CS 70　　　Discrete Mathematics and Probability Theory
Fall 2013　　Vazirani　　　　　　　　　　Week 12 Discussion

Distributions and Continuous Probability

Geometric Distribution

1. One of the most important properties of the geometric distribution $X \approx Geom(p)$ is the memoryless property: $\Pr[X = j | X > i] = \Pr[X = j - i]$ for all $j > i$. Intuitively, what does this property say? Another way to write it is

$$\Pr[X - i = j - i | X > i] = \Pr[X = j - i].$$

   Imagine that you are flipping a coin repeatedly and you have just observed that the first i flips were all tails. Verify that this property holds.

2. Recall the elevator question from homework 10. Now assume that n people are getting on the elevator, but there are still 10 floors. Each person still gets off at a randomly selected floor, and each person's destination is independent of everyone else's. What is the expected number of people who need to get on the elevator until the elevator is required to stop at every floor?

Poisson Distribution

On average, .7 cars drive by a gas station per hour. Each car stops independently at the gas station with probability p.

1. Let X be the number of cars that drive by the gas station. For $i \geq 0$, what is $\Pr[X = i]$?

2. Let Y be the number of cars that stop at the gas station in the next hour. For $i \geq 0$, what is $\Pr[Y = i]$? You can use the following identity: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$.

Continuous Probability

Let's revisit the bus problem from the previous discussion and from homework 10. Let's say there are two bus lines, 1 and 2, and each stops at your house once per hour. A line 1 bus stops at your house at the hour and a line 2 bus stops at your house half past the hour. However, line 1 takes you to work in 15 minutes and line 2 takes you to work in 45 minutes. You just get on whichever bus stops at your house first. What is the expected time it takes you to get to work?

1. What is the probability density function?

2. What is the expectation?

3. What is the variance?

4. Now let's say line 1 takes 5 minutes and line 2 still takes 45 minutes. What is the density function? The expectation?