

Conditional Probabilities and Applications

The Birthday Paradox

You are recording the birthday of every person you happen to encounter. Assume for convenience that there are 30 days in each month (and 12 months).

1. What is the probability that the i^{th} person you encounter has a birthday in March?
2. What is the probability that it takes more than n people to find two people who have the same birthday?
3. What is the probability that it takes exactly n people to find two people who have the same birthday?
4. Now you're just looking for someone with the same birthday as you. What is the probability that it takes exactly n people for this to occur?

Let's say you stop recording birthdays after you encounter n people.

1. What is the probability that exactly 4 of these people were born on March 1?
2. What is the probability that none of these people have birthdays which occur in the months of January, February or March?
3. What is the probability that there exists a person among those n people with a birthday in July?

Bayesian Inference

The next CS 70 midterm will be cancelled if there is a power outage on campus. Assume the probability of a power outage on any given day is $\frac{1}{100}$. However, one of your GSIs claims to have the ability to predict power outages. When there is actually a power outage, your GSI is correct 90% of the time. When there is not, your GSI is correct only 50% of the time. If your GSI says there will be a power outage on November 5th, what is the probability that he or she is correct?