Total points = 10

Question 1 (1 point)

The 32 receives are shown connected to the sender in the binary tree configuration shown in Figure 1. With network-layer broadcast, a copy of the message is forwarded over each link exactly once. There are thus 62 link crossings (2+4+8+16+32). With unicast emulation, the sender unicasts a copy to each receiver over a path with 5 hops. There are thus 160 link crossings (5×32).

A topology in which all receivers are in a line, with the sender at one send of the line, will have the largest disparity between the cost of network-layer broadcast and unicast emulation.

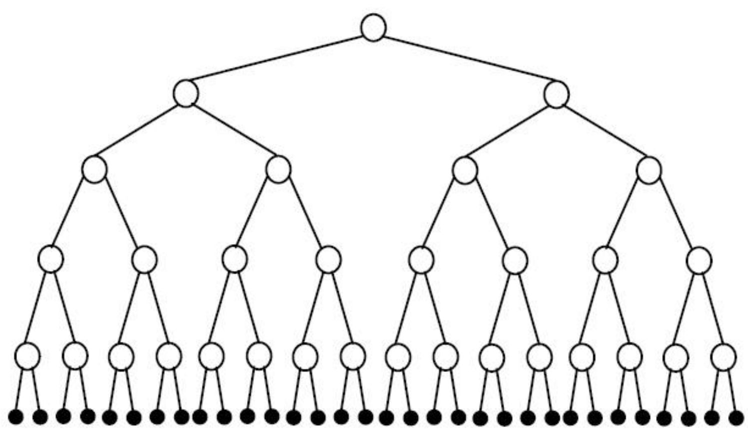


Figure 1: Tree for Question 1

Question 2 (1 point)

Dijkstra's algorithm for the network in Figure 2, with node A as the source, results in a leastunicast- cost path tree of links AC, AB, and BD, with an overall free cost of 20. The minimum spanning tree contains links AB, BD, and DC, at a cost of 11.

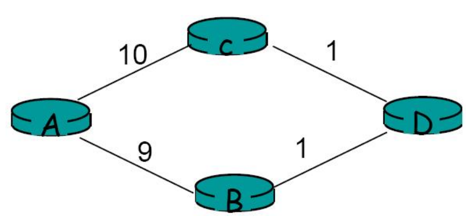
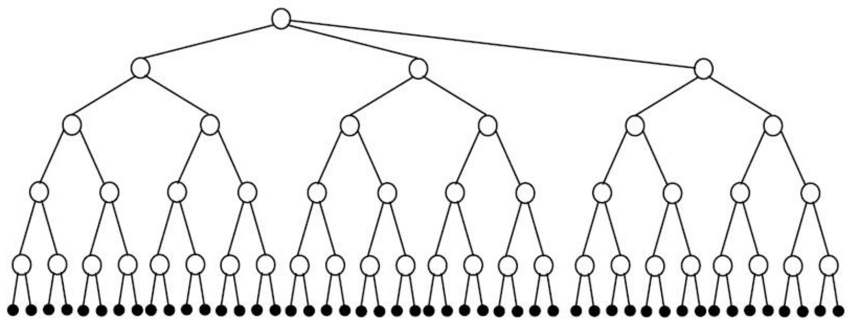


Figure 2: Network for Question 2

Question 3 (1 point)

As shown in the figure below, after 1 step 3 copies are transmitted, after 2 steps 6 copies are transmitted. After 3 steps, 12 copies are transmitted, and so on. After k steps, $3 \cdot 2^{k-1}$ copies will be transmitted in that step.



Question 4 (1 point)

(a) $r_1 - t_1 + r_2 - t_2 + \ldots + r_{n-1} - t_{n-1} = (n-1) \cdot d_{n-1}$. Substituting this into the expression for d_n gives $d_n = \frac{n-1}{n} \cdot d_{n-1} + \frac{r_n - t_n}{n}$.

(b) The delay estimate in part (a) is an average of the delays. It gives equal weight to recent delays and to "old" delays. The delay estimate in Section 6.3 gives more weight to recent delays; delays in the distant past have relatively little impact on the estimate.

Question 5 (1 point)

(a) Both schemes require 25% more bandwidth. The first scheme has a playback delay of 5 packets. The second scheme has a delay of 2 packets.

(b) The first scheme will be able to reconstruct the original high-quality audio encoding. The second scheme will use the low quality audio encoding for the lost packets and will therefore have lower overall quality.

(c) For the first scheme, many of the original packets will be lost and audio quality will be very poor. For the second scheme, every audio chunk will be available at the receiver, although only the low quality version will be available for every other chunk. Audio quality will be acceptable.

Question 6 (2 points)

Let τ be a time at which flow 1 traffic starts to accumulate in the queue. We refer to τ as the beginning of a flow-1 busy period. Let $t > \tau$ be another time in the same flow-1 busy period. Let $T_1(\tau, t)$ be the amount of flow-1 traffic transmitted in the interval $[\tau, t]$. Clearly,

$$T_1(\tau, t) \quad \geq \quad \frac{W_1}{\sum W_j} \cdot R \cdot (t - \tau)$$

Let $Q_1(t)$ be the amount of flow-1 traffic in the queue at time t. Clearly,

$$\begin{aligned} Q_1(t) &= b_1 + r_1(t-\tau) - T_1(\tau,t) \\ &\leq b_1 + r_1(t-\tau) + \frac{W_1}{\sum W_j} \cdot R \cdot (t-\tau) \\ &= b_1 + (t-\tau) \cdot \left[r_1 - \frac{W_1}{\sum W_j} \cdot R \right] \end{aligned}$$

Since $r_1 < \frac{W_1}{\sum W_j} \cdot R$, $Q_1(t) \leq b_1$. Thus, the maximum amount of flow-1 traffic in the queue is b_1. The minimal rate at which this traffic is served is $\frac{W_1 \cdot R}{\sum W_j}$.

Thus, the maximum delay for a flow-1 bit is

$$\frac{b_1}{W_1 \cdot R / \sum W_j} = d_{max}.$$

Question 7 (3 points)

1. Total number of information bits per codeword = $m \times m = m^2$. Total number of bits per codeword = $m^2 + m + m = m^2 + 2m$. Therefore, data rate = $\frac{m^2}{m^2+2m} = \frac{1}{1+(2/m)}$. Data rate for the single parity code is $\frac{m^2}{m^2+1} = \frac{1}{1+(1/m^2)} > \frac{1}{1+(2/m)}$. So, the single parity check code has less redundancy compared to the 2-D code.
2. For $m = 3$, there are $3^2 = 9$ data bits. Since each data bit can independently be 0 or 1 and fixing the data bits fixes the corresponding codeword, we get $2^9 = 512$ possible codewords.

   The minimum hamming distance for this 2-D code is 3. To see this, consider an arbitrary codeword. If we flip one message bit, then the corresponding row parity and column parity bits get flipped. So, the minimum hamming distance is more than 1. If we flip two message bits, then they must be in distinct columns or distinct rows. So, at least those two distinct column parity or row parity bits will flip resulting in at least 4 bits getting flipped. Thus, $d_{min} > 2$. Furthermore, the single bit flip case shows that $d_{min} = 3$.
3. Note that in the argument in part (b), we never used the fact that $m = 3$, so it applies to all m. Thus, $d_{min} = 3$ for general m.
4. Since the minimum hamming distance $d_{min} = 3$, we are guaranteed to detect up to $d_{min} - 1 = 2$ bit errors. An example of an undetected 3-bit error pattern is a message bit and the corresponding row parity bit and the column parity bit flipping. Since, in all rows and columns, even number of bits flip, the resulting word is a codeword and hence the error is undetected.
5. Since $d_{min} = 3$, we can correct up to $(3-1)/2 = 1$ bit errors. An example of an uncorrectable 2-bit error pattern is when a message bit and its corresponding row parity bit

get flipped. Suppose, two bits flip in a codeword c in such a manner. For the resulting word, flipping the corresponding column bit gives us another codeword (call it c'). Now, we cannot figure out whether c was transmitted and 2 bits flipped or c' was transmitted and 1 bit flipped. Thus, this error pattern cannot be corrected.

6. (a) Data rate now = $\frac{m^2}{(m+1)^2} < \frac{1}{1+(1/m^2)}$. So, this code has more redundancy than the single parity code.

   (b) Since the number of message bits remains unchanged, we again have 512 codewords. The minimum hamming distance $d_{min} = 4$. To see this, consider an arbitrary codeword. If we flip 1 message bit, the corresponding row parity and column parity bits get flipped. Furthermore, the extra bit also flips since exactly one row parity bit flipped. So, 4 bits flipped in this case. For the case of 2 message bits flipping, the argument from part (2) still applies (because we only added an extra bit without modifying the other bits). This means that if 2 message bits flip, then at least 4 bits flip in total. Now consider the case of 3 message bits flipping. So, it must be the case that at least one row or column has exactly one message bit flipped. For this row or column, the corresponding parity bit also gets flipped. This means that at least 4 bits get flipped in total. So, $d_{min} > 3$. Furthermore, the case of 1 message bit flipping implies that $d_{min} = 4$.

   (c) Note that the argument in part (b) did not use the fact that $m = 3$. So, we have $d_{min} = 4$ for all m.

   (d) Since the minimum hamming distance $d_{min} = 4$, we are guaranteed to detect up to $d_{min} - 1 = 3$ bit errors. An example of an undetected 4-bit error pattern is a message bit and the corresponding row parity bit, the column parity bit and the extra redundancy bit flipping. Since, in all rows and columns, even number of bits flip, the resulting word is a codeword and hence the error is undetected.

   (e) Since $d_{min} = 4$, we can correct up to $\lfloor (4-1)/2 \rfloor = 1$ bit errors. An example of an uncorrectable 2-bit error pattern is when a message bit and its corresponding row parity bit get flipped. Suppose, two bits flip in a codeword c in such a manner. For the resulting word, flipping the corresponding column bit and the extra redundancy bit gives us another codeword (call it c'). Now, we cannot figure out whether c was transmitted and 2 bits flipped or c' was transmitted and 2 bits flipped. Thus, this error pattern cannot be corrected.