CS 70 Discrete Mathematics and Probability Theory
Fall 2013 Week 2 Discussion

Induction and Stable Marriage

1. Suppose that I start with 0 written on a piece of paper. Each minute, I choose a digit written on the paper and erase it. If it was 0 I replace it with 010. If it was 1 I replace it with 1001. Prove that no matter which digits I choose and no matter how long the process continues, I never end up with two 1's in a row.

2. Suppose that in a certain city there are N intersections, some pairs of which are connected by bidirectional roads. The roads are such that it is possible to travel from one intersection to any other intersection over the roads. Sometimes civil engineers will close a road for construction (which they will later reopen). However, whenever they close a road, it will be part of some *loop*: a sequence of connected roads $R_1, R_2, \ldots, R_k$ which aren't under construction, which start and end at the same location, and in which each road appears at most once. Prove that it is always possible to get from any intersection to any other intersection.

3. Run the Stable Marriage Algorithm on the following preference table:

Man	Women			
1	B	A	C	D
2	D	C	B	A
3	A	C	B	D
4	A	D	B	C

Woman	Men			
A	2	1	3	4
B	3	4	2	1
C	1	3	4	2
D	1	4	2	3

4. Suppose Alice, Bob, and Charlie are three people participating in the propose-and-reject algorithm. Suppose Alice rejects bob's proposal and ends up matched with Charlie. Prove that if Alice rejects Bob's proposal and ends up matched with Charlie, then Alice prefers Charlie to Bob.

5. Prove that the matching produced by the propose-and-reject algorithm is *stable*: there is no pair of people (Alice, Bob), such that Alice prefers Bob to her partner under the matching *and* Bob prefers Alice to his partner under the matching.

   (Hint: consider separately the cases where Alice and Bob are paired, where Alice rejected Bob, and where Bob never proposed to Alice.)

6. Based on the preference tables from problem 3, indicate whether each of the following pairings is stable. For each person, which of the stable matchings do they most prefer? Which do they least prefer?

   (a) $\{(1,B),(2,C),(3,A),(4,D)\}$
   (b) $\{(1,D),(2,B),(3,C),(4,A)\}$
   (c) $\{(1,C),(2,A),(3,B),(4,D)\}$
   (d) $\{(1,A),(2,B),(3,C),(4,D)\}$