Roundoff Noise issues in Cascade Structure

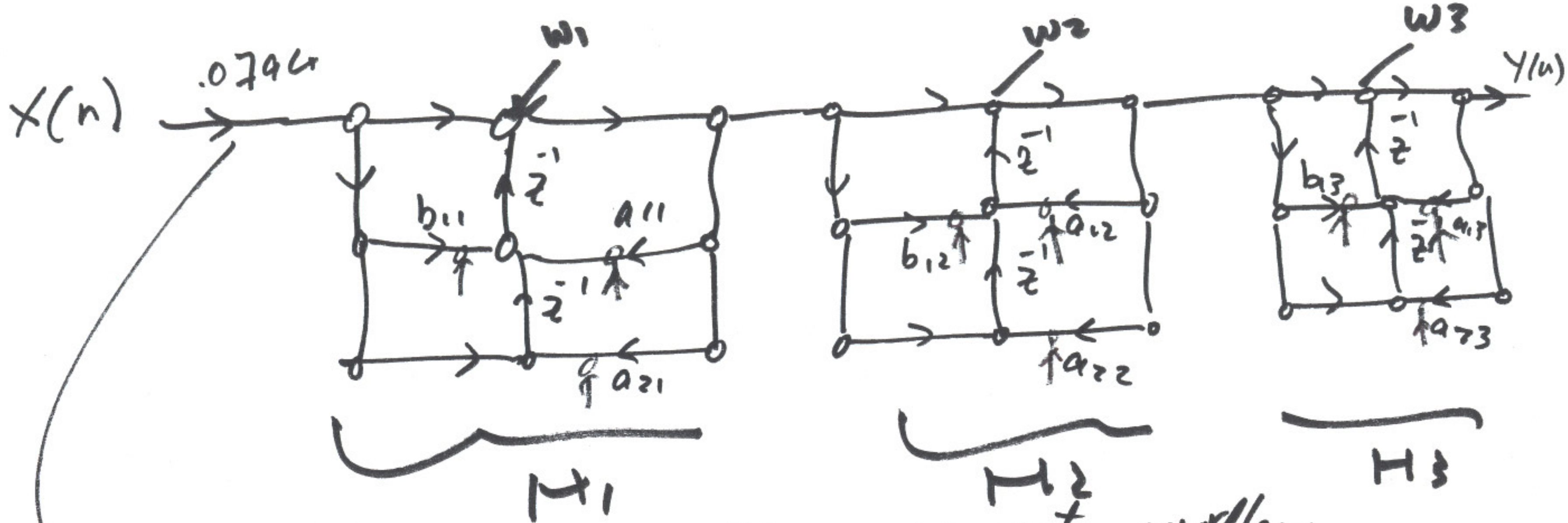
Specs: $.99 \leq |H(\omega)| \leq 1.01 \qquad |\omega| < 0.5\pi$

$|H(\omega)| \leq 0.01 \qquad .56\pi \leq |\omega| < \pi$

$$H(z) = 0.0794 \prod_{k=1}^{3} H_k(z) = 0.0794 \prod_{k=1}^{3} \frac{(1 + b_{1k} z^{-1} + b_{2k} z^{-2})}{(1 - a_{1k} z^{-1} - a_{2k} z^{-2})}$$

unity gain
at passband

K	a_{1k}	a_{2k}	b_{1k}
1	0.47	-0.17	1.7
	0.14	-0.6	.78
	-0.05	-0.90	.411



Can show: 0.0794 prevents overflow in the above implementation.

Add ~~noise~~ Quantization noise sources

$$H(z) = s_1 \; H_1(z) \quad s_2 \; H_2(z) \quad s_3 \; H_3(z)$$

$s_1 s_2 s_3 = .0794$

Choose s_1, s_2, s_3 so that no overflows in H_1, H_2, H_3

If you apply Technique #2 from last lecture

$$S \cdot X_{max} < \frac{1}{\max\limits_{k,|\omega|<\pi} |H_k(\omega)|}$$

Transfer fn. between input and all intermediate nodes.

$\Rightarrow$ S_1 only worry internal nodes of H_1

S_2 only considering internal nodes of H_2

$\Rightarrow$

$\boxed{S_1 S_2 S_3 = .0794}$

$S_1 = .186$ $S_2 = .52$ $S_3 = .80$

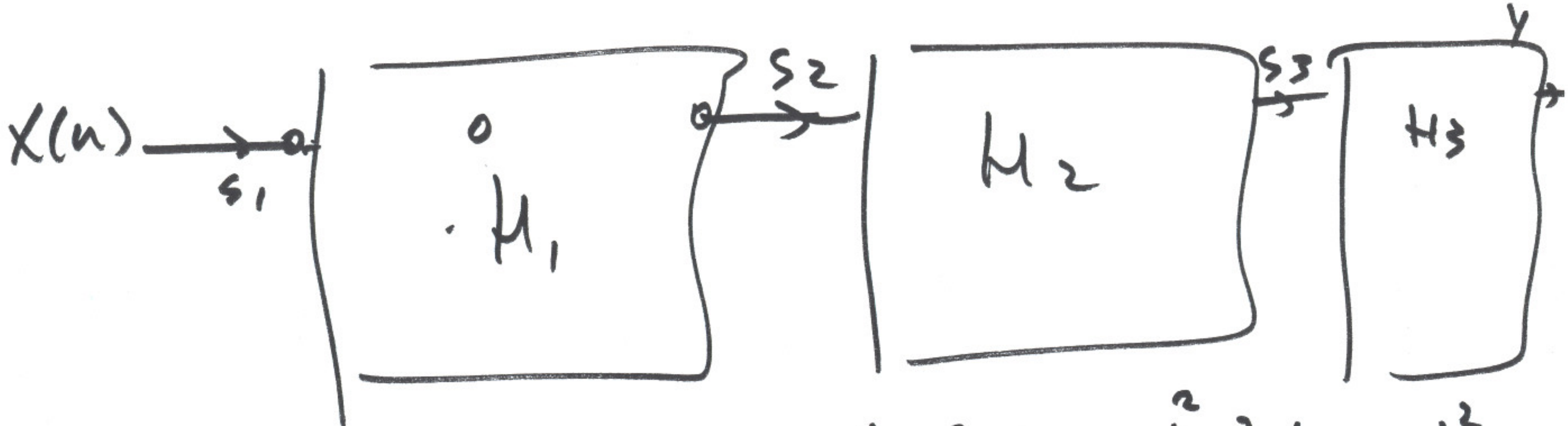
$$S_1 \max_{|\omega|<\pi,\, m} \left| H_{1m}(\omega) \right| < 1$$

$H_{1m}(\omega)$ are transfer fn between input to H_1 and all of its nodes, m.

$$S_1 S_2 \max_{|\omega|<\pi,\, m,\, n} \left| H_{1m}(\omega)\, H_{2n}(\omega) \right| < 1$$

$$S_1 S_2 S_3 = .0794$$

$\Rightarrow S_1\ S_2\ S_3$, three eqns three unknowns



$$\text{noise variance at output} = 5\,\frac{2^{-2B}}{12}\left[\frac{1}{2\pi}\int_{-\pi}^{+\pi}\frac{s_2^2\,|H_2(\omega)|^2\,s_3^2\,|H_3(\omega)|^2}{|A_1(\omega)|^2}\,d\omega + \frac{1}{2\pi}\int_{-\pi}^{+\pi}\frac{s_3^2\,|H_3(\omega)|^2}{|A_2(\omega)|^2}\,d\omega + \frac{1}{2\pi}\int_{-\pi}^{+\pi}\frac{1}{|A_3(\omega)|^2}\,d\omega\right]$$

observation: $\left.\begin{array}{lll} H_1 & \text{doesn't appear} & \\ H_2 & \text{appears} & 1 \\ H_3 & \text{appear} & 2 \end{array}\right\} \to$ lots of different σ_f^2 depending on how we pair up zeros + poles.

Jackson's rule for choosing zero/pole pair:

① pair up a pole that is closest to the unit circle with the zero that is also closest to the unit circle.

② Repeat rule 1 until poles + zeros are exhausted.

③ The resulting 2nd order section should be ordered according to either: increasing closeness to the unit circle or decreasing closeness. to the unit circle.

noise process $n(t) \longrightarrow$ stationary.

Auto correlation

$$A(\tau) = E[n(t)\,n(t+\tau)]$$

$\Downarrow$ F.T.

Powerspectrum of noise

EE 126
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Fixed Point

int
long
short

$\downarrow$

Floating Point

float
double

$$f = 2^{c}\,\hat{X}_M$$

characteristic
exponent

mantissa

overflow
non issue.

c , $\hat{X}_M$

fixedpoint

fixed point.

$0.5 < \hat{X}_M < 1$

floating point Arith

(1) no overflows

(2) Addition **AND** multipl
Both introduce errors

Both error are signal dependent

$$\hat{x} = x(1+\epsilon) = x + \epsilon x$$

signal dependent