Introduction to Graphs

Note: you aren't expected to complete even all of the non-challenge problems. Extra problems are included to help with practice.

1. Give the necessary and sufficient conditions for an undirected graph to have an Eulerian walk.
2. A *Hamiltonian path* is a path that visits each vertex exactly once. A *tournament graph* is a directed graph such that for all vertices u, v in the graph, either $(u, v) \in E$ or $(v, u) \in E$. Show that a tournament graph has a Hamiltonian path.
3. **Challenge problem**: In lecture, you learned that an undirected graph $G = (V, E)$ has an Eulerian tour if and only if the graph is connected (except for isolated vertices) and even degree. Prove the following alternate characterization of Eulerian graphs: A connected graph G has an Eulerian tour if and only if its edge set can be decomposed into disjoint cycles (two cycles are disjoint if they share no edges). Hint: try using induction on the number of edges in the graph.