CS70 Fall 2013
Discrete Math and Probability Theory

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Lecture 1: Introduction, Induction

Goals of cs70

1. Mathematical crystals: highly symmetric mathematical objects

Reliable storage and communication
Security and cryptography
Distributed computation
…

Goals of cs70

2. Probability theory – one of the great achievements of 20th century

   Bayesian inference
   Statistical machine learning
   Hashing and load balancing
   Probabilistic algorithms
   …

Goals of cs70

3. Infinity and uncountability

   Diagonalization
   Liar paradox
   …

Goals of cs70

4. Proofs:

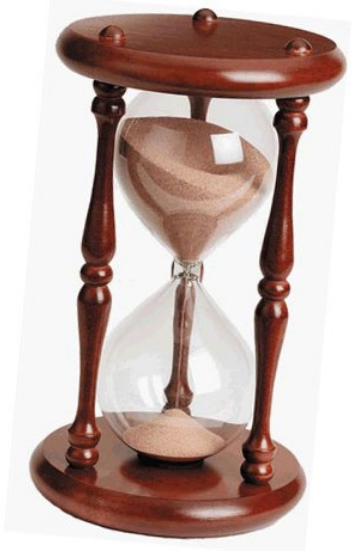
   Proof by induction – basic technique for analyzing algorithms
   Induction and recursion
   Idea of proof vs. sketch of proof vs. proof

   …

Goals of cs70

5. Problem solving skills:

- Google problem: Using only a four-minute hourglass and a seven-minute hourglass, measure exactly nine minutes – without the process taking longer than nine minutes.



About cs70

- Homework 30%, Midterms 20% each, Final 30%
- Tentative midterm dates: Midterm 1: Oct 1, Midterm 2: Nov 5
- Homework: online 15%, written 15%
- Written homework out Monday, due 5 pm following Monday
- Scan and submit as pdf
- Homework 0 out today, due Tuesday September 3 at 10:30 am.
- Homework party: Friday 1:30 – 4:30 pm Wozniak lounge
- Lowest homework score dropped. Late homeworks not accepted.

About cs70

- Strict honor code:
  You are allowed to (encouraged to!) discuss and work on homework problems with your fellow students.
  You **must** write up your solutions by yourself. You should also cite any online or other sources you consulted.
  Failure to do this will be considered cheating.

- Course notes

- Course website: inst.eecs.berkeley.edu/~cs70

- Discussion forum: Piazza

- If you wish to consult a book, “Discrete Math” by Rosen.

Ask a grown-up: why don't I like maths?

Who better to answer eight-year-old Connie's question than the mathematics wiz from TV's Countdown?

Rachel Riley
The Guardian, Saturday 2 February 2013

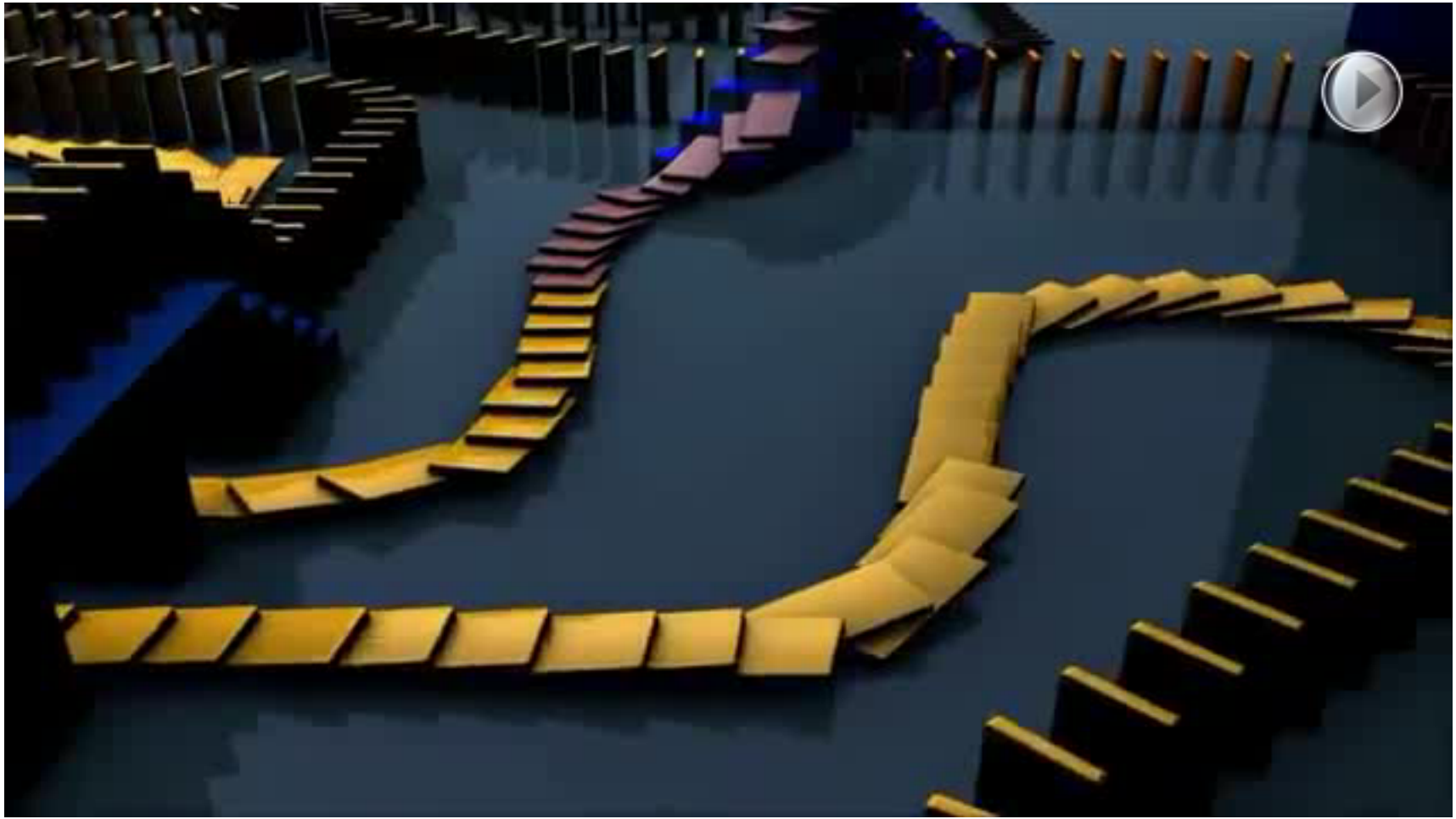
Rachel Riley: 'Hitting a brick wall in maths isn't fun, but look for a new way around a problem and keep trying.' Photograph: Eamonn McCabe/Jaime Turner/GNM Imaging

Countdown's Rachel Riley replies: If you don't enjoy maths, I imagine it's down to one of two reasons: what you're trying is too advanced or too easy. I had a Homer Simpson poster up at university that said, "If something's hard to do, it's not worth doing", which I found funny because my love of maths has always been about the mental challenge. It's supposed to test you and get your brain working.

Hitting a brick wall in maths isn't fun, but look for a new way around a problem and keep trying, and you'll get enormous satisfaction from understanding something that at first has you stumped. Once you've had that eureka moment, you can move on to the next challenge with confidence.

Learning maths is a bit like building a Jenga tower with unlimited pieces. If you try to put pieces on the top too quickly, it'll come crashing down. Mastering the basics is important, as with solid foundations your enjoyment (like a Jenga tower) will grow and grow, with only the sky as your limit.

Proof by Induction



Proof by Induction:

$$P(k) : 0 + 1 + \cdots + k = \frac{k(k+1)}{2}$$

$$\forall k \in \mathcal{N} \quad P(k)$$

$$P(k) : 0 + 1 + \cdots + k = \frac{k(k+1)}{2}$$

Assume $0 + \cdots + k = \frac{k(k+1)}{2}$

Step: $(0 + \cdots + k) + k + 1$

$= \frac{k(k+1)}{2} + k + 1$

$= (k+1)\left[\frac{k}{2} + 1\right] = \frac{(k+1)(k+2)}{2}$