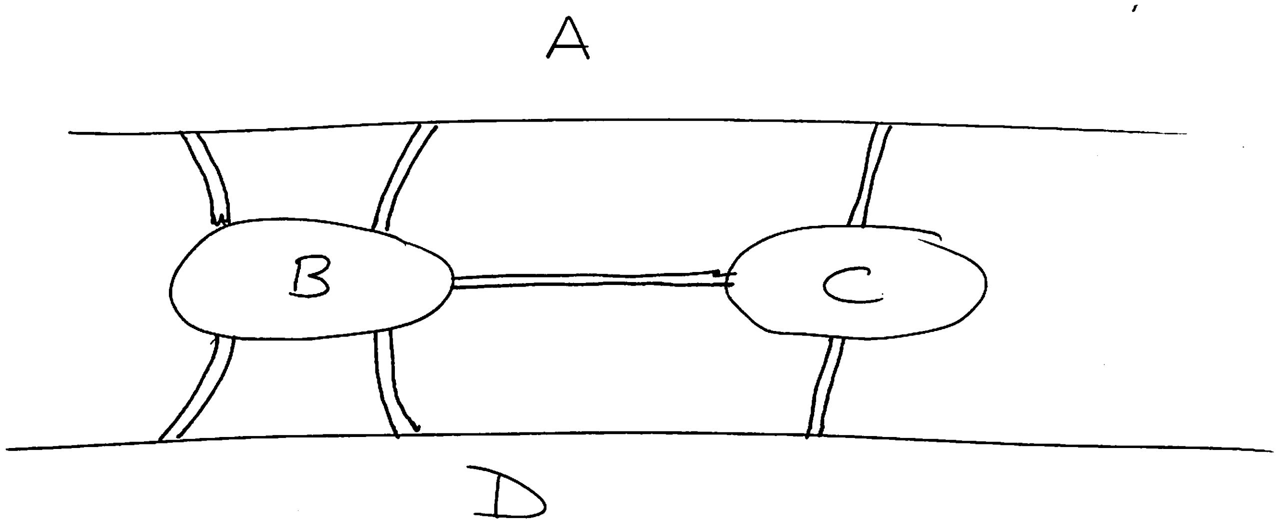
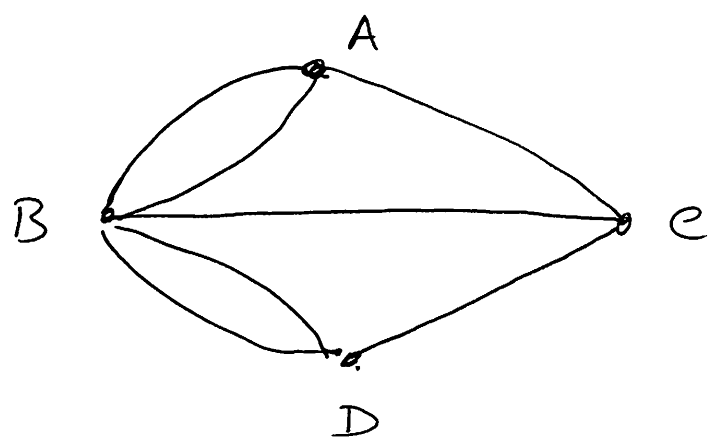
Graphs and Eulerian Tours.

Euler 1735

Königsberg.



Abstraction:



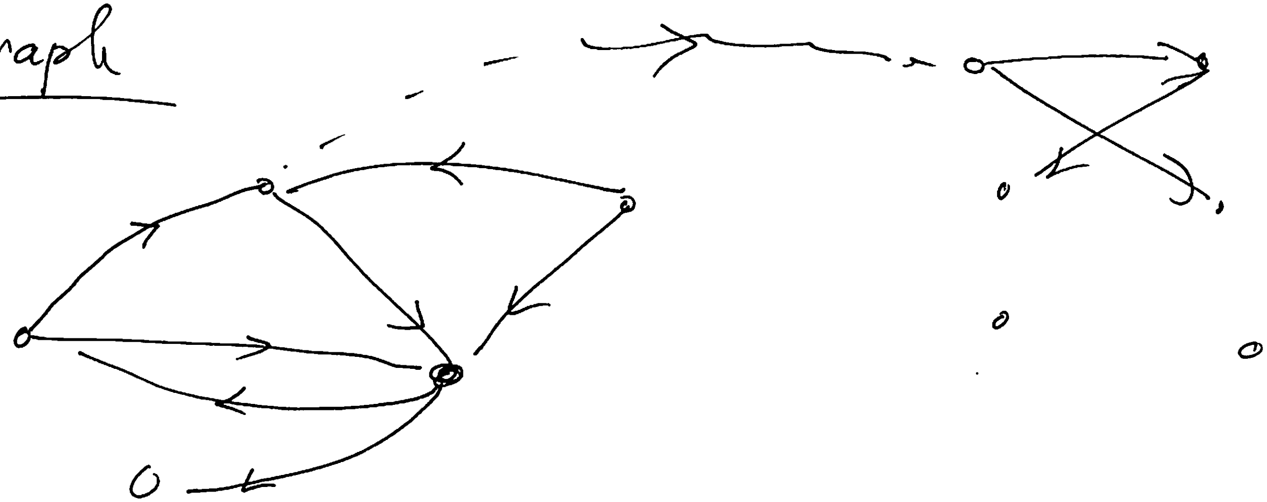
A, B, C, D vertices.

lines

{A, B} {B, D} edges.

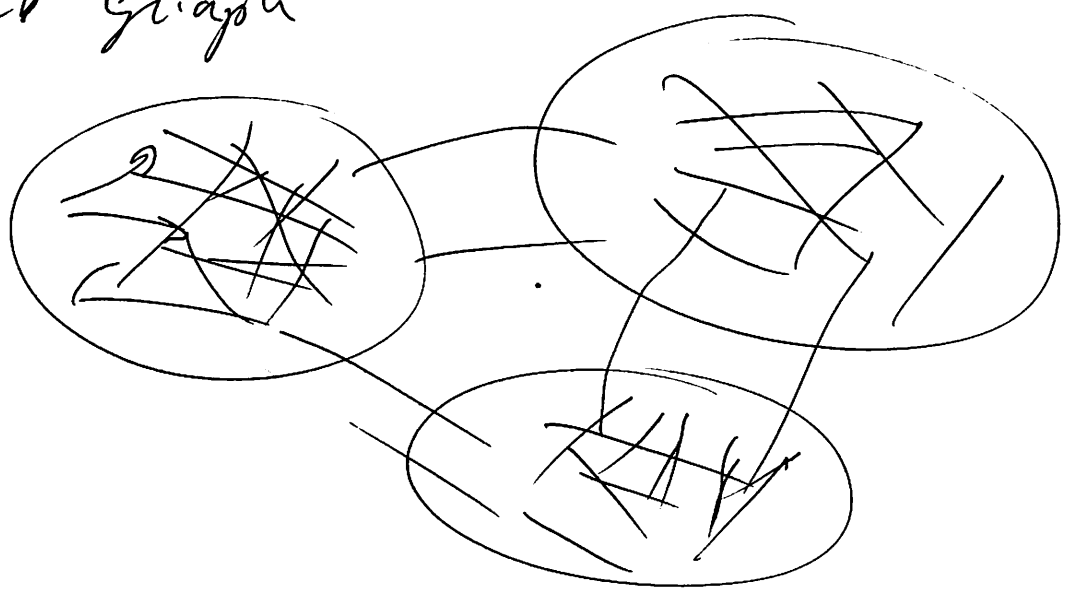
Google

Web graph



Page rank.

Web graph



EU Buys Large Graph for 1.2 Billion Euro.

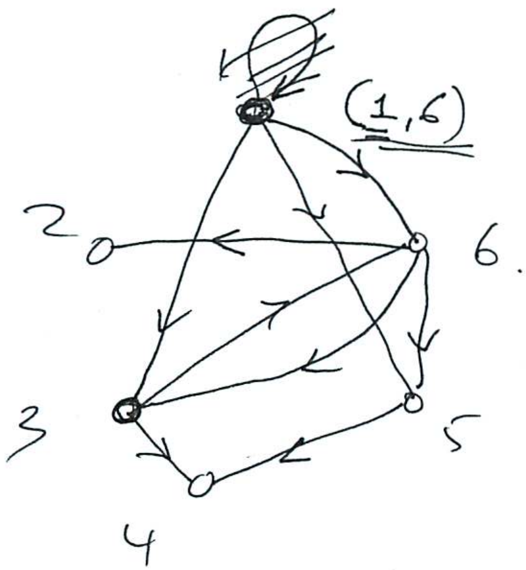
Connection pattern of the Brain:

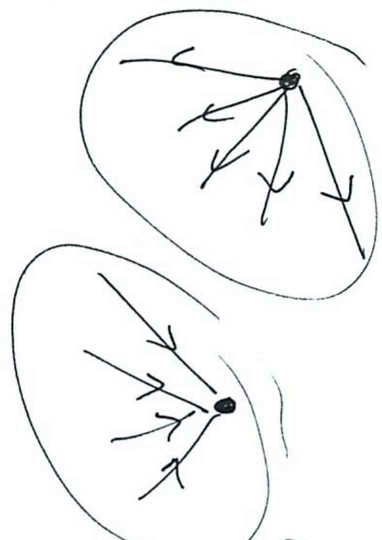
Evolution: Phylogeny trees.

$G(V, E)$ graph G with vertex set V and edge set E.

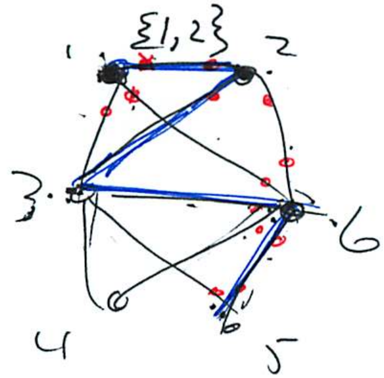
$V = \{1, 2, 3, 4, 5, 6\}$.

Directed





a Undirected.



$\deg(6) = 5$

$\deg(1) = 3$

$\deg(3) = 5$.

$\deg(4) = 2$

$E = \{\{1,2\}, \{1,3\}, \{1,6\}, \{2,3\}, \{2,6\}, \{3,6\}, \{3,4\}, \{3,5\}, \{4,6\}, \{5,6\}\}$.

$U \times W = \{(u, w) : u \in U \;\&\; w \in W\}$.

$U = \{1, 2, 3\}$ $\quad W = \{a, b\}$

$U \times W = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$.

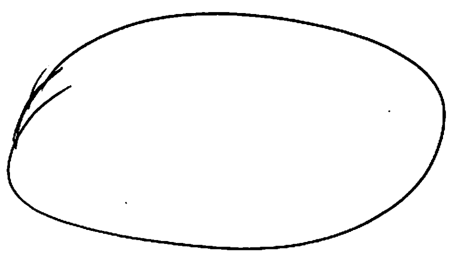
$E \subseteq V \times V$ $\quad \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$.

$\{\cancel{(1,1)}, (1,2), \cdots, (1,6)$
$(2,1), \cancel{(2,2)} \cdots\quad (2,6)\}$

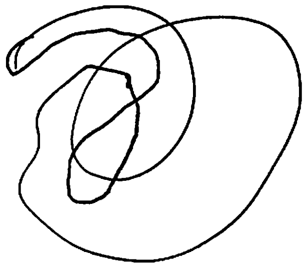
in-deg(6) = 2

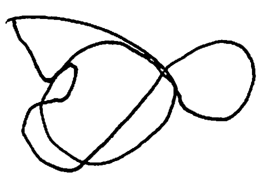
out-deg(6) = 3

Simple cycle



tour





$$\sum_{v \in V} \deg(v) = ? = 2|E|$$

Cor Number of odd degree vertices in any graph $G(V, E)$ must be even.

if vertex v has no neighbors i.e. $\deg(v) = 0$ the v is called an isolated vertex.

length = 4

Path : 1, 2, 3, 6, 5

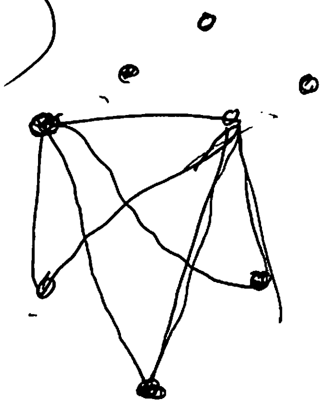
Simple path. no repititions.

walk : 1, 2, 3, 1, 6, 3, 5

(simple) cycle : 1, 2, 3, 6, 1

tour : 1, 2, 6, 1, 3, 5, 6, 1

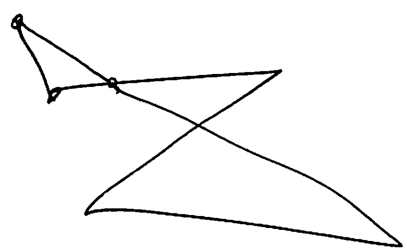
Theorem: A graph has an Eulerian tour iff ~~on~~ it is connected (up to isolated vertices) and every vertex has even degree.



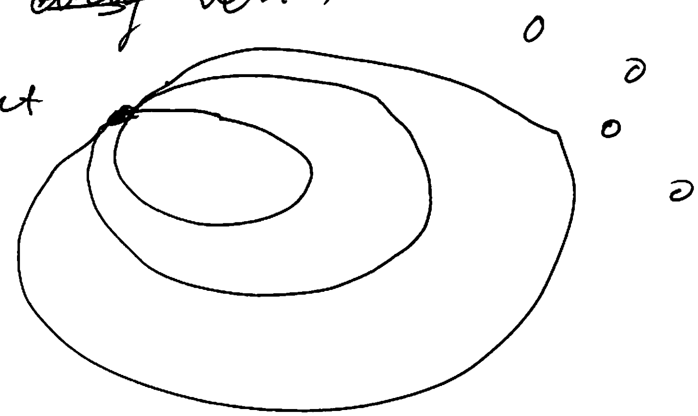
Eulerian Graph.

Idea: every time tour enters a vertex it must leave. Each such visit ~~at~~ uses up two edges out of its total ~~of~~ degree.

$\Rightarrow$



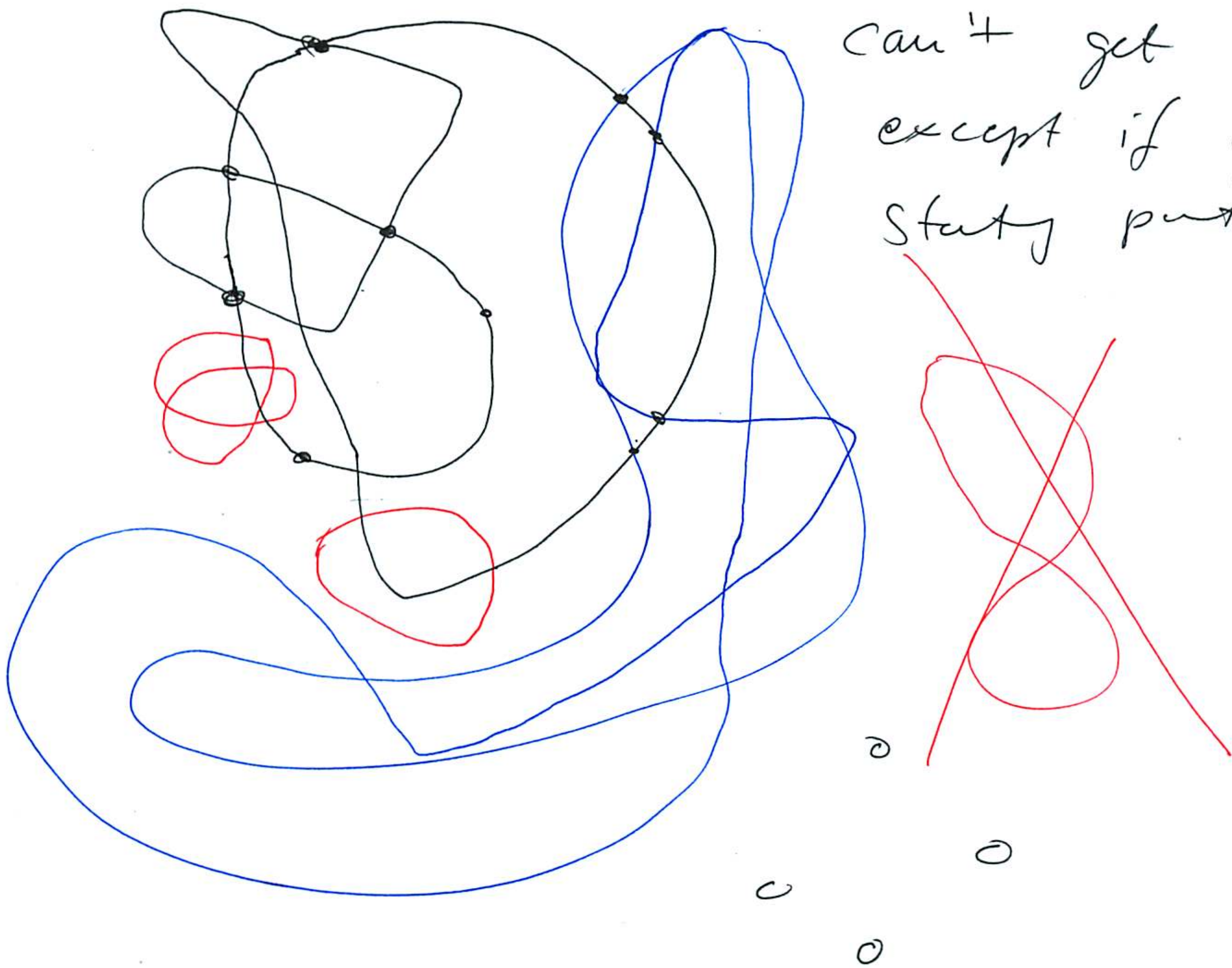
leave every time
enter ~~every~~ vertex } even degree.



$\Leftarrow$

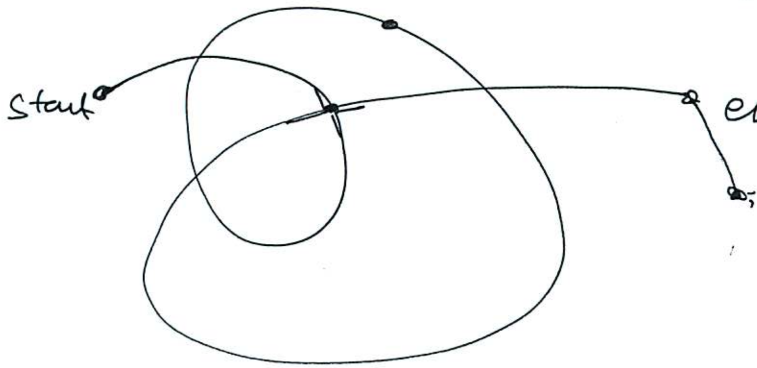
Given: connected graph (up to isolated vertices) with all vertices even degree.

Start walking from some vertex.

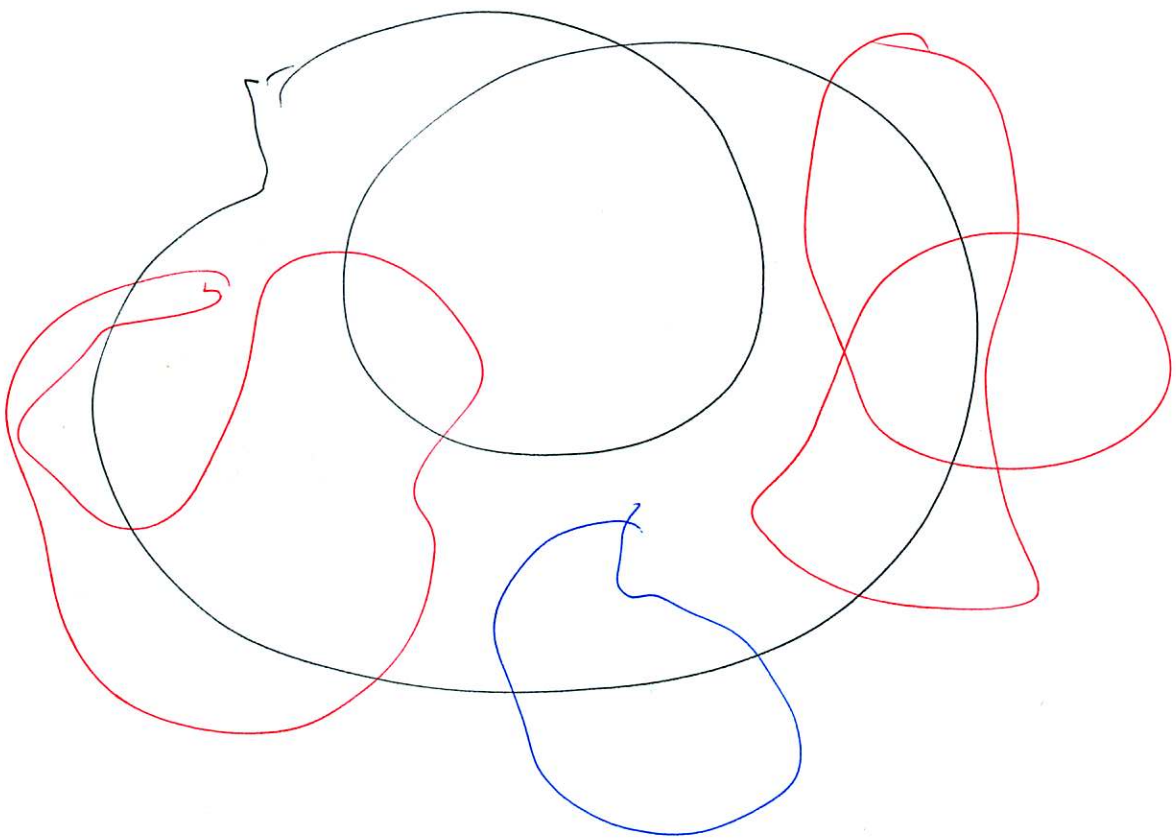


Can't get stuck anywhere except if you return to starting point. ▨

only stuck ~~at~~ when return to starting pt:



only two odd degree vertices in tour.



Already traversed black part.

All other edges are connected to black part of graph & have even degree.