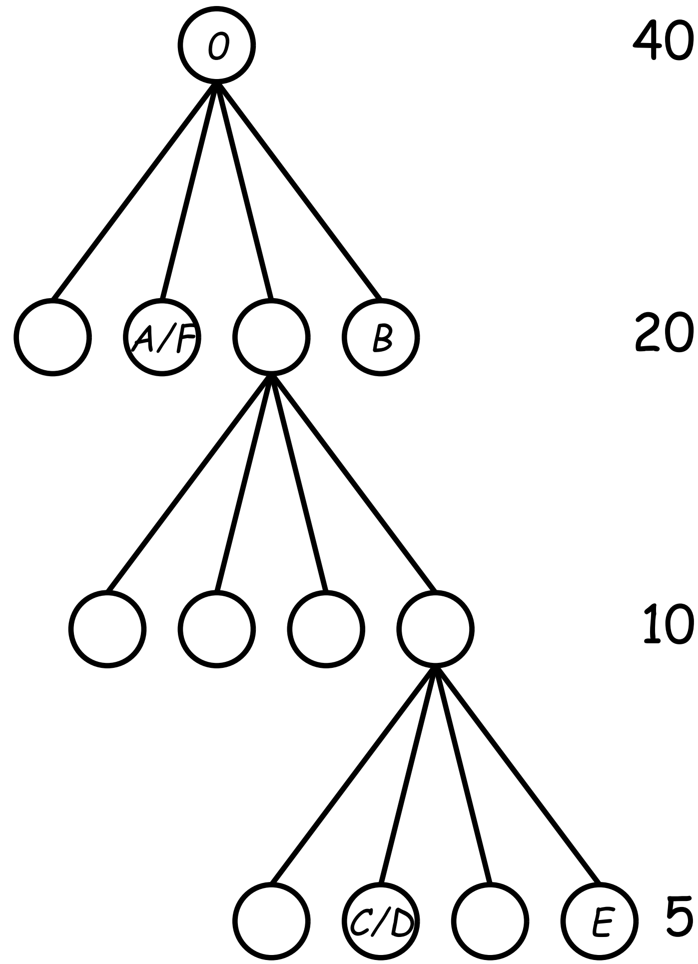
Point-region (PR) Quadtrees

- If we use a Quadtree to track moving objects, it may be useful to be able to *delete* items from a tree: when an object moves, the subtree that it goes in may change.
- Difficult to do with the classical data structure above, so we'll define instead:
- A quadtree consists of a bounding rectangle, B and either
  - Zero up to a small number of items that lie in that rectangle, or
  - Four quadtrees whose bounding rectangles are the four quadrants of B (all of equal size).
- A completely empty quadtree can have an arbitrary bounding rectangle, or you can wait for the first point to be inserted.

Example of PR Quadtree



(≤ 2 points per leaf)



Navigating PR Quadtrees

- To find an item at (x, y) in quadtree T,
  1. If (x, y) is outside the bounding rectangle of T, or T is empty, then (x, y) is not in T.
  2. Otherwise, if T contains a small set of items, then (x, y) is in T iff it is among these items.
  3. Otherwise, T consists of four quadtrees. Recursively look for (x, y) in each (however, step #1 above will cause all but one of these bounding boxes to reject the point immediately).
- Similar procedure works when looking for all items within some rectangle, R:
  1. If R does not intersect the bounding rectangle of T, or T is empty, then there are no items in R.
  2. Otherwise, if T contains a set of items, return those that are in R, if any.
  3. Otherwise, T consists of four quadtrees. Recursively look for points in R in each one of them.

Insertion into PR Quadtrees

Various cases for inserting a new point N, assuming maximum occupancy of a region is 2, showing initial state $\Longrightarrow$ final state.