Expectation and Variance

Graphs

Consider a random graph as in the homework, but slightly different: we still have n nodes, but each edge appears independently with probability p. Let X be the number of isolated nodes (nodes with degree 0). What is $E(x)$?

Balls and Bins again!

Let's go back to another one of the homework questions, regarding balls and bins. In this question, you throw n balls and you lose \$1 for every ball that was already in the bin. In the homework question, you computed how much you expect to lose. We will now show that with probability at least $\frac{1}{2}$, no bin has more than $\sqrt{2n}$ balls.

To do this, let's first consider a simpler question which will help. Suppose that the average number of mistakes made on midterm 2 was 10. Prove that at most $\frac{1}{3}$ of students made 30 or more mistakes. We can now use this idea to solve prove the above statement.

Variance

In the homework, you were asked how long it would take you to go to your office given that you were taking a very slow bus to work. Let's change this problem in several ways. First of all, the bus that you are taking is now very, very fast- the bus ride to work takes a negligible amount of time. Also, the bus passes your house once every hour (and it is going towards your office). One last detail: you have a friend who sometimes shows up to take you to work. This friend shows up with probability .5, and when he does, he is at your door exactly when you wake up and drops you off at work in a negligible amount of time.

1. Given this, how long will it be in expectation between when you wake up and when you arrive at your office?
2. What is the variance?
3. Now let's say your friend becomes more or less reliable- he shows up with probability p. How does the variance change?