Oct. 24, 2006

Overlap Save

Thought Exp ..

$x_1 \longrightarrow L$ pt.

$x_2 \longrightarrow P$ pt

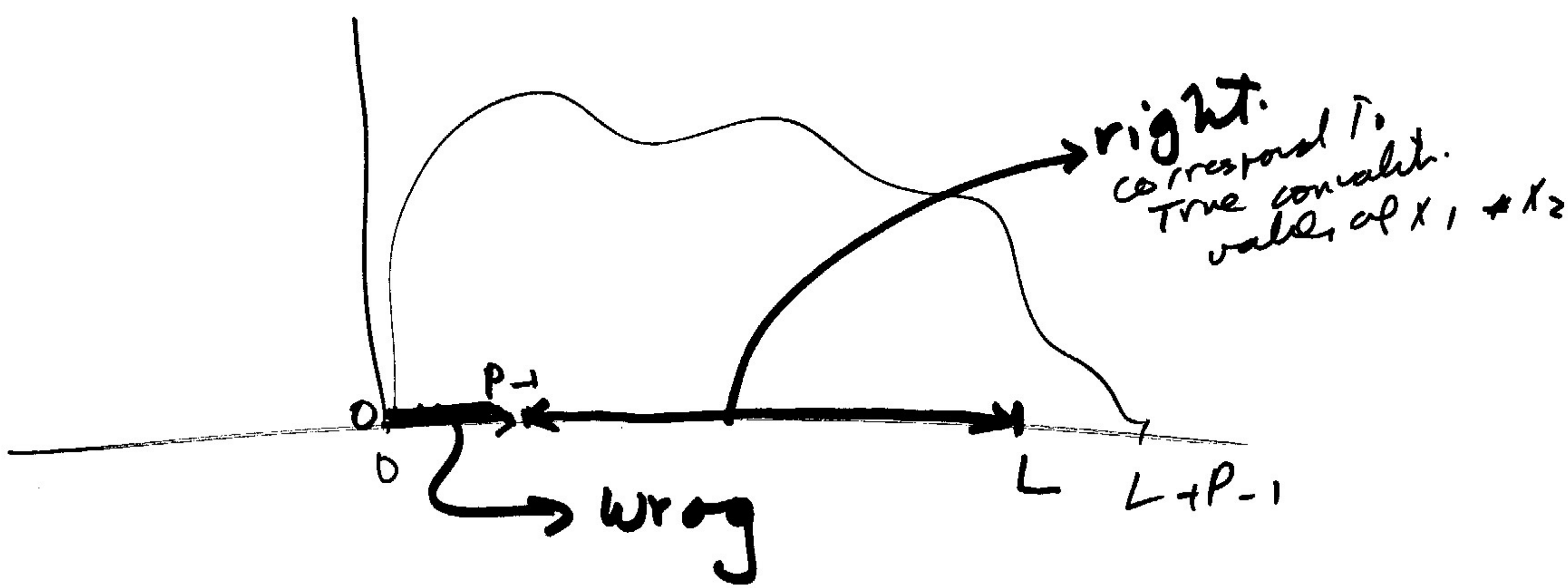
$x_1 * x_2 \longrightarrow L+P-1$ pt seq.

- Correct way:
  - Pad both seq with enough zeros to get $L+P-1$ pt sequence.
  - Multiply DFT of those $L+P-1$ pt seq.
  - Take IDFT of the product.

- Assume $L >> P$

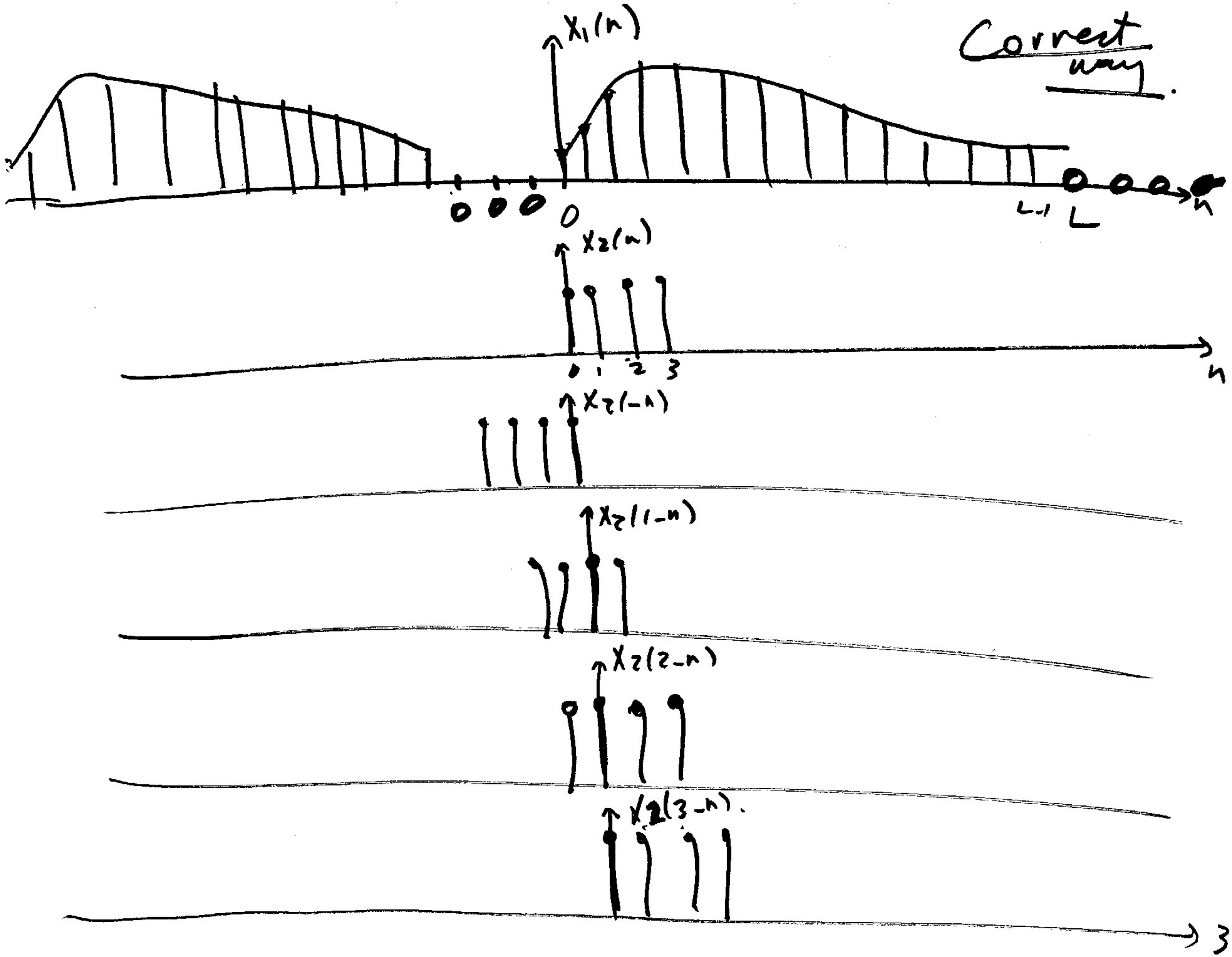
"Wrong Way" –
- Suppose take L pt DFT of x_1 and x_2
- Multiply two L pt DFT
- Take L point IDFT of the product

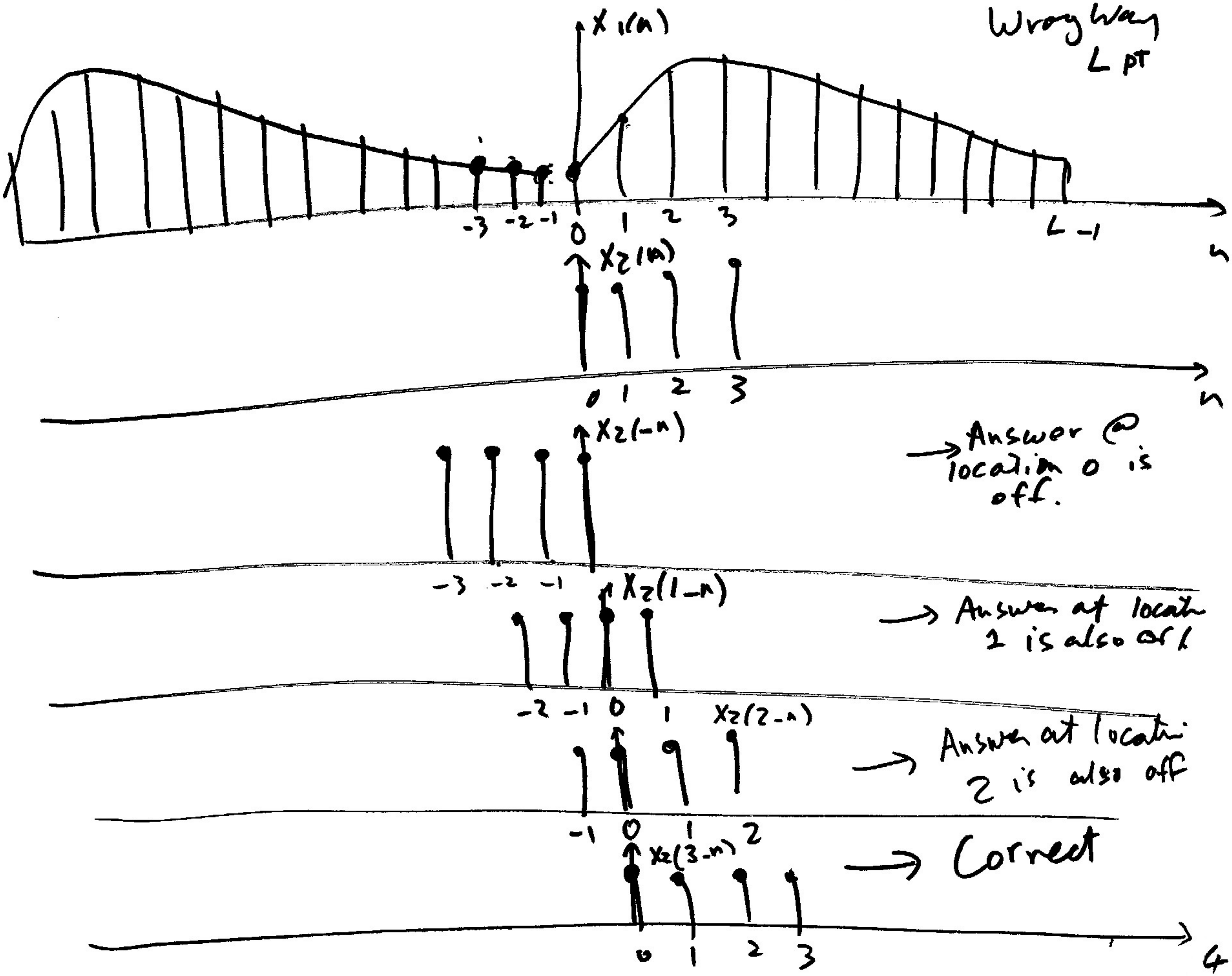
__Claim__ Only the first P points of the answer obtained via the "wrong way" is really wrong. The rest are good



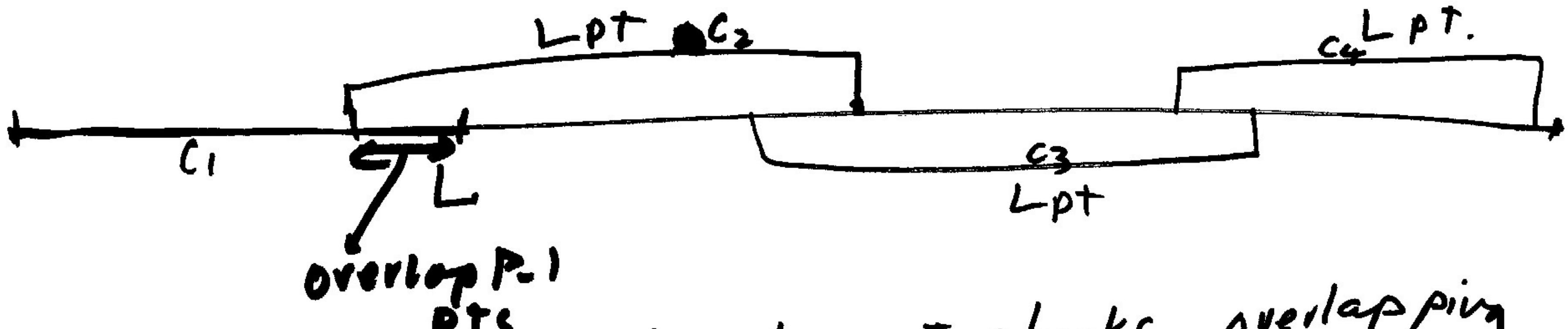
first $P-1$ are wrong.
remaining points up to index $L-1$
or Lth output is Right.

Correct way.




Wrong Way
L pt
$x_1(n)$
-3 -2 -1 0 1 2 3 $L-1$ n
$x_2(n)$
0 1 2 3 n
$x_2(-n)$
→ Answer @ location 0 is off.
-3 -2 -1
$x_2(1-n)$
→ Answer at locati 1 is also off
-2 -1 0 1
$x_2(2-n)$
→ Answer at locati 2 is also off
-1 0 1 2
$x_2(3-n)$
→ Correct
0 1 2 3 4

Overlap Save



① Segment sequence into L point chunks, overlapping with each other by P-1 points

② L pt circular convolution of each chunk.
multiply L pt DFT of chunk by L PT DFT of X_2 $\Big\}$ $\longrightarrow$ IDFT L pt.

③ throw away the first P-1 points of the ~~answer~~ IDFT in part 2, replace it with ~~pr~~ answer obtained from previous segment.

Fast Fourier Transform

Decimation in Time

Decimation in Freq.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n k}{N}}$$

For each k $\longrightarrow$ N adds, N mults.

N values of k $\longrightarrow$ N^2 adds, N^2 mults.

Direct way of computing DFT.

$O(N^2)$

Decimation in time

$$X(k) = \sum_{\substack{n \text{ even} \\ 0 \to N-1}} x(n) e^{-j\frac{2\pi nk}{N}} + \sum_{\substack{n \text{ odd} \\ 0 \to N-1}} x(n) e^{-j\frac{2\pi nk}{N}}$$

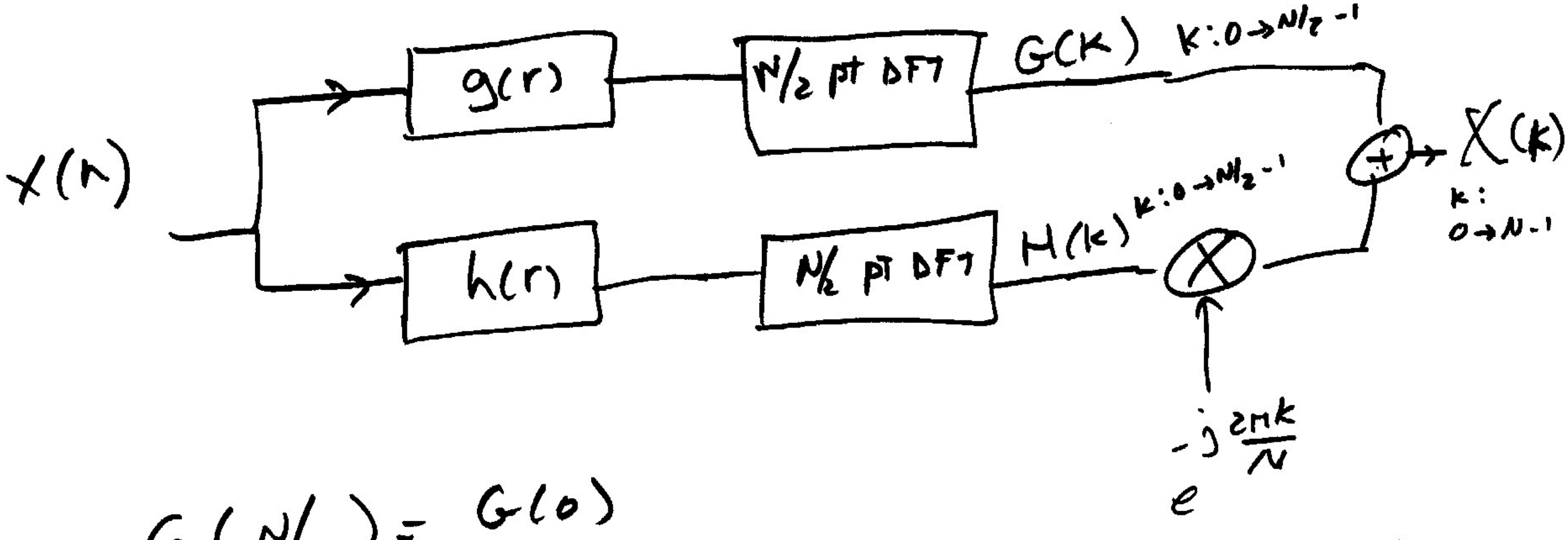
$n = 2r \quad r: 0 \to N/2 - 1$

$n = 2r+1$

$r: 0 \to N/2 - 1$

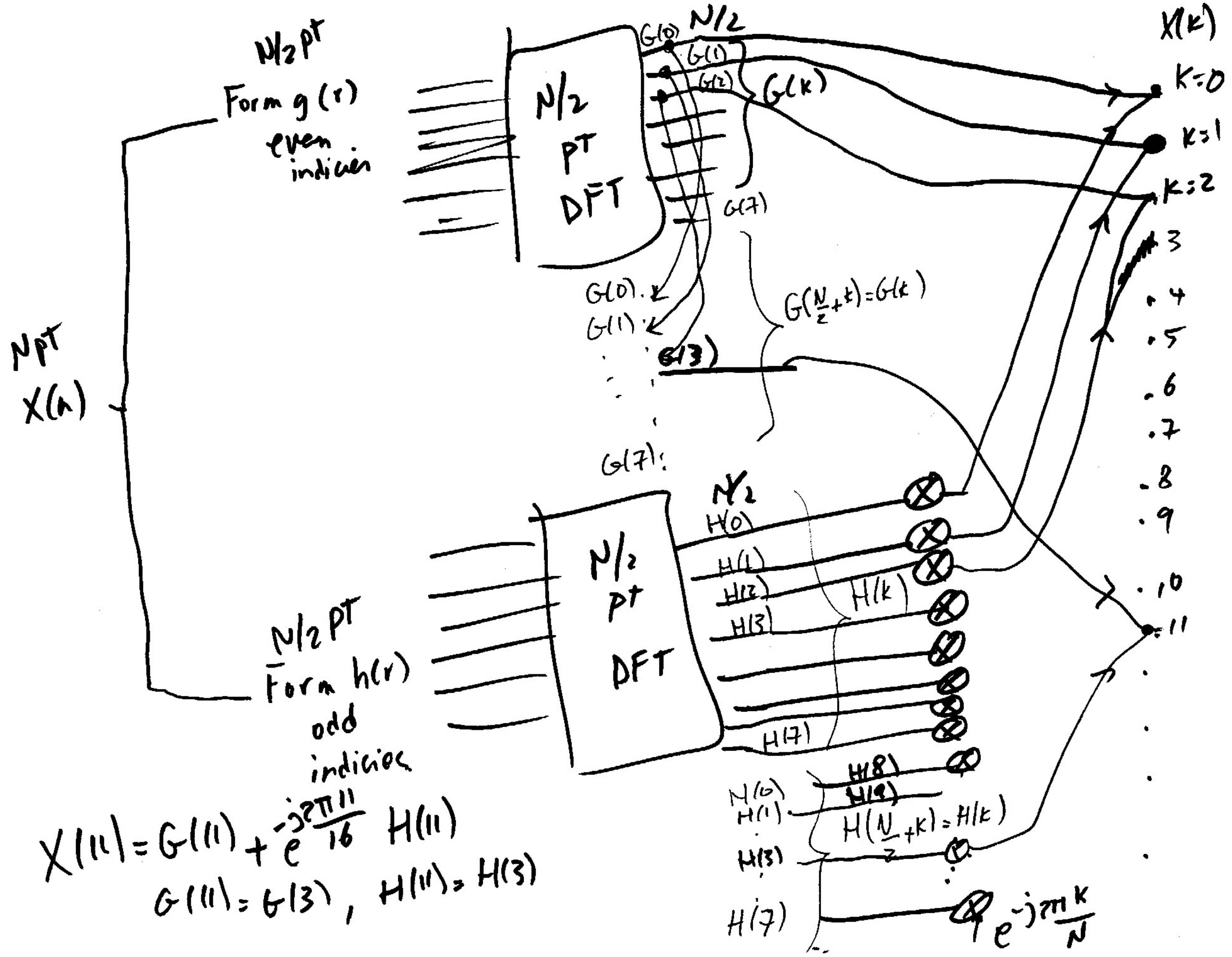
$$X(k) = \sum_{r=0}^{N/2-1} \underbrace{x(2r)}_{g(r)} e^{-j\frac{2\pi\, 2rk}{N}} + \sum_{r=0}^{N/2-1} \underbrace{x(2r+1)}_{h(r)} e^{-j\frac{2\pi(2r+1)k}{N}}$$

$$X(k) = \underbrace{\sum_{r=0}^{N/2-1} g(r) e^{-j\frac{2\pi kr}{N/2}}}_{N/2 \text{ pt DFT of } g(r)} + e^{-j\frac{2\pi k}{N}} \underbrace{\sum_{r=0}^{N/2-1} h(r) e^{-j\frac{2\pi kr}{N/2}}}_{N/2 \text{ pt DFT of } h(r)}$$



$$G(N/2) = G(0)$$
$$G(N/2+1) = G(1)$$
$$G(N/2+k) \triangleq G(K)$$

$$H(K) = H\left(\frac{N}{2}+k\right)$$



$$X(11) = G(11) + e^{-j\frac{2\pi 11}{16}} H(11)$$

$$G(11) = G(3), \quad H(11) = H(3)$$

Repeat the above process until I get to a 2 pt DFT.

$$\sum_{n=0}^{1} x(n) e^{-j\frac{2\pi nk}{2}} = x(0) e^{-j(0)} + x(1) e^{-j\frac{2\pi k}{2}}$$

$$= x(0) + x(1) e^{-j\pi k}$$

$k = 0$: $e^{-j\pi k} = 1 \Longrightarrow X(0) = x(0) + x(1)$

$k = 1$: $e^{-j\pi k} = -1 \Longrightarrow X(1) = x(0) - x(1)$

New Notation

$$W_N^k \triangleq e^{-j\frac{2\pi k}{N}}$$

Twiddle factor.

Use twiddle factor to redraw the basic flow graph for Dec. in Time.

Fig 9.3 of O&S.

$\downarrow$

9.7 of O&S.