University of California, Berkeley
Department of Electrical Engineering and Computer Sciences
EE123: DIGITAL SIGNAL PROCESSING

Fall 2006

Discussion #7

(1) For each of the following systems, determine if the system is (i) stable, (ii) causal, (iii) linear, and (iv) time-invariant.

a. $T(x[n]) = (\cos \pi n)x[n]$

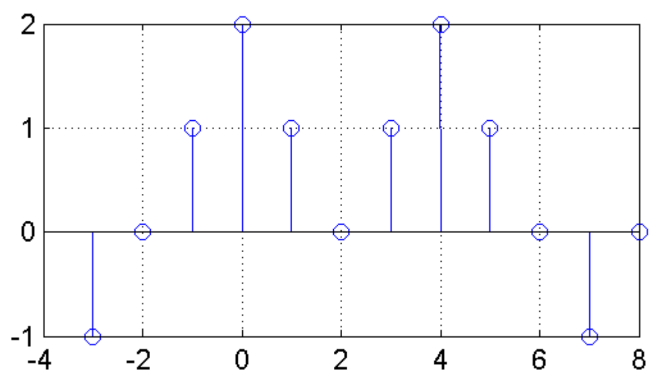
b. $T(x[n]) = x[n]\sum_{k=0}^{\infty}\delta[n-k]$

c. $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$

(2) Consider a system with input *x[n]* and output *y[n]*. The input-output relation for the system is defined by the following properties: (i) $y[n] - a \cdot y[n-1] = x[n]$, and (ii) $y[0] = 1$.

a. Determine if the system is time-invariant.
b. Determine if the system is linear.
c. Assume that (i) stays the same, but (ii) is changed to $y[0] = 0$. Does this change your answer to either part (a) or (b)?

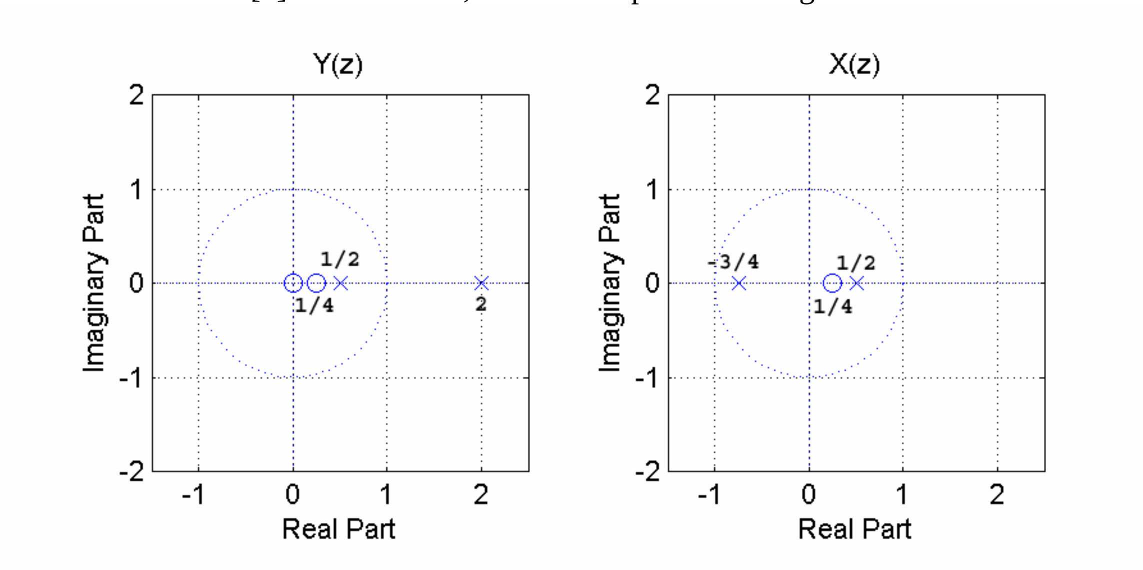
(3) Consider the following sequence, *x[n]*, with Fourier transform *X(ω)*.



Find the following without explicitly evaluating *X(ω)*.

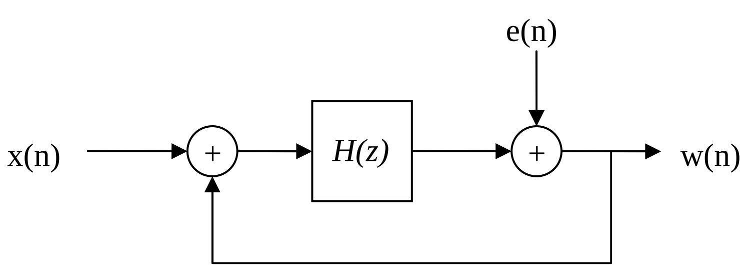
a. X(0)
b. X(π)
c. Arg X(ω)
d. $\int_{-\pi}^{\pi} X(\omega)d\omega$
e. Determine and sketch the signal whose Fourier transform is *X(-ω)*.
f. Determine and sketch the signal whose Fourier transform is *Re{X(ω)}*.

(4) The signal *y[n]* is the output of an LTI system with impulse response *h[n]* for a given input *x[n]*. *y[n]* is stable, and has a z-transform *Y(z)* with pole-zero diagram shown below. *x[n]* is also stable, and has the pole-zero diagram shown below.



a. What is the ROC of *Y(z)*?
b. Is *y[n]* left sided, right sided or two sided?
c. What is the ROC of *X(z)*?
d. Is *x[n]* a causal (i.e. $x[n]=0$ for $n<0$) sequence?
e. What is $x[0]$?
f. Draw the pole-zero plot of *H(z)*, and specify its ROC.
g. Is *h[n]* an anti-causal (i.e. $h[n]=0$ for $n>0$) sequence?

(5) *H(z)* is the system function of a causal LTI system. Given the following system:

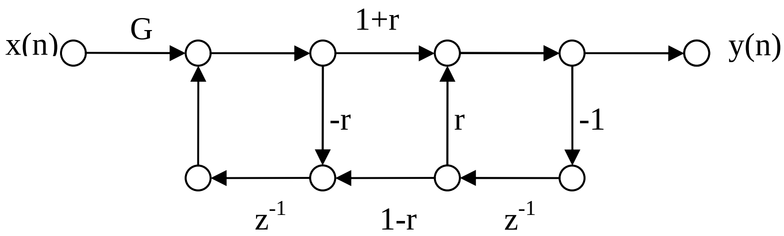


a. Obtain an expression for *W(z)* in the form:

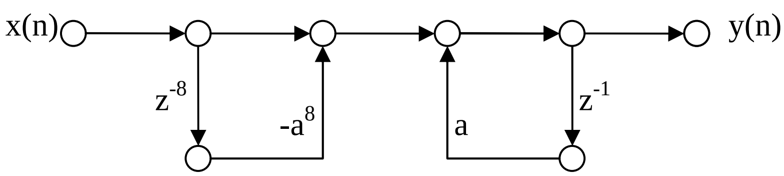
$$W(z) = H_1(z)X(z) + H_2(z)E(z)$$

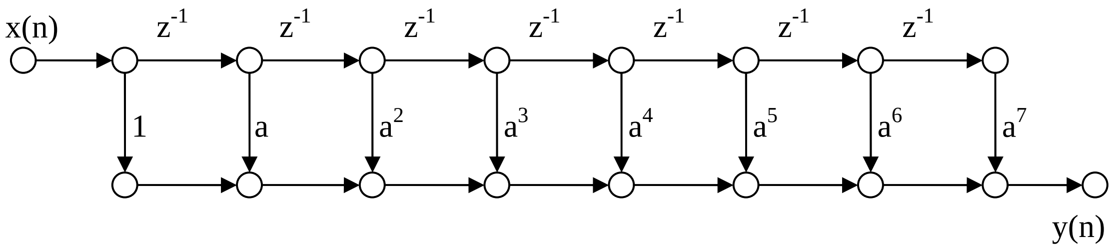
where both $H_1(z)$ and $H_2(z)$ are expressed in terms of *H(z)*.
b. If $H(z)=z^{-1}/(1-z^{-1})$, determine $H_1(z)$ and $H_2(z)$.
c. Is *H(z)* stable? Are $H_1(z)$ and $H_2(z)$ stable?

(6) Consider the following discrete-time system. Determine its system function $H(z)=Y(z)/X(z)$, and determine the magnitude of the poles of $H(z)$ as a function of r for $|r|<1$.



(7) When implemented with infinite-precision arithmetic, the flow graphs below have the same system function.





a. Show that the two systems have the same overall system function from $x[n]$ to $y[n]$.
b. Assume that the preceding systems are implemented with two's complement fixed-point arithmetic and that products are *rounded before* additions are performed. Draw signal flow graphs that insert round-off noise sources at appropriate locations in the signal flow graphs.
c. Circle the nodes in your figure from (b) where overflow can occur.
d. Assume that $|a|<1$. Find the total noise variance at the output of each system and determine the maximum value of $|a|$ such that Network 1 has lower total noise variance than Network 2.