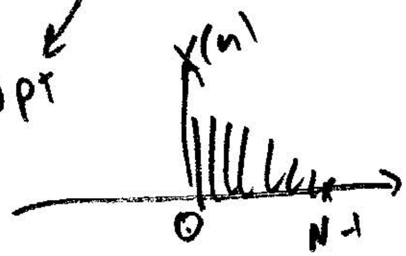
Properties of DFT:

① Shift property

$0 \le n < N$

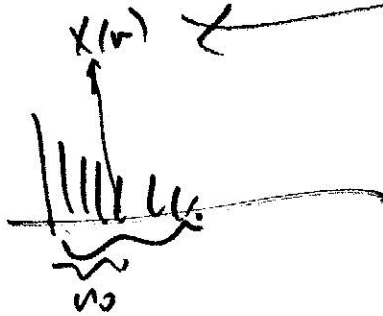
$$x(n) \xleftrightarrow{\text{DFT}} X(k) \qquad 0 \le k < N$$

N pt   N pt.



Wrong.

$$\cancel{x(n-n_0)} \xleftrightarrow{\text{IDFT}} X(k)\, e^{-j\frac{2\pi n_0 k}{N}}$$

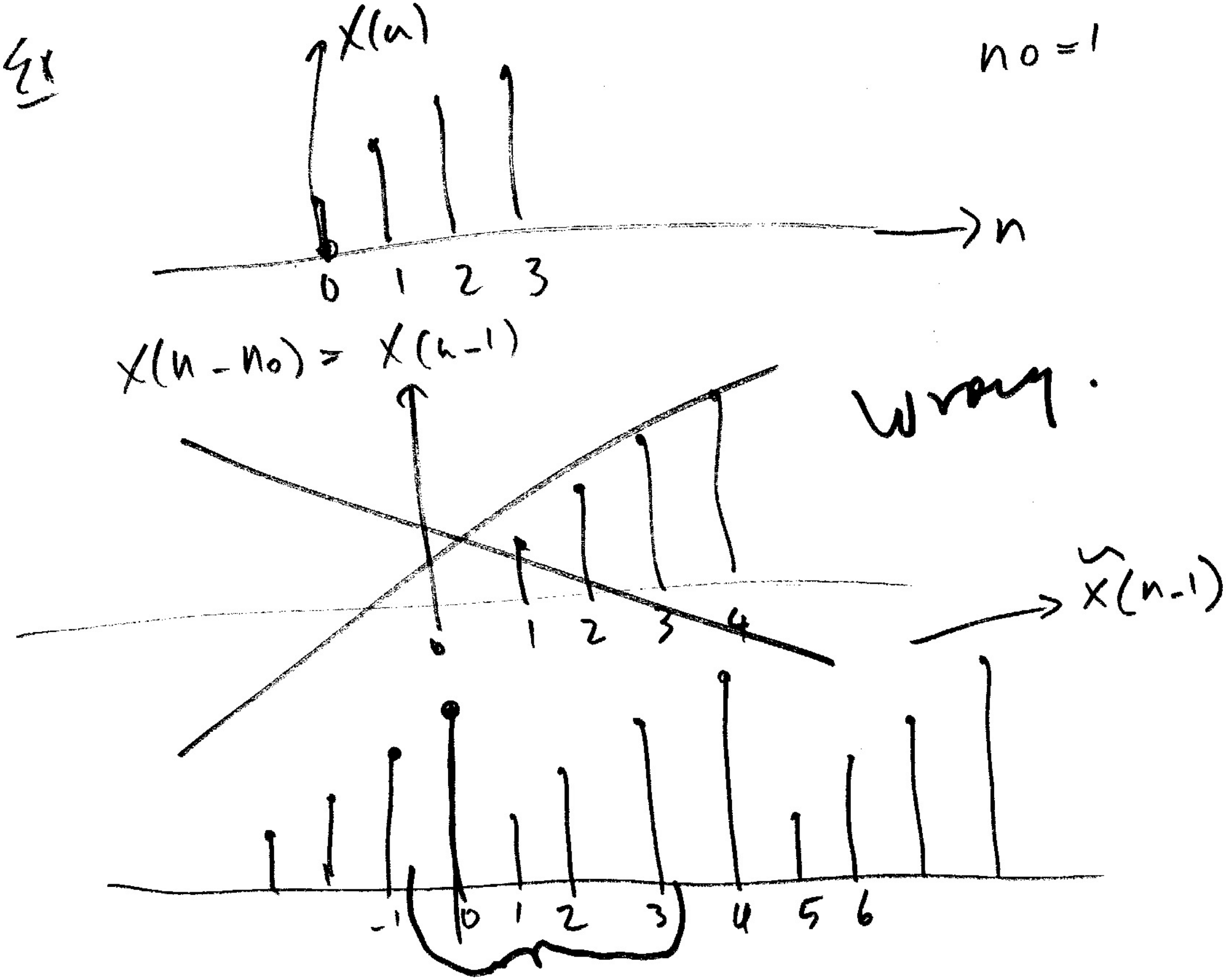


Answer:

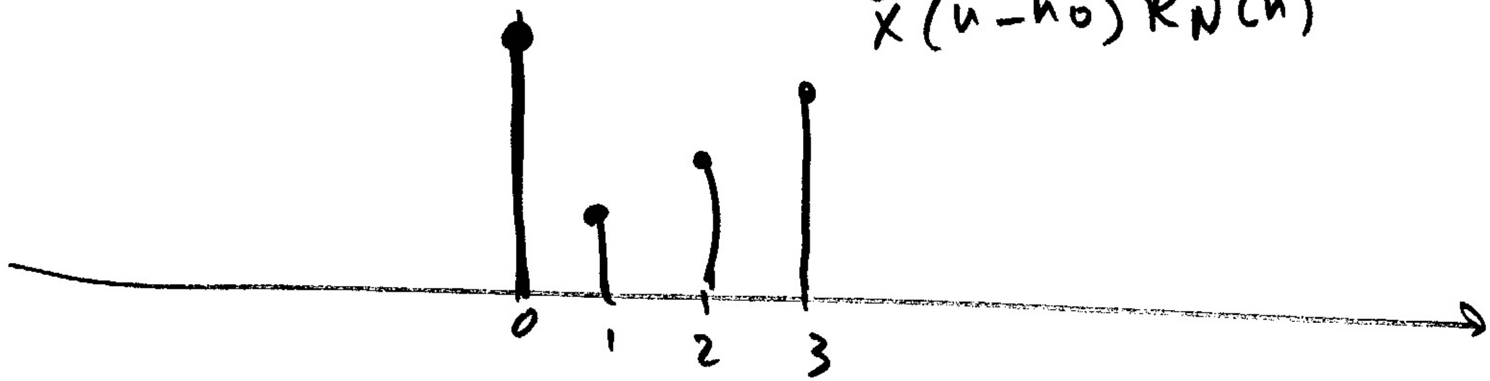
$$\tilde{x}(n-n_0)\, R_N(n) \xleftrightarrow{\text{DFT}} X(k)\, e^{-j\frac{2\pi n_0 k}{N}}$$

Ex

$n_0 = 1$



$\tilde{x}(n-n_0)R_N(n)$



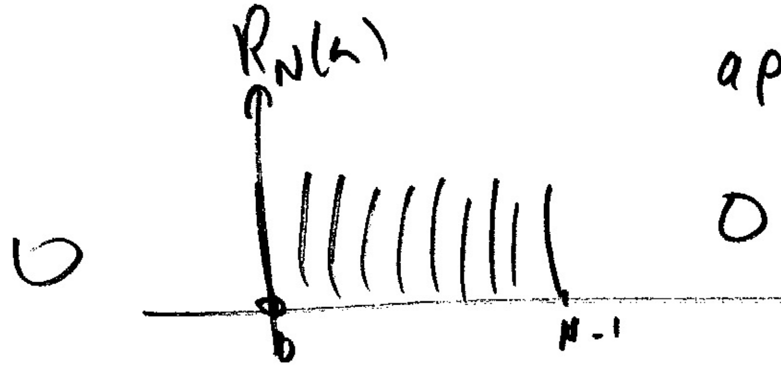
Why:

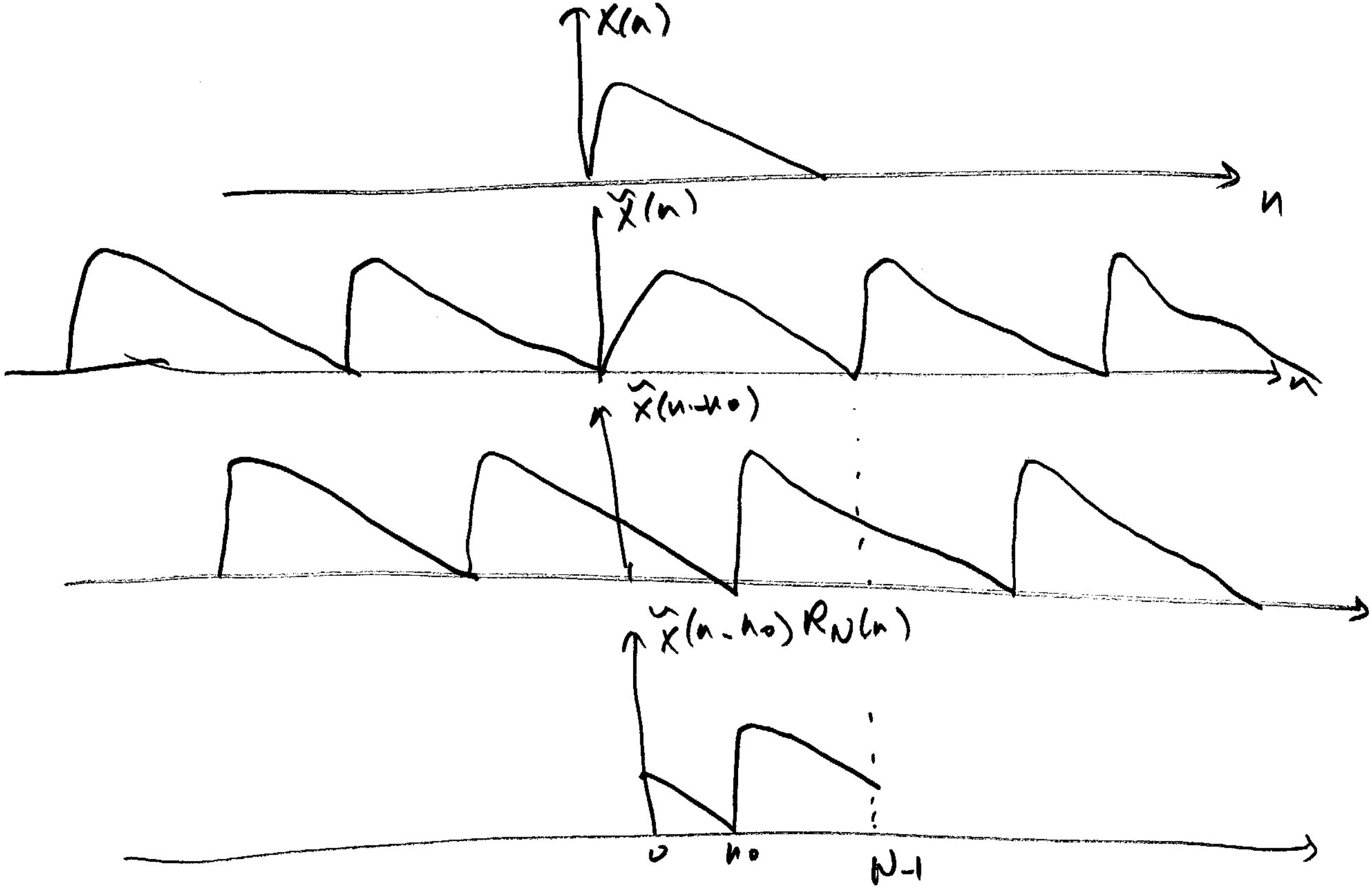
$x(n) \xleftrightarrow{\text{Periodic}} \tilde{x}(n) = \sum_{r=-\infty}^{+\infty} x(n+rN) \xleftrightarrow{\text{DFS}} \tilde{X}(k) \longleftrightarrow X(k)$

$X(k)e^{-j\frac{2\pi n_0 k}{N}} \longrightarrow \text{DFS} \quad \tilde{X}(k)e^{-j\frac{2\pi n_0 k}{N}} \longrightarrow$

periodic seq. $\quad \tilde{x}(n-n_0) \longrightarrow$

aperiodic $\quad \tilde{x}(n-n_0)R_N(n)$




$x(n)$
n
$\tilde{x}(n)$
n
$\tilde{x}(n+n_0)$
$\tilde{x}(n-n_0)R_N(n)$
0
n_0
$N-1$

Property of DFT

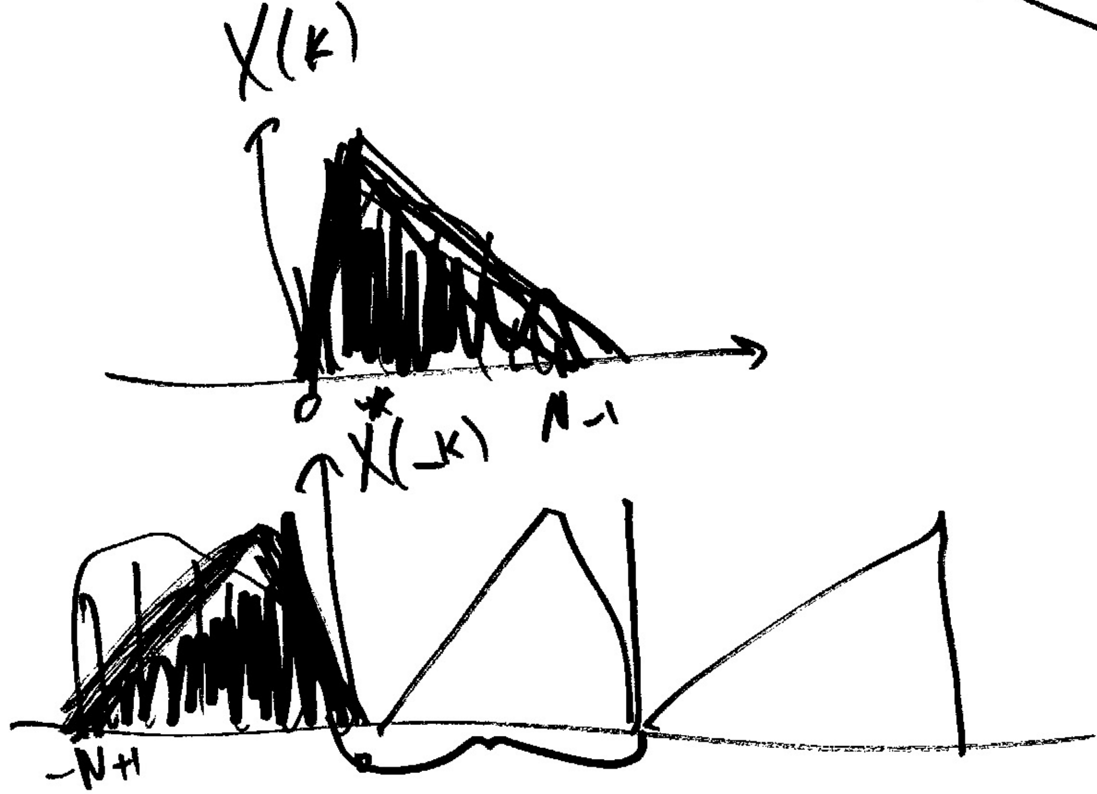
$0 \le k < N$

$x(n) \xleftrightarrow{DFT} X(k)$

N points, N PT

$y(n) = x^*(n) \xleftrightarrow{DFT} \cancel{X^*(-k)}$ wrong

$X^*(-k)\, R_N(k)$



$x^*(n) \longrightarrow \tilde{x}^*(n)$

make it periodic

$\xrightarrow{\text{DFS}} \quad \tilde{X}^*(-k)$

$\xrightarrow{\text{Take one period}} \quad \tilde{X}^*(-k)\, R_N(k)$

Symmetry Properties

$$x(n) \overset{DFT}{\longleftrightarrow} X(k)$$

$$\mathrm{Re}\{x(n)\} \longleftrightarrow R_N(k)\,\overbrace{X_e(k)}$$

$$j\,\mathrm{Im}\{x(n)\} \longleftrightarrow R_N(k)\,\overbrace{X_o(k)}$$

$$R_N(n)\,\overbrace{x_e(n)} \longleftrightarrow \mathrm{Re}\{X(k)\}$$

$$R_N(n)\,\overbrace{x_o(n)} \longleftrightarrow j\,\mathrm{Im}\{X(k)\}$$

How To use DFT To do Convolution:

Convolution $\longrightarrow$ linear convolution.

$$x_1 * x_2 = x_3$$

$$x_3(n) = \sum_k x_1(k)\, x_2(n-k)$$

LTI

$$x(n) \rightarrow \boxed{\begin{matrix} h(n) \\ LTI \end{matrix}} \rightarrow y(n) = x * h$$

$$= \sum_k x(k) h(n-k)$$

Periodic Convolution: for convolving 2 periodic sequences

$$\tilde{x}_1 \circledast \tilde{x}_2 = \tilde{x}_3(n)$$

periodic N $\quad$ N $\quad$ N

Circular Convolution:

2 finite length sequence

x_1 ← N pt

x_2 ← N pt.

$$\underbrace{x_1}_{N\text{ pt.}} \circledast \underbrace{x_2}_{N\text{ pt}} = \underbrace{x_4}_{N\text{ pt}}$$

Def of circular Convolution

$$x_4(n) \triangleq x_1 \underbrace{\circledast}_{\text{circular}} x_2 = \left[\tilde{x}_1 \underbrace{\circledast}_{\text{periodic conv}} \tilde{x}_2 \right] R_N(n)$$

Can show: Npt DFT of x_4 is the product of Npt DFT of x_1 and x_2.

$x_4 \longrightarrow$ Npt DFT $X_4(k)$

$x_1 \longrightarrow$ " " $X_1(k)$

$x_2 \longrightarrow$ " " $X_2(k)$

$$X_4(k) = X_1(k)\, X_2(k)$$

why?

Apply DFS properties

$$\tilde{x}_1(n) \circledast \tilde{x}_2(n) = \tilde{x}_4(n)$$

$$\tilde{X}_1(k) = \text{DFT}\{\tilde{x}_1(n)\}$$

$$\tilde{X}_1(k)\, \tilde{X}_2(k) = \tilde{X}_4(k)$$

$$R_N(k)\left\{ \tilde{X}_1(k)\tilde{X}_2(k) \right\} = R_N(k)\ \tilde{X}_4(k)$$

$$X_1(k)\,X_2(k) = X_4(k)$$

If we multiply N pt DFT of x_1 with N pt DFT of x_2, you get DFT of "circular convolution" of x_1 and x_2

not necessarily their linear convolution

<u>Ex</u> Fig 8.14 in O&S

$x_1(n) = \delta(n-1)$

$x_2(n)$ N point seq.

Circular convolution.

See Fig 8.3 first for periodic convolution.

Conclude.

$$x_3(n) = x_1(n) \circledast x_2$$

$$x_3(n) = R_N(n) \left[\tilde{x}_1(n) \circledast \tilde{x}_2(n) \right]$$

$$= \sum_{m=0}^{N-1} x_2(m) \left[\tilde{x}_1(n-m) R_N(m) \right]$$