

Probability

CS70 Fall 2013.
Lecture 13

Coin: Flip it 10,000 times.

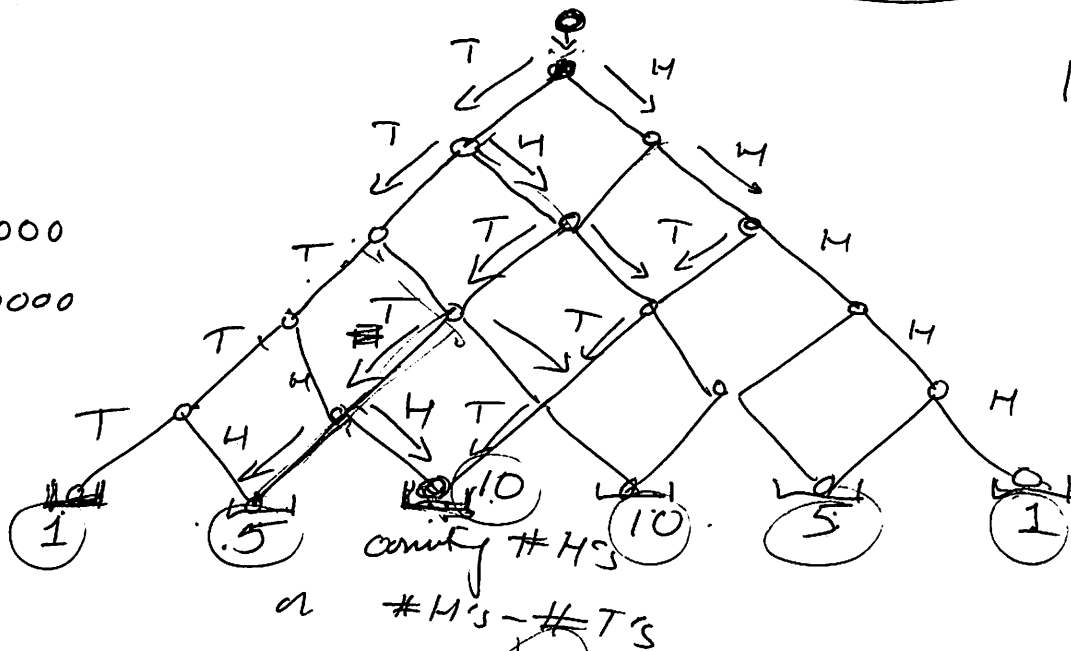
How many H's? 5000

Most of time 4900 - 5100

Galton - Watson Board

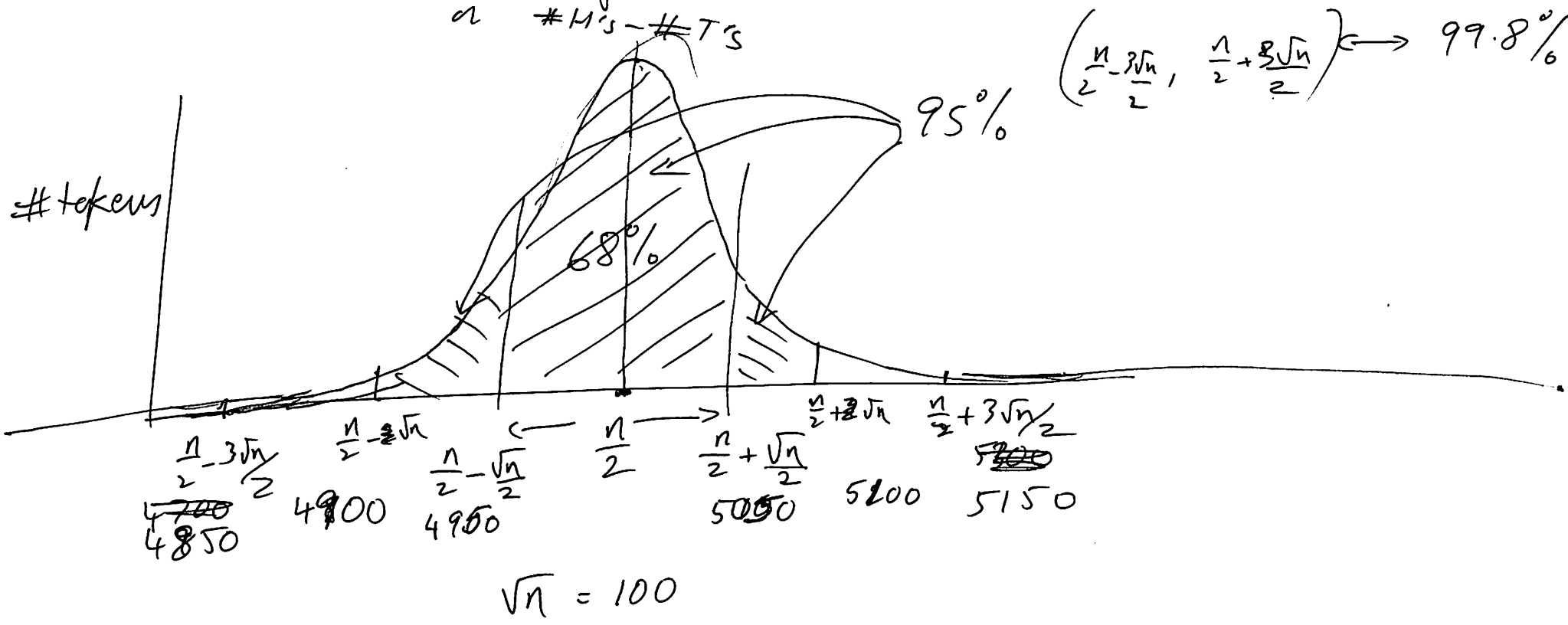
Flippy a coin $\frac{n}{5}$ times.

$n = 10,000$
 $\# \text{ paths} = 2^{10000}$



$= 32 = 2^5$

tokens



Fair coin 50-50 chance. $p = \frac{1}{2}$

Biased coin: 20-80
.2 vs .8 $\leftrightarrow P = .8$

10,000 times

8000 H's.

7900 - 8100

Suppose you didn't know p .

7971 H's.

$$\hat{p} = \frac{7971}{10,000} = \underline{\underline{.7971}}$$

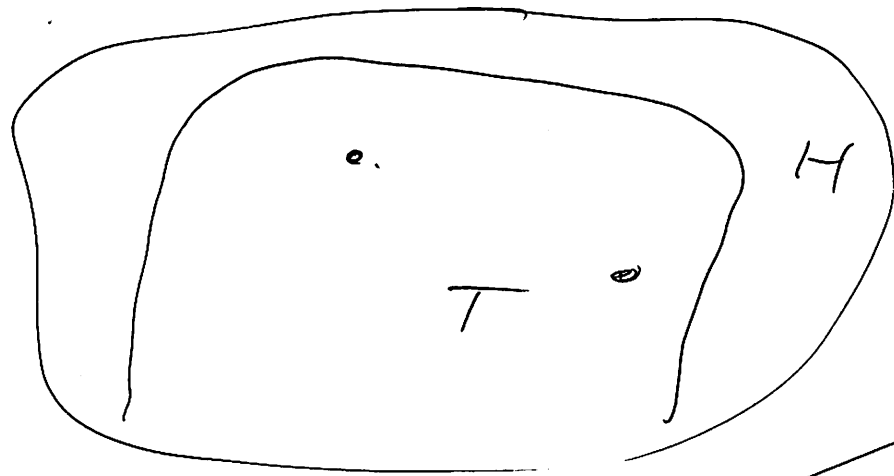
$$\text{error} = |\hat{p} - p| = |.7971 - .8| = \underline{\underline{.0029}}$$

~~error~~ if #H's is between 7900 & 8100

$$\text{then } |\hat{p} - p| \leq \frac{2}{100} \cdot 0.01 \quad |.7900 - .8| = 0.01$$

error $|\hat{p} - p| \leq 0.01$ with confidence 95%
 $\rightarrow 1 - \underline{\underline{.05}}$

Opinion Polling



$n = 500$ times.

How many phone calls?

How much error can you tolerate? $\epsilon = .05$

How much confidence? $\delta = .01$

Based on ϵ, δ can pick a number n .

$|\hat{p} - p| \leq \epsilon$ with confidence $1 - \delta$.

$n = \frac{1}{4\epsilon^2\delta}$ is sufficient.

Counting

$$S = \{1, 2, 3, 4\}$$

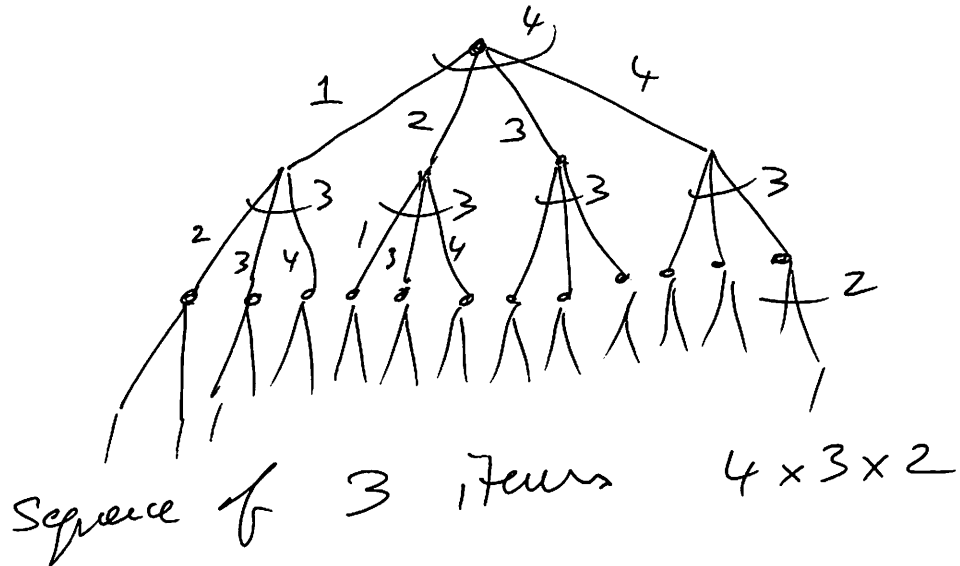
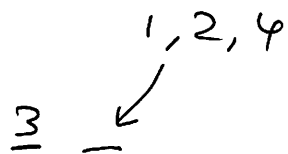
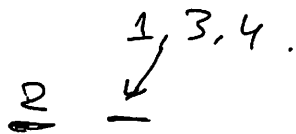
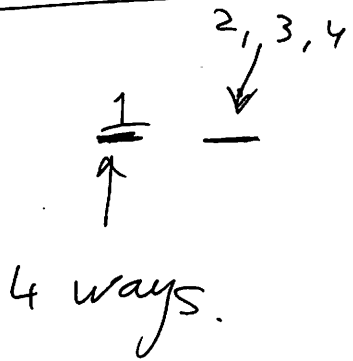
2, 4, 1,

1, 4, 2

3, 1, 4

How many ways?

co $4 \times 3 = 12.$



$$S = \{1, 2, \dots, n\} \quad n = 52.$$

picking a sequence of $k = 5$ items.

$$\frac{n(n-1)(n-2)\dots(n-k+1)}{(n-k)(n-k-1)\dots 2 \times 1} = \frac{n!}{(n-k)!}$$

$n(n-1)\dots(n-k+1)$ = total number of sequences of length k

$$\underbrace{52 \times 51 \times 50 \times 49 \times 48}$$

$$n! = n(n-1)(n-2)\dots \times 2 \times 1.$$

If we ~~can~~ create an object by making a sequence of k ~~distinct~~ choices: n_1 ways of making 1st choice, n_2 ways of making 2nd choice, no matter what the 1st choice, n_k ways of making k th choice.

objects = $n_1 \times n_2 \times \dots \times n_k$.

$\{1, 2, \dots, n\}$

$n=52$

Q1: Pick a sequence of $k=5$ of these items.

Order matters.

$2, 1, 4, 5, 7 \neq 1, 2, 4, 5, 7.$

How many ways?
" "
sequences

Q2 Pick a subset of $k=5$ of these items.

$\{2, 1, 4, 5, 7\} = \{1, 2, 4, 5, 7\}.$

↑ How many ways?