University of California, Berkeley
Department of Electrical Engineering and Computer Sciences
EE123: DIGITAL SIGNAL PROCESSING

Fall 2006

Discussion #15

1. The even part of a real sequence *x[n]* is defined by
$$x_e[n] = \frac{x[n] + x[-n]}{2}$$
Suppose that *x[n]* is a real finite-length sequence defined such that *x[n]=0* for *n<0* and *n≥N*. Let *X[k]* denote the *N*-point DFT of *x[n]*.
(a) Is *Re{X[k]}* the DFT of $x_e[n]$?
(b) What is the inverse DFT of *Re{X[k]}* in terms of *x[n]*?

2. Suppose that *x[n]* is an infinite-length, stable (i.e. absolutely summable) sequence with z-transform given by:
$$X(z) = \frac{1}{1 - \frac{1}{3} z^{-1}}$$
Suppose that *y[n]* is a finite-length sequence of length *N*, and the *N*-point DFT of *y[n]* is:
$$Y[k] = X(z)\big|_{z=\exp(j2\pi k/N)}, \qquad k = 0,1,\ldots,N-1$$
Determine *y[n]*.

3. Consider an infinite duration sequence *x[n]*, which is non-zero for all n. We wish to filter *x[n]* with the impulse response *h[n]*. The sequence *h[n]* is a finite duration sequence that is zero outside *0≤n≤200*. The filtered signal is denoted by *y[n]* and is given by *y[n]=x[n]*h[n]*. Suppose we wish to determine *y[n]* for *0≤n≤N-1*.

   To do this, a student obtained *x[n]* for *0≤n≤N-1*, extended it to the left with 200 zeros, and performed a *N+200* point circular convolution of the extended segment of *x[n]* with *h[n]*. The student then eliminated the first 200 points of the results and claimed that the remained is *y[n]* for *0≤n≤N-1*.

   Is the student correct? If the student is incorrect, what portion of *y[n]* can we get from the result of the circular convolution?

4. Suppose that a finite-length sequence $x[n]$ has the N-point DFT $X[k]$, and suppose that the sequence satisfies the symmetry condition:

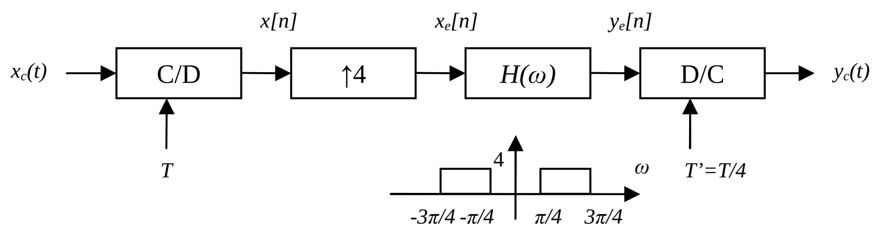
$$x[n] = -x[((n + N/2))_N], \quad 0 \le n \le N-1$$

where N is even and $x[n]$ is complex.

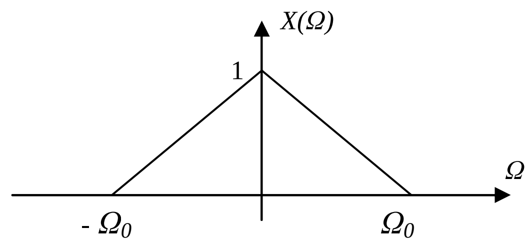
(a) Show that $X[k]=0$ for $k=0,2,\ldots,N-2$.

(b) Show how to compute the odd-indexed DFT values $X[k]$, $k=1,3,\ldots,N-1$ using only one $N/2$-point DFT plus a small amount of extra computation.

5. Consider the system shown in the figure below.



The input to this system is the bandlimited signal whose Fourier transform is shown below with $\Omega_0=\pi/T$.



(a) Sketch the Fourier transform $X(\omega)$, $X_e(\omega)$, $Y_e(\omega)$, and $Y_c(\Omega)$.

(b) For the general case when $X_c(\Omega)=0$ for $|\Omega|\ge\pi/T$, express $Y_c(\Omega)$ in terms of $X_c(\Omega)$. Also, give a general expression for $y_c(t)$ in terms of $x_c(t)$ when $x_c(t)$ is band-limited in this manner.

6. The system function $G(z)$ represents a type II FIR generalized linear-phase system with impulse response $g[n]$. This system is cascaded with an LTI system whose system function is $(1-z^{-1})$ to produce a third system with system function $H(z)$ and impulse response $h[n]$. Prove that the overall system is a generalized linear-phase system, and determine what type of linear phase system it is.

7. Suppose we wish to design a highpass linear phase FIR filter $h[n]$ using the window method with the following specifications:

$$|H(\omega)| \le \delta_s, \qquad |\omega| \le \omega_s$$
$$1-\delta_p \le |H(\omega)| \le 1+\delta_p, \quad \omega_p \le |\omega| \le \pi$$

where $\omega_s = 0.3\pi$, $\omega_p = 0.45\pi$, $\delta_s = 0.04$ and $\delta_p = 0.01$.

Note the table below of properties for various windows:

Window	**Peak Amplitude of Side Lobe (dB)**	**Approximate Width of Main Lobe**	**Peak Approximation Error (dB)**
Rectangular	-13	$4\pi/N$	-21
Bartlett	-25	$8\pi/N$	-25
Hanning	-31	$8\pi/N$	-44
Hamming	-41	$8\pi/N$	-53
Blackman	-57	$12\pi/N$	-74

(a) Suppose we can use either Bartlett or Hamming windows. Which type of window would you choose? Why?

(b) Using the window $w[n]$ you chose in part (a), what is a reasonable initial choice of the window length N? Let $H_d(\omega)$ be the DTFT of the ideal filter $h_d[n]$, where $h[n]=h_d[n]w[n]$. Determine $H_d(\omega)$ for $-\pi \le \omega \le \pi$.

(c) Suppose that you can use the Kaiser window. Determine a reasonable initial choice of parameters (i.e. shape β, and filter length M). Determine the corresponding ideal response $H'_d(\omega)$ for $-\pi \le \omega \le \pi$.

8.

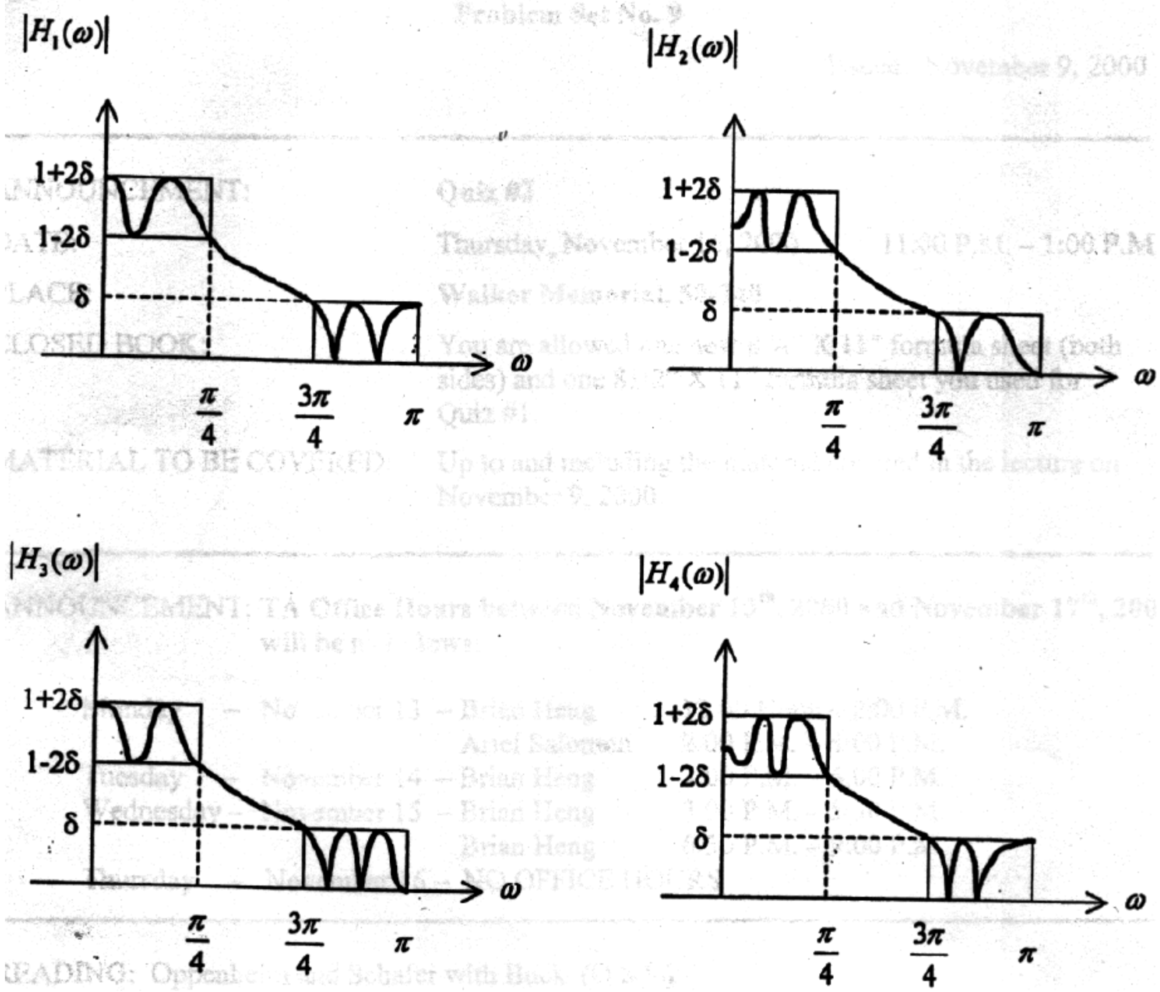
You are given the following optimal FIR lowpass filter design problem:

$$\text{Given } \delta_1 = 2\delta, \quad \delta_2 = \delta, \quad \omega_p = \frac{\pi}{4}, \quad \omega_s = \frac{3\pi}{4}, \text{ and } N \text{ (the length of the filter),}$$

$$\text{minimize } \delta$$

On the next page, the magnitude of the frequency response of four filters $|H_i(\omega)|$, $i = 1, \ldots, 4$ is shown. Let N_i represent the length of the impulse response $h_i(n)$ of the filter $H_i(\omega)$, i.e., $h_i(n)$ is nonzero only for $0 \le n \le N_i - 1$. Assume N_i is odd and $h_i(n) = h_i(N_i - 1 - n)$

For each frequency response, determine whether it could have been generated by the Parks-McClellan algorithm. Justify your reasoning and determine the possible values of N_i for each filter $h_i(n)$ that could have been generated by the Parks-McClellan algorithm.


|H₁(ω)|
1+2δ
1-2δ
δ
ω
π/4
3π/4
π
|H₂(ω)|
1+2δ
1-2δ
δ
ω
π/4
3π/4
π
|H₃(ω)|
1+2δ
1-2δ
δ
ω
π/4
3π/4
π
|H₄(ω)|
1+2δ
1-2δ
δ
ω
π/4
3π/4
π