# CS61B Lecture #31

**Today:**

- More balanced search structures (*DS(IJ)*, Chapter 9

# Really Efficient Use of Keys: the Trie

- Haven't said much about cost of comparisons.
- For strings, worst case is length of string.
- Therefore should throw extra factor of key length, $L$, into costs:
  - $\Theta(M)$ comparisons really means $\Theta(ML)$ operations.
  - So to look for key $X$, keep looking at same chars of $X$ $M$ times.
- Can we do better? Can we get search cost to be $O(L)$?

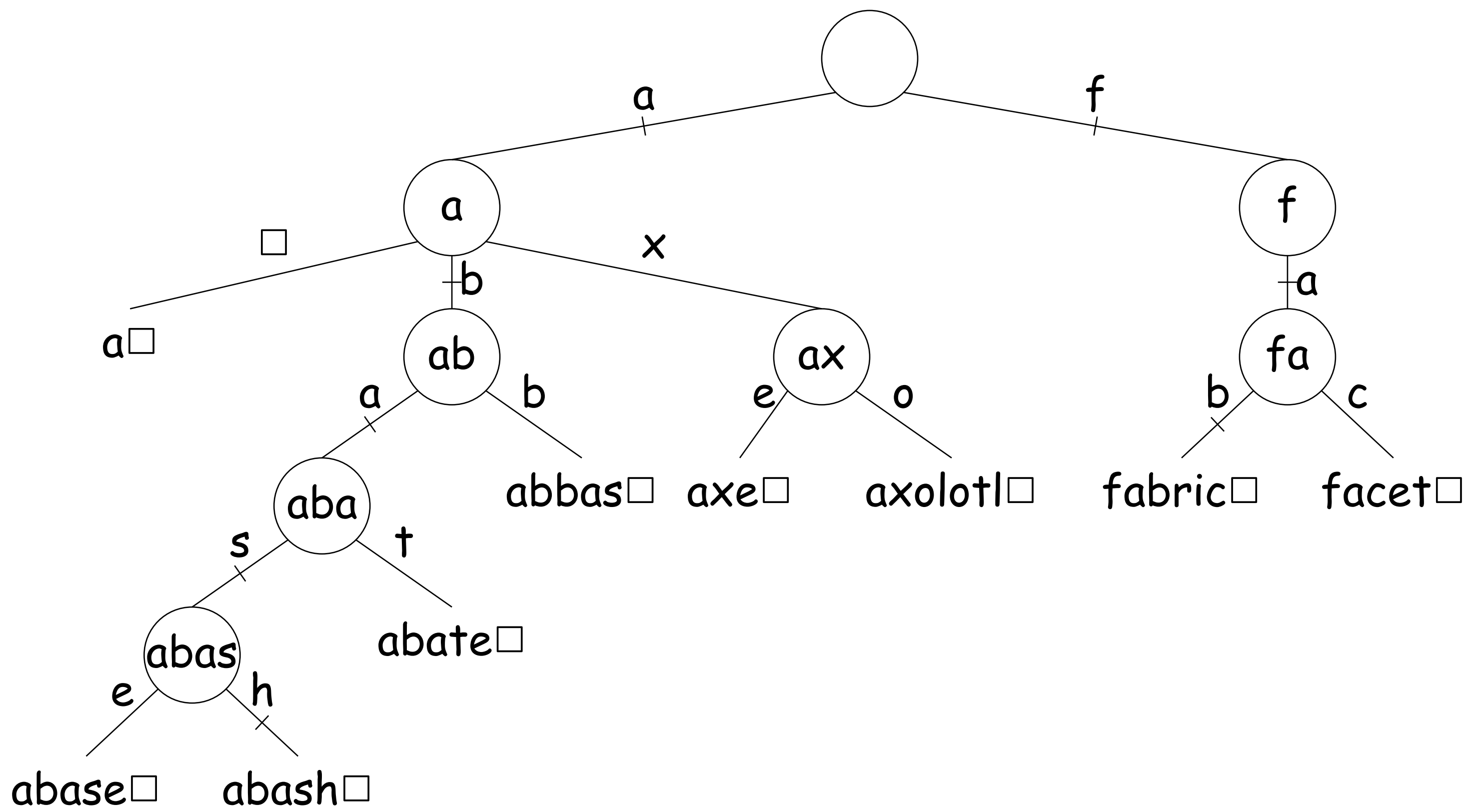
**Idea:** Make a *multi-way decision tree,* with one decision per character of key.

# The Trie: Example

- Set of keys

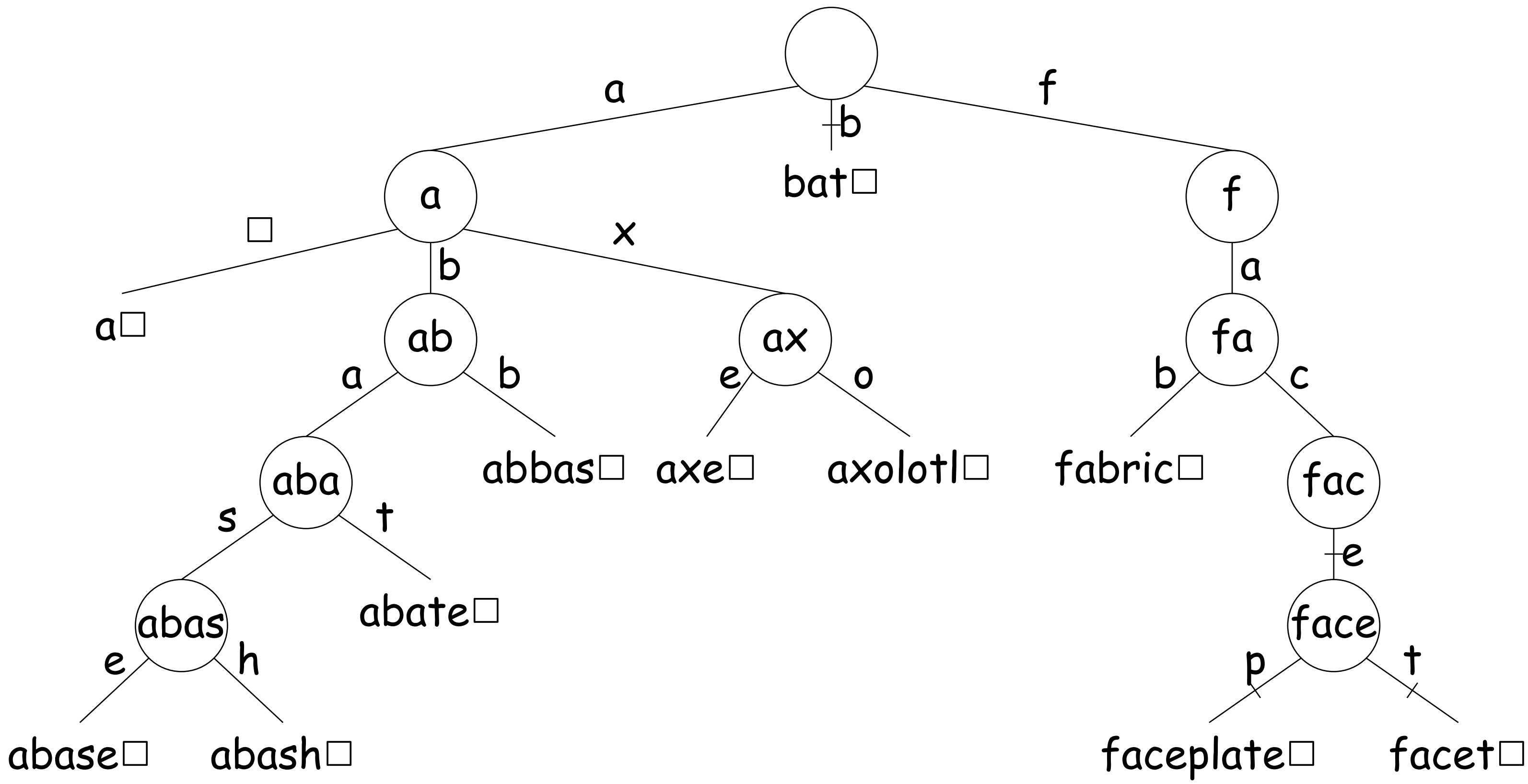
  {a, abase, abash, abate, abbas, axolotl, axe, fabric, facet}

- Ticked lines show paths followed for "abash" and "fabric"
- Each internal node corresponds to a possible prefix.
- Characters in path to node = that prefix.



# Adding Item to a Trie

- Result of adding `bat` and `faceplate`.
- New edges ticked.



# A Side-Trip: Scrunching

- For speed, obvious implementation for internal nodes is array indexed by character.
- Gives $O(L)$ performance, $L$ length of search key.
- [Looks as if independent of $N$, number of keys. Is there a dependence?]
- **Problem:** arrays are *sparsely populated* by non-null values—waste of space.

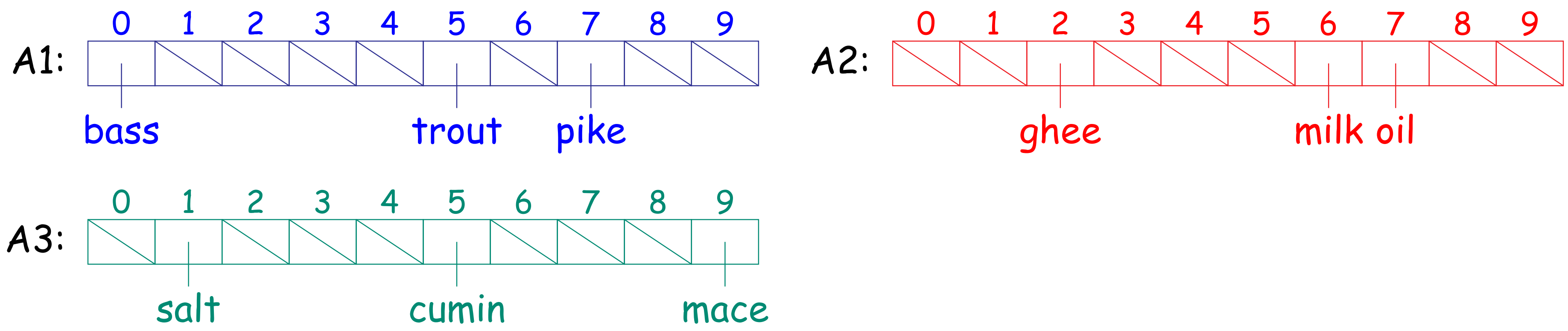
**Idea:** Put the arrays on top of each other!

- Use null (0, empty) entries of one array to hold non-null elements of another.
- Use extra markers to tell which entries belong to which array.

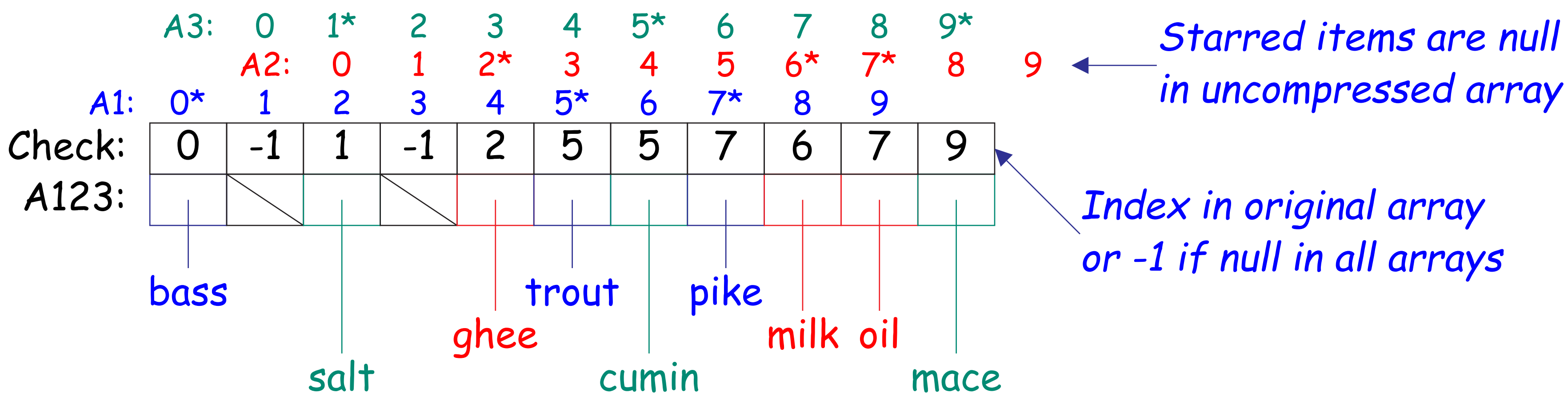
# Scrunching Example

**Small example:** (unrelated to Tries on preceding slides)

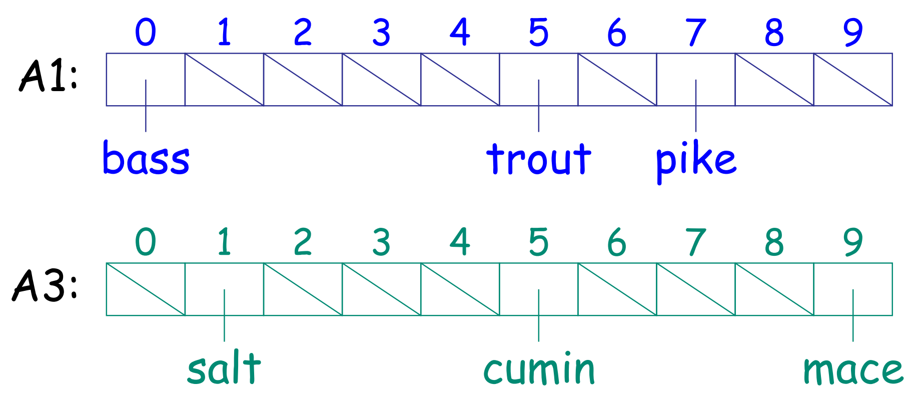
- Three arrays, each indexed 0..9

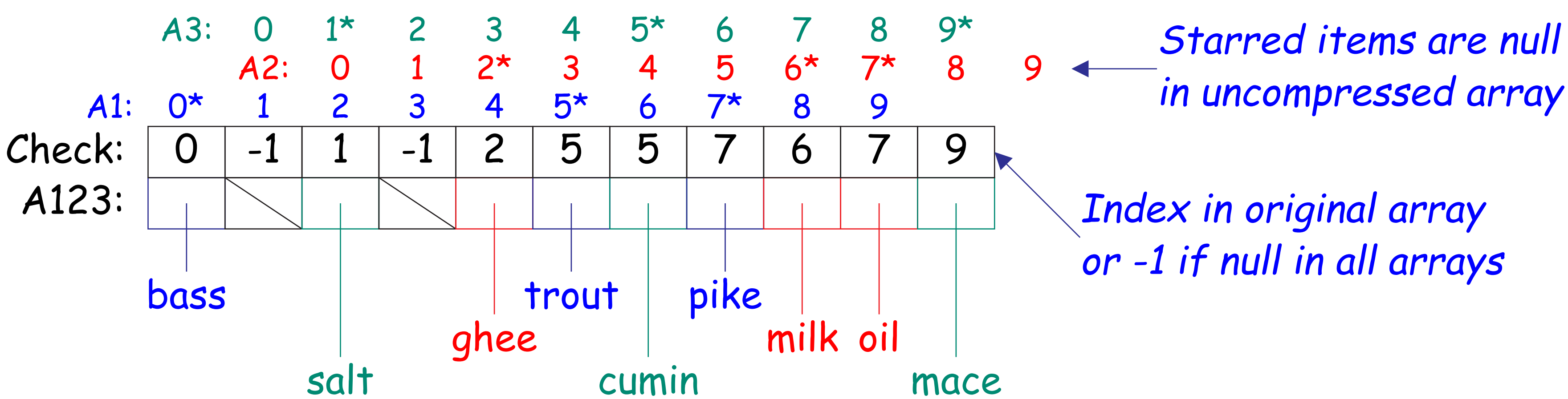



- Now overlay them, but keep track of the original index of each item:



# Scrunching Example (contd.)



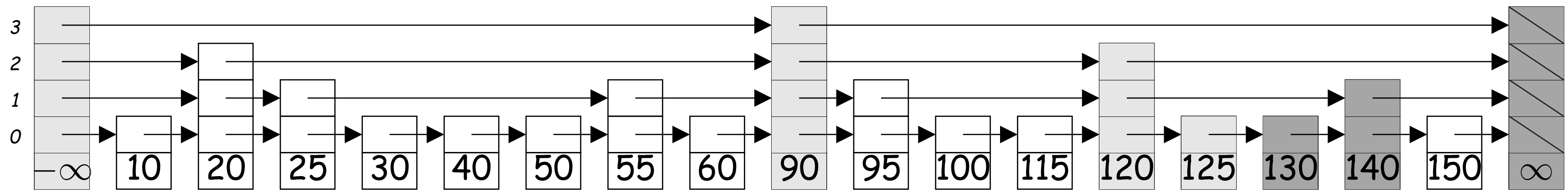

```
/* A1[i] == */    (Check[i] == i)      ? A123[i]      : null;
/* A2[i] == */    (Check[i + 2] == i) ? A123[i + 2] : null;
/* A3[i] == */    (Check[i + 1] == i) ? A123[i + 1] : null;
```

# Practicum

- The scrunching idea is cute, but
  - Not so good if we want to expand our trie.
  - A bit complicated.
  - Actually more useful for representing large, sparse, fixed tables with many rows and columns.
- Furthermore, number of children in trie tends to drop drastically when one gets a few levels down from the root.
- So in practice, might as well use linked lists to represent set of node's children...
- ...but use arrays for the first few levels, which are likely to have more children.

# Probabilistic Balancing: Skip Lists

- A *skip list* can be thought of as a kind of n-ary search tree in which we choose to put the keys at "random" heights.
- More often thought of as an ordered list in which one can skip large segments.
- Typical example:




- To search, start at top layer on left, search until next step would overshoot, then go down one layer and repeat.
- In list above, we search for 125 and 127. Gray nodes are looked at; darker gray nodes are overshoots.
- Heights of the nodes were chosen randomly so that there are about 1/2 as many nodes that are $> k$ high as there are that are $k$ high.
- Makes searches fast *with high probability.*

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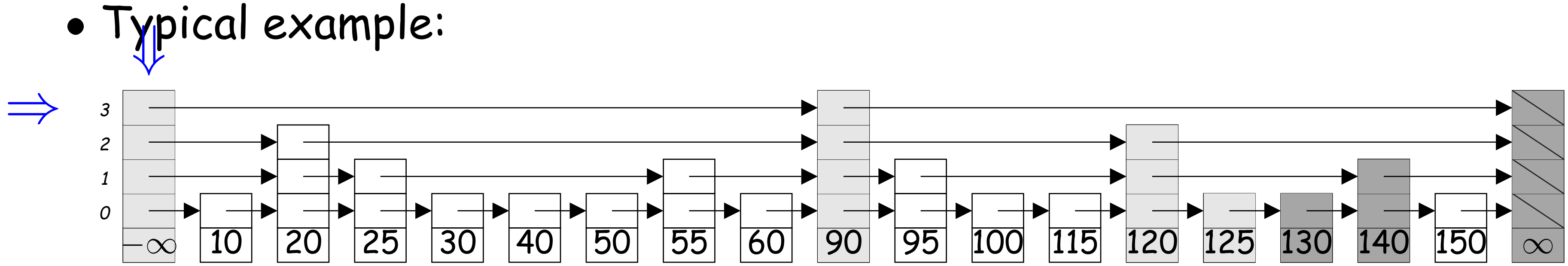


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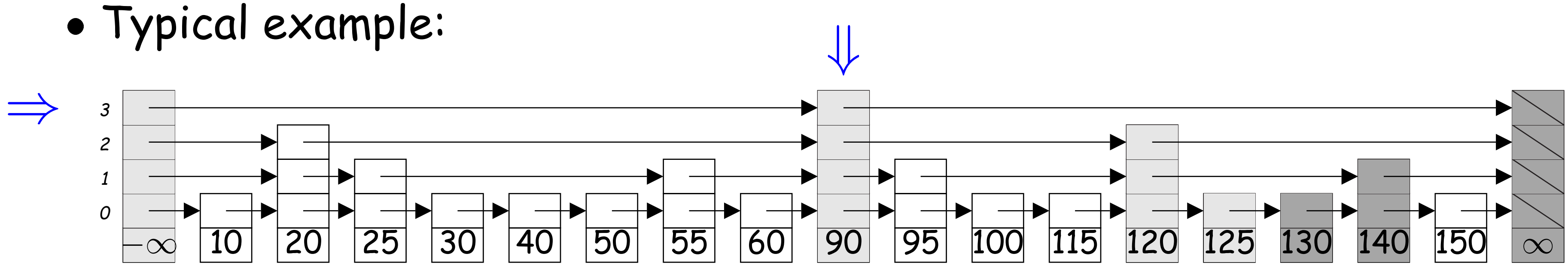


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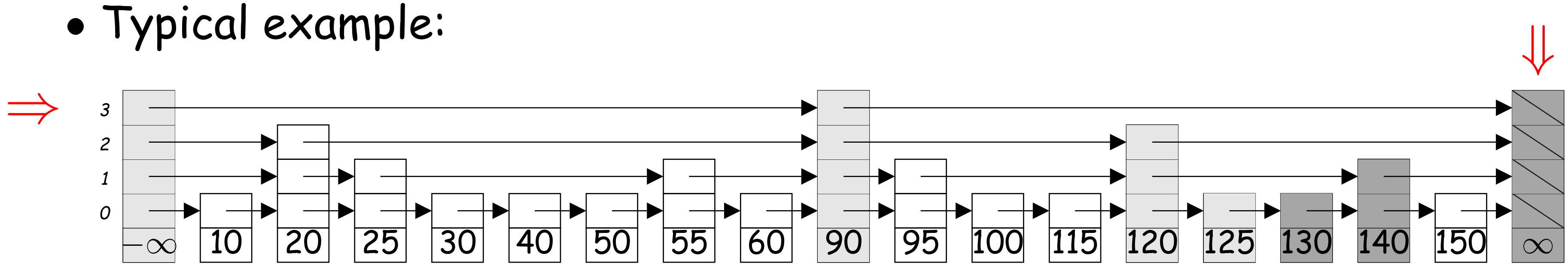


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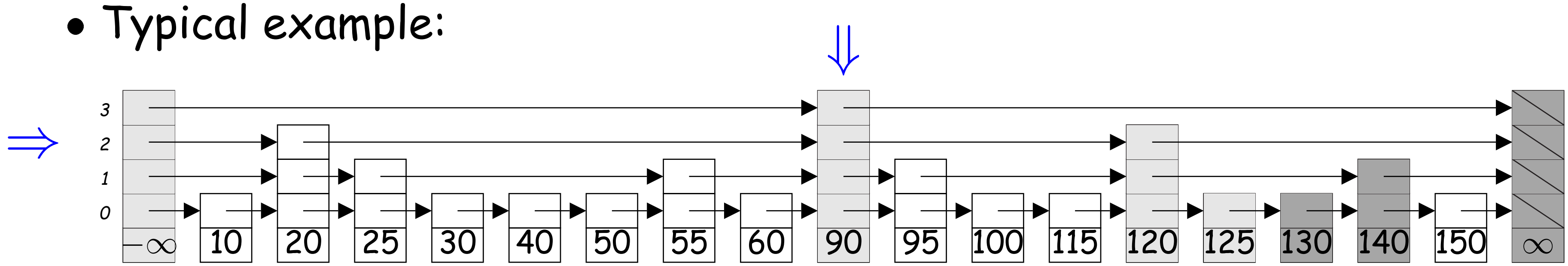


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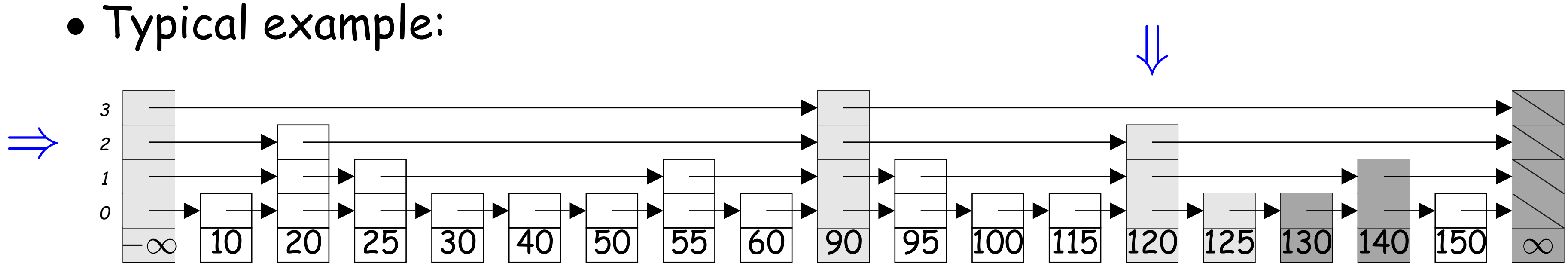


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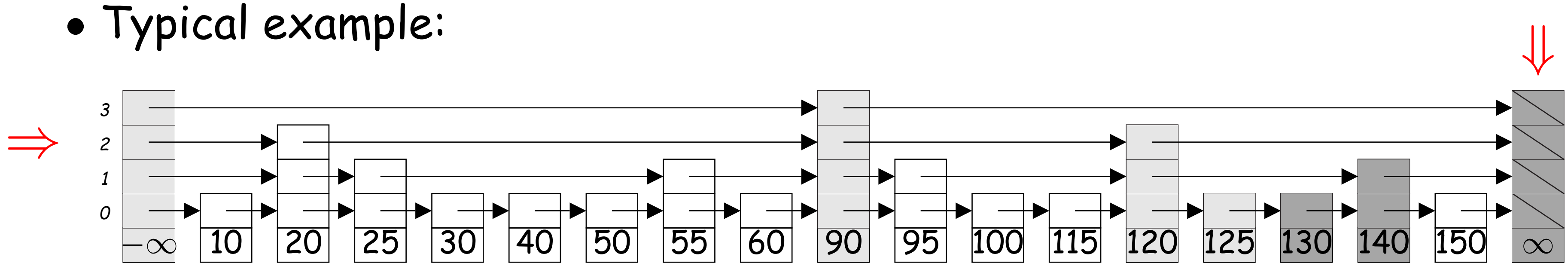


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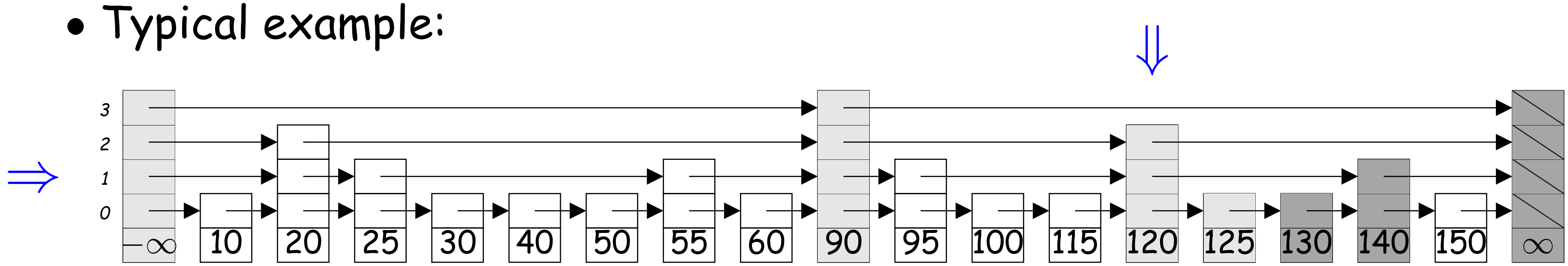


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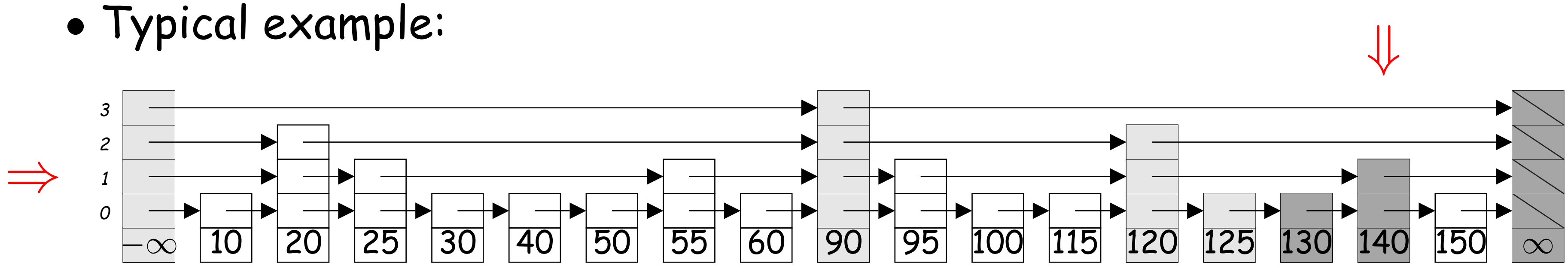


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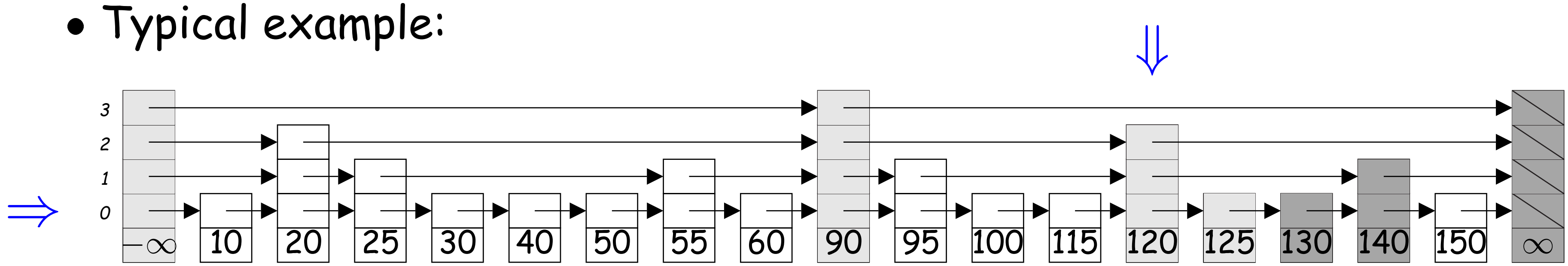


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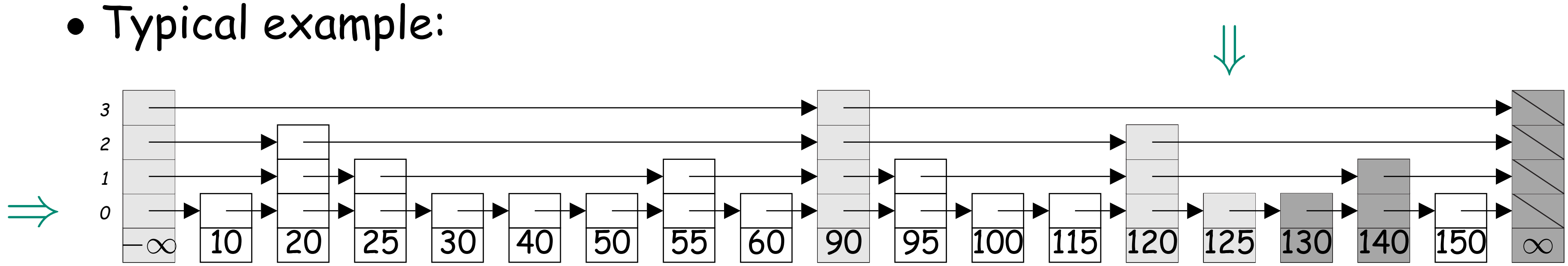


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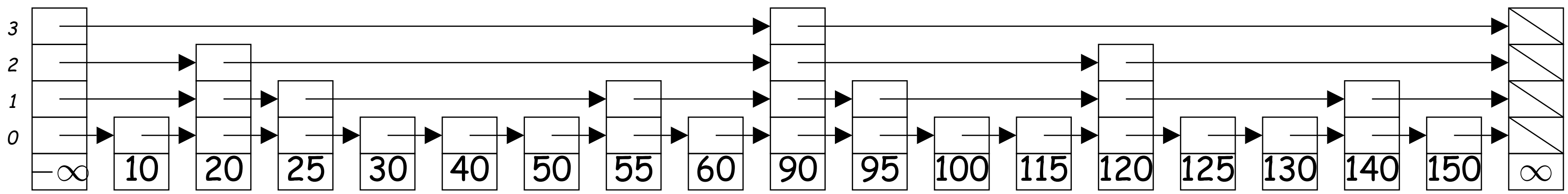
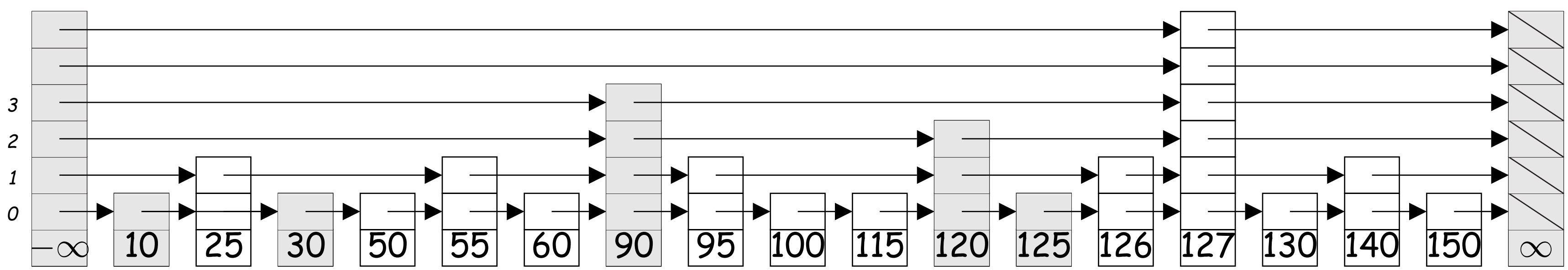
# Example: Adding and deleting

- Starting from initial list:




- In any order, we add 126 and 127 (choosing random heights for them), and remove 20 and 40:

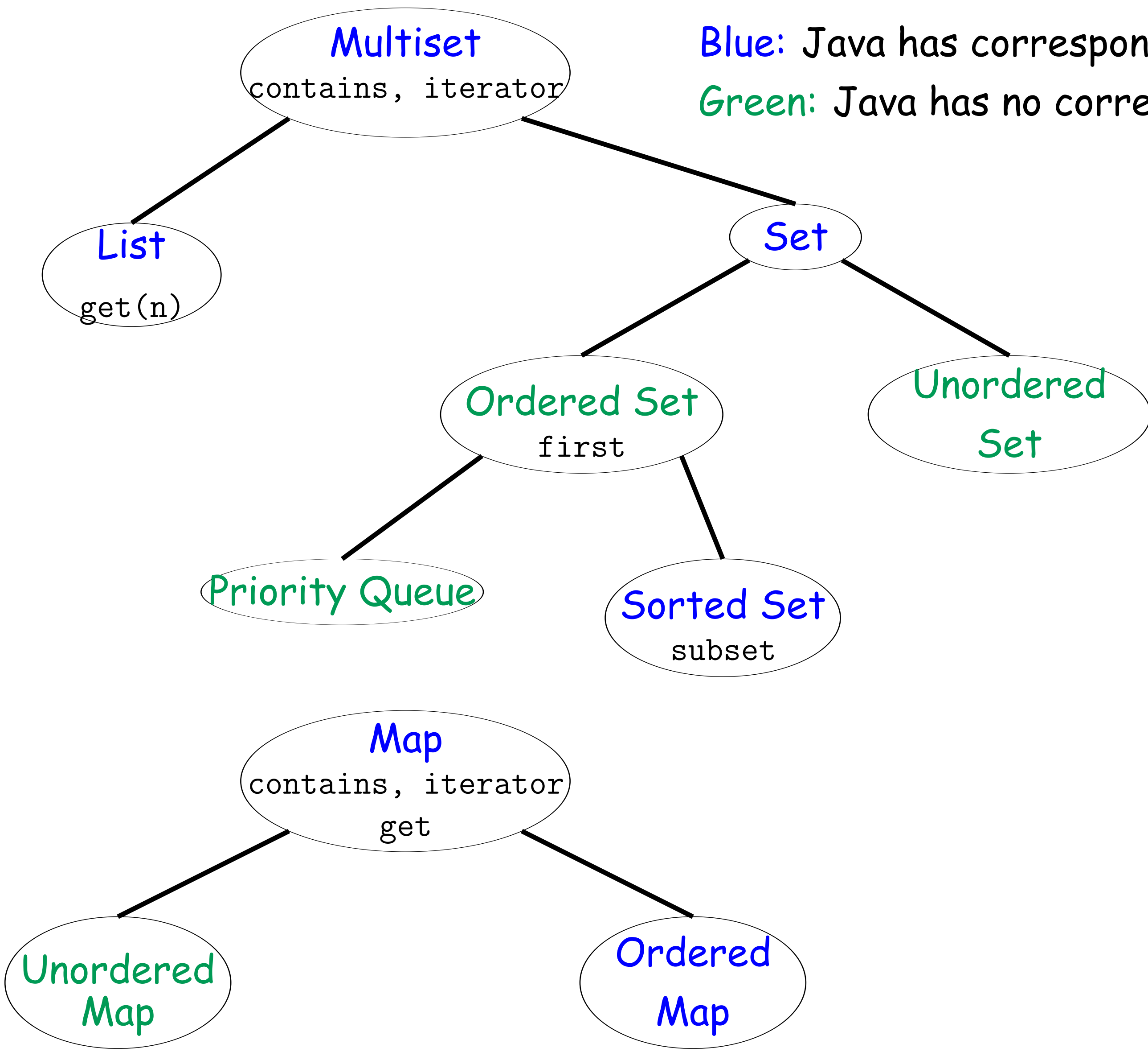



- Shaded nodes here have been modified.

## Summary

- Balance in search trees allows us to realize $\Theta(\lg N)$ performance.
- B-trees, red-black trees:
  - Give $\Theta(\lg N)$ performance for searches, insertions, deletions.
  - B-trees good for external storage. Large nodes minimize # of I/O operations
- Tries:
  - Give $\Theta(B)$ performance for searches, insertions, and deletions, where $B$ is length of key being processed.
  - But hard to manage space efficiently.
- *Interesting idea:* scrunched arrays share space.
- Skip lists:
  - Give probable $\Theta(\lg N)$ performace for searches, insertions, deletions
  - Easy to implement.
  - Presented for *interesting ideas:* probabilistic balance, randomized data structures.

# Summary of Collection Abstractions




Blue: Java has corresponding interface

Green: Java has no corresponding interface

# Data Structures that Implement Abstractions

## Multiset

- List: arrays, linked lists, circular buffers
- Set
  - OrderedSet
    - Priority Queue: heaps
    - Sorted Set: binary search trees, red-black trees, B-trees, sorted arrays or linked lists
  - Unordered Set: hash table

## Map

- Unordered Map: hash table
- Ordered Map: red-black trees, B-trees, sorted arrays or linked lists

# Corresponding Classes in Java

**Multiset** (Collection)

- List: ArrayList, LinkedList, Stack, ArrayBlockingQueue, ArrayDeque
- Set
  - OrderedSet
    - Priority Queue: PriorityQueue
    - Sorted Set (SortedSet): TreeSet
  - Unordered Set: HashSet

**Map**

- Unordered Map: HashMap
- Ordered Map (SortedMap): TreeMap