CS61B Lecture #23

Today: Backtracking searches, game trees (*DSIJ*, Section 6.5)

Searching by "Generate and Test"

- We've been considering the problem of searching a set of data stored in some kind of data structure: "Is $x \in S$?"
- But suppose we *don't* have a set S, but know how to recognize what we're after if we find it: "Is there an x such that $P(x)$?"
- If we know how to enumerate all possible candidates, can use approach of *Generate and Test:* test all possibilities in turn.
- Can sometimes be more clever: avoid trying things that won't work, for example.
- What happens if the set of possible candidates is infinite?

Backtracking Search

- Backtracking search is one way to enumerate all possibilities.
- Example: *Knight's Tour.* Find all paths a knight can travel on a chessboard such that it touches every square exactly once and ends up one knight move from where it started.
- In the example below, the numbers indicate position numbers (knight starts at 0).
- Here, knight (N) is stuck; how to handle this?

6							
		5					
4	7						
	10		2				
8	3	0					
N		9		1			

General Recursive Algorithm

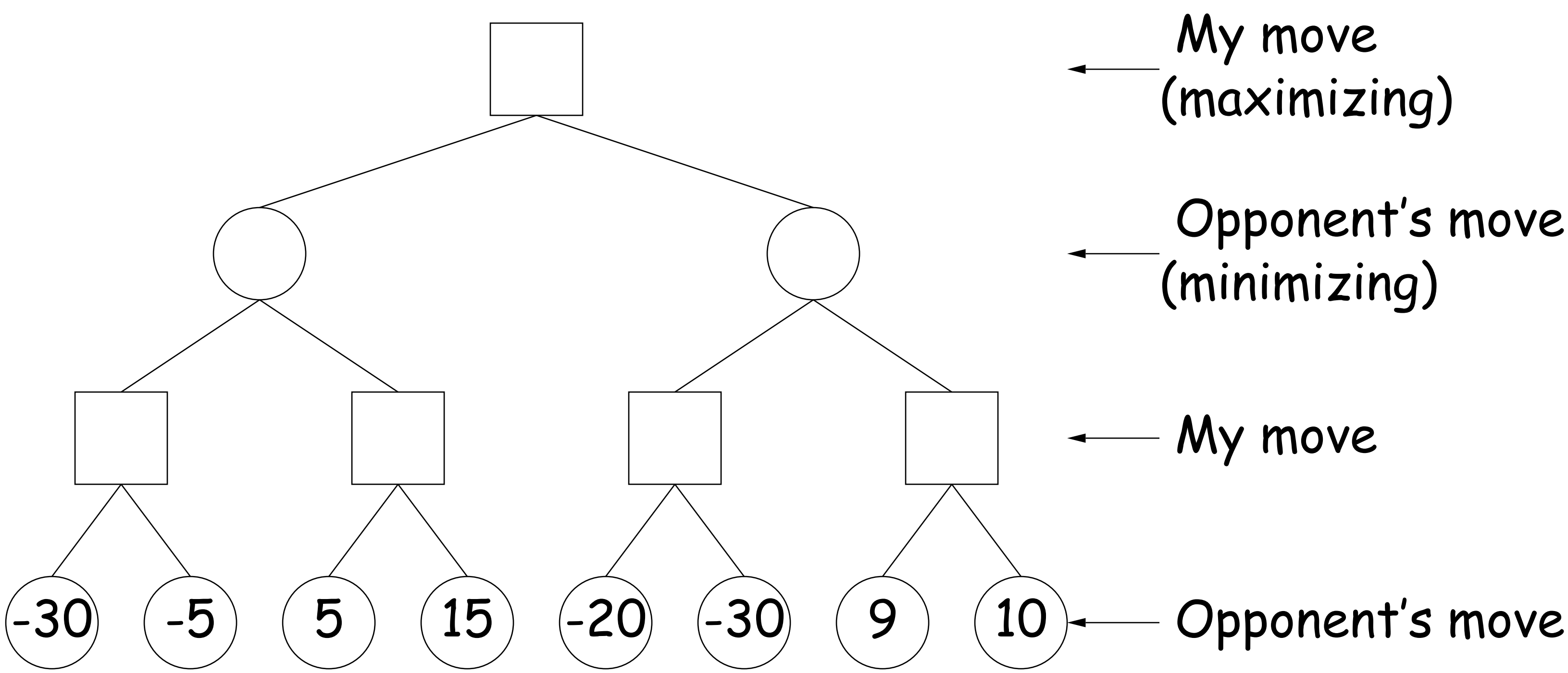
```
/** Append to PATH a sequence of knight moves starting at ROW, COL
 *  that avoids all squares that have been hit already and
 *  that ends up one square away from ENDROW, ENDCOL. B[i][j] is
 *  true iff row i and column j have been hit on PATH so far.
 *  Returns true if it succeeds, else false (with no change to PATH).
 *  Call initially with PATH containing the starting square, and
 *  the starting square (only) marked in B. */

boolean findPath(boolean[][] b, int row, int col,
                 int endRow, int endCol, List path) {
  if (path.size() == 64)   return isKnightMove(row, col, endRow, endCol);
  for (r, c = all possible moves from (row, col)) {
    if (!b[r][c]) {
      b[r][c] = true; // Mark the square
      path.add(new Move(r, c));
      if (findPath(b, r, c, endRow, endCol, path))  return true;
      b[r][c] = false; // Backtrack out of the move.
      path.remove(path.size()-1);
    }
  }
  return false;
}
```

Another Kind of Search: Best Move

- Consider the problem of finding the *best* move in a two-person game.
- One way: assign a *heuristic value* to each possible move and pick highest (aka *static valuation*).
- Otherwise, we can use a variety of heuristics. Some examples of static valuations:
  - assign a maximal or minimal value to a won position (depending on side.)
  - number of black pieces − number of white pieces in checkers.
  - (weighted sum of white piece values) − (weighted sum of black pieces in chess), such as queen=9, rook=5, knight=bishop=3, pawn=1.
  - Nearness of pieces to strategic areas (center of board).
- But this is misleading. A move might give us more pieces, but set up a devastating response from the opponent.
- So, for each move, look at *opponent's* possible moves, use the best move that results *for the opponent* as the value.
- But what if *you* have a great response to opponent's response?
- How do we organize this sensibly?

Game Trees

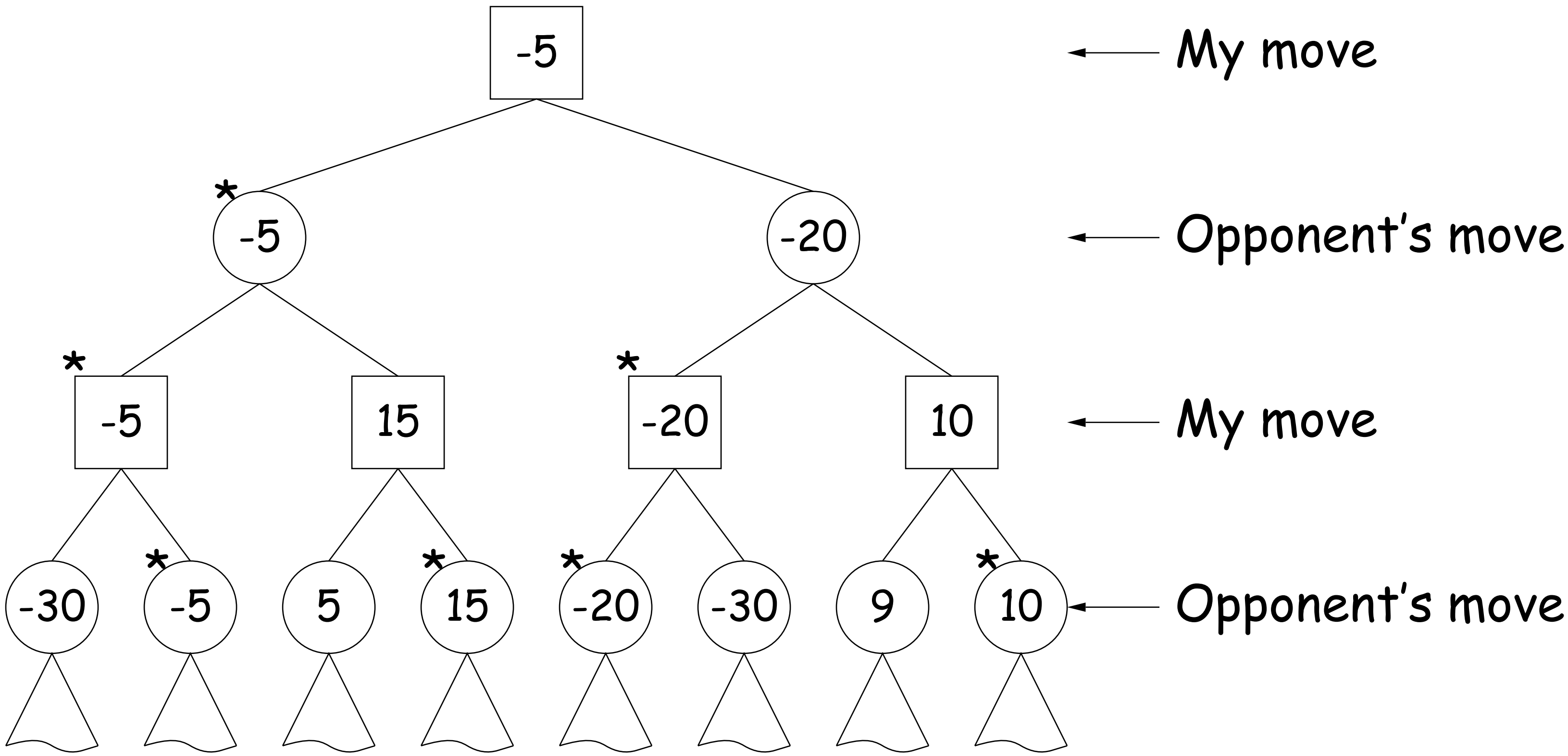
- Think of the space of possible continuations of the game as a tree.
- Each node is a position, each edge a move.



- Suppose numbers at the bottom are the values of those final positions *to me*. Smaller numbers are of more value to *my opponent*.
- What should I move? What value can I get if my opponent plays as well as possible?

Game Trees, Minimax

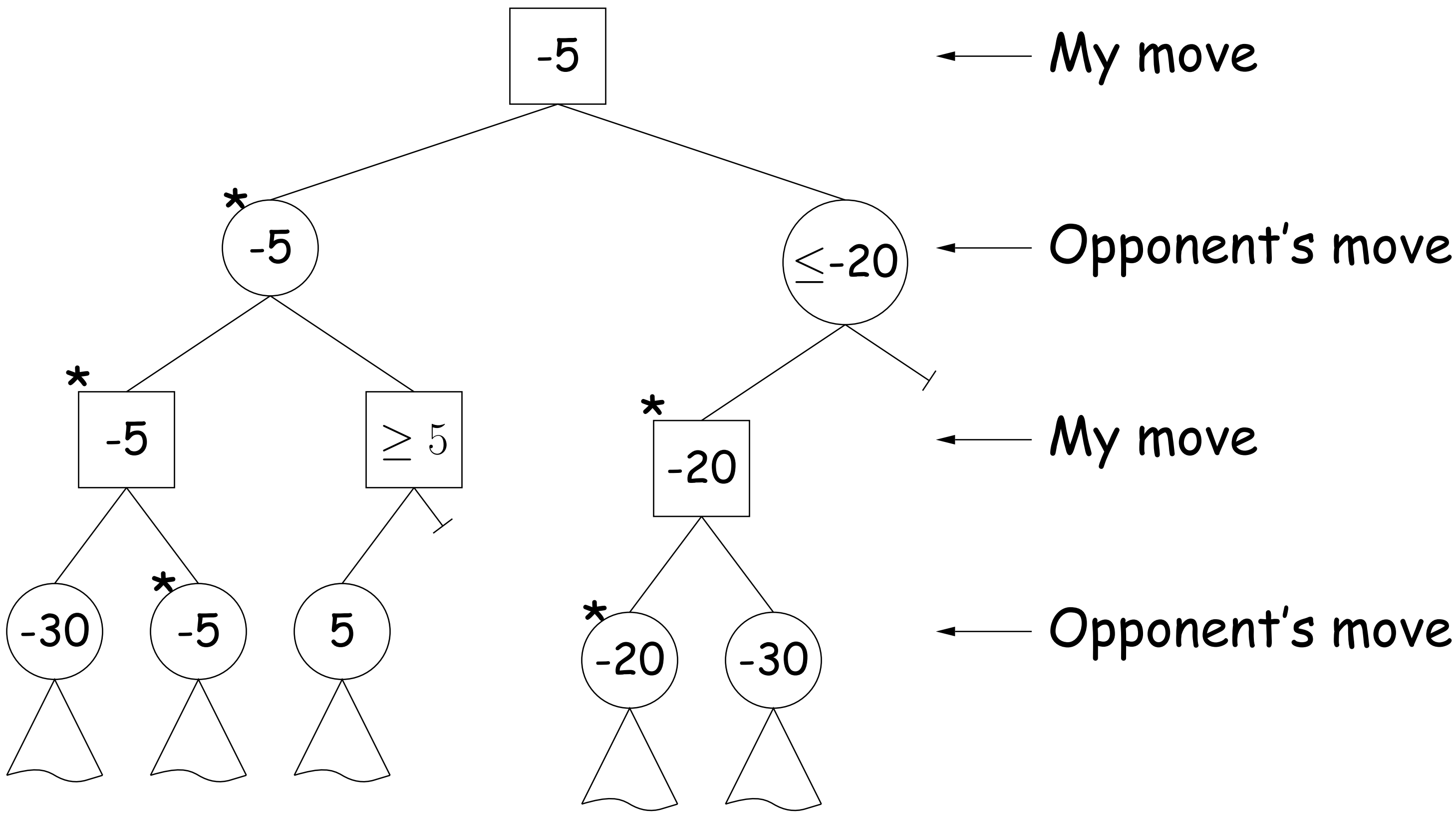
- Think of the space of possible continuations of the game as a tree.
- Each node is a position, each edge a move.



- Numbers are the values we guess for the positions (larger means better for me). Starred nodes would be chosen.
- I always choose child (next position) with maximum value; opponent chooses minimum value—the *minimax algorithm*.

Alpha-Beta Pruning

- We can *prune* this tree as we search it.



- At the '≥ 5' position, I know that the opponent will not choose to move here (already has a -5 move).
- At the '≤ -20' position, my opponent knows that I will never choose to move here (since I already have a -5 move).

Cutting off the Search

- If you could traverse game tree to the bottom, you'd be able to force a win (if it's possible).
- Sometimes possible near the end of a game.
- Unfortunately, game trees tend to be either infinite or impossibly large.
- So, we choose a maximum *depth,* and use a heuristic static valuation as the value at that depth.
- Or we might use *iterative deepening*, repeating the search at increasing depths until time is up.
- Much more sophisticated searches are possible, however (take CS188).

Overall Search Algorithm

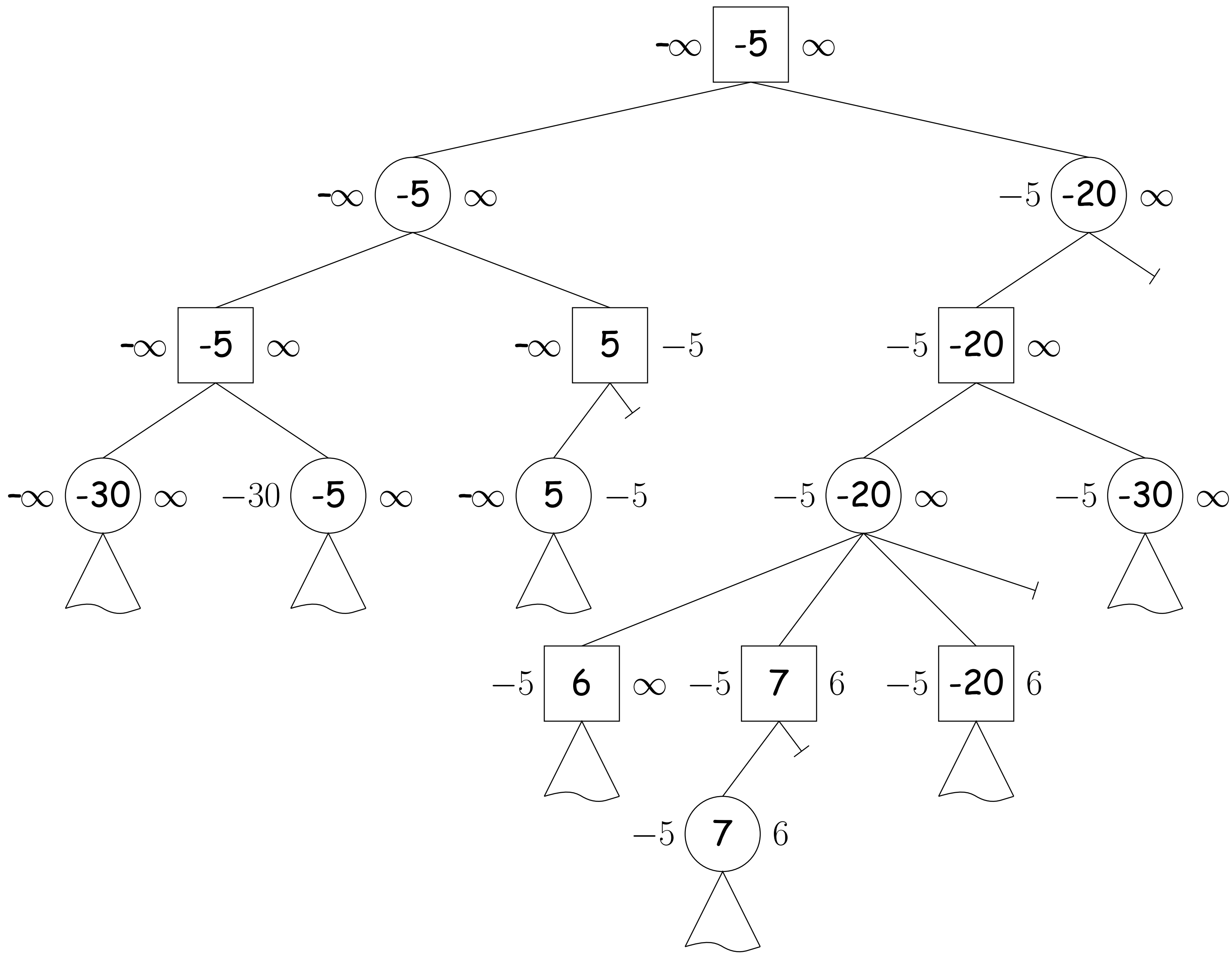
- Depending on whose move it is (maximizing player or minimizing player), we'll search for a move estimated to be optimal in one direction or the other.
- Search will be exhaustive down to a particular depth in the game tree; below that, we guess values.
- Also pass α and β limits:
  - High player does not care about exploring a position further after finding that its value will be larger than a position the minimizing player has already found, because the minimizing player will simply not choose a position with that larger value.
  - Likewise, minimizing player won't explore a positions whose value is less than what the maximizing player can get (α).
- To start, a maximizing player will find a move with the call

  `maxPlayerValue(`*current position*, *search depth*, $-\infty$, $+\infty$`)`

- minimizing player:

  `minPlayerValue(`*current position*, *search depth*, $-\infty$, $+\infty$`)`

Sample Tree with Alpha and Beta Values



Some Pseudocode for Searching (Maximizing Player)

```
/** The estimated minimax value of position POSN, searching up to
 *  DEPTH moves ahead, assuming it is the maximizing player's move.
 *  If the value is determined to be <=ALPHA, then the function
 *  may return any value <=ALPHA, even if inaccurate.  Likewise if the
 *  value is >=BETA, it may return any value >=BETA.  Assumes ALPHA<BETA. */
int maxPlayerValue(Position posn, int depth, int alpha, int beta)
{
    if (posn is a final position of the game || depth == 0)
        return staticGuess(posn);
    int bestSoFar = −∞;
    for (each legal move, M, in position posn) {
        Position next = makeMove(posn, M);
        int response = minPlayerValue(next, depth-1, alpha, beta);
        if (response > bestSoFar) {
            bestSoFar = response;
            alpha = max(alpha, bestSoFar);
            if (alpha >= beta)
                return bestSoFar;
        }
    }
    return bestSoFar;
}
```

Some Pseudocode for Searching (Minimizing Player)

```
/** The estimated minimax value of position POSN, searching up to
 *  DEPTH moves ahead, assuming it is the minimizing player's move. */
int minPlayerValue(Position posn, int depth, int alpha, int beta)
{
    if (posn is a final position of the game || depth == 0)
        return staticGuess(posn);
    int bestSoFar = +∞;
    for (each legal move, M, in position posn) {
        Position next = makeMove(posn, M);
        int response = maxPlayerValue(next, depth-1, alpha, beta);
        if (response < bestSoFar) {
             bestSoFar = response;
             beta = min(beta, bestSoFar);
             if (alpha >= beta)
                 return bestSoFar;
        }
    }
    return bestSoFar;
}
```