

Infinity and Uncountability

1. For each of the following functions from \mathbb{R} to \mathbb{R} , determine whether it is an injection, surjection, bijection, none of the above, or more than one of the above.
 - (a) $f(x) = 2^x$
 - (b) $f(x) = x^2$
 - (c) $f(x) = 2x + 1$
2. For each of the following sets, decide whether it is countable or uncountable, and justify your answer.
 - (a) The set of all prime numbers
 - (b) The set of all finite sequences of integers
 - (c) The set of all real numbers in the range $[0, 0.1]$
 - (d) The set of all real numbers that are roots of polynomials with natural number coefficients
3. Prove that if A is uncountable and B is a countable subset of A , then $A - B$ is uncountable.