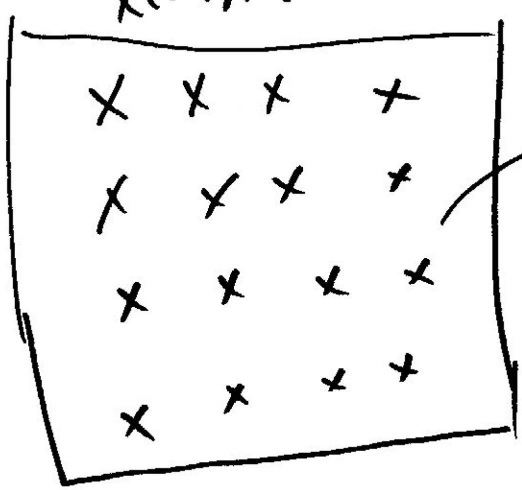
Oct. 31, 2006

Discrete Cosine Trafom

$\sum x^2(n)$

$x(n_1, n_2)$



DFT

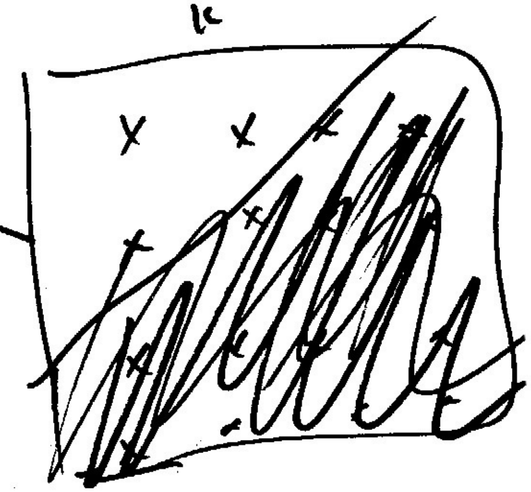
DCT

DHT.

Mellon.

⋮

$\sum_k |X(k)|^2$



Transform Coding: Basic idea Trafom. the pixels into Trafom domain, set a bunch of coeffs equal to zero.

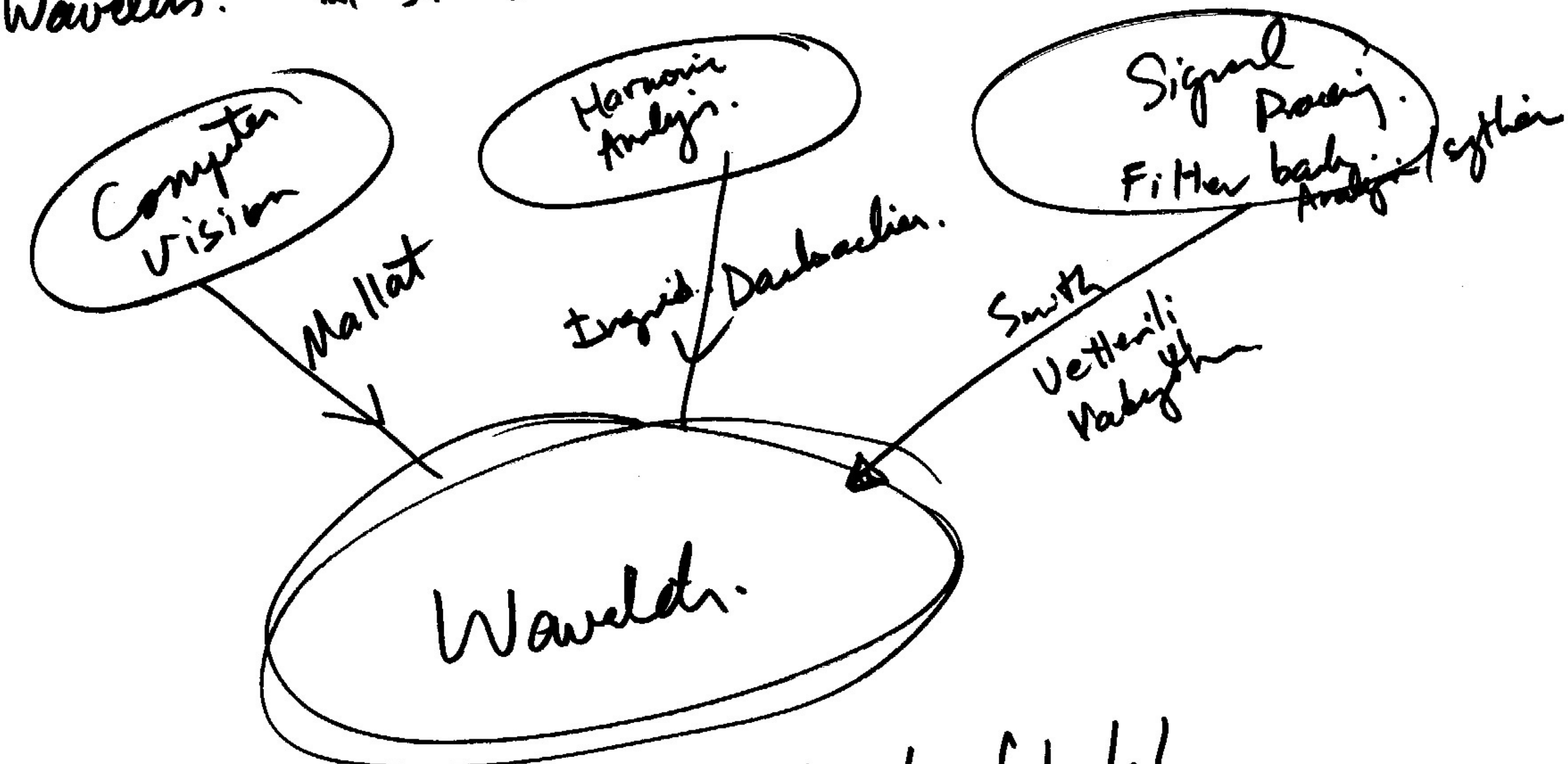
DCT has better "compaction property"

coeffs in Trafom domain are stat indp.

1

↳ Used in JPEG, MPEG1, MPEG2, MPEG4
H.261, H.263, H.264

Wavelets: JPEG 2000 ← still image coding



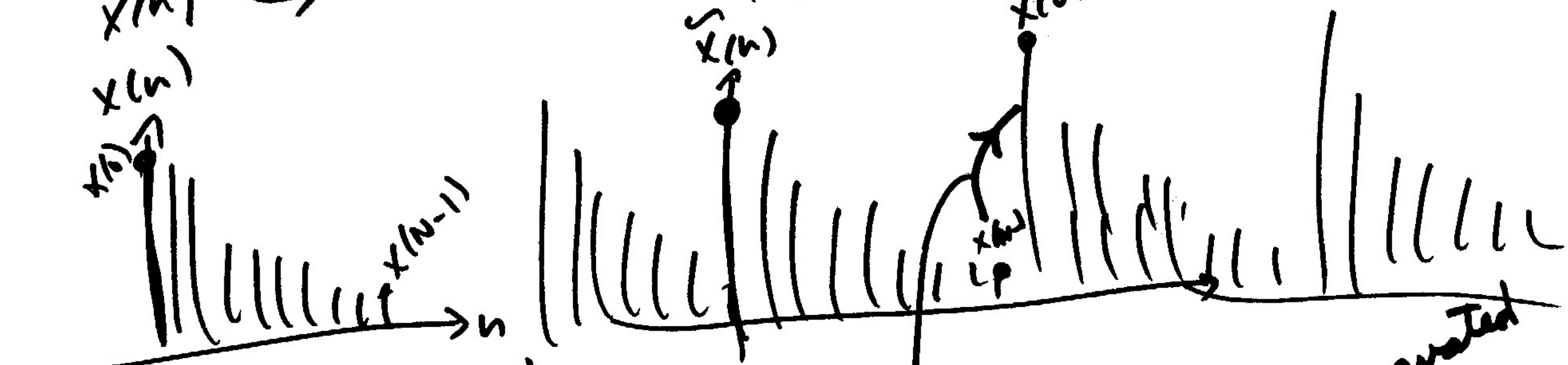
Haar, Hartley, Hadamard, Walsh

DCT: Real coefficients. Real arithmetic

DFT: complex coeff. complex arithmetic.

Why DCT has better compaction:

DFT: $x(n) \rightarrow \tilde{x}(n) \xrightarrow{DFS} \tilde{X}(k) \xrightarrow{\text{one period}} X(k)$ (DFT)

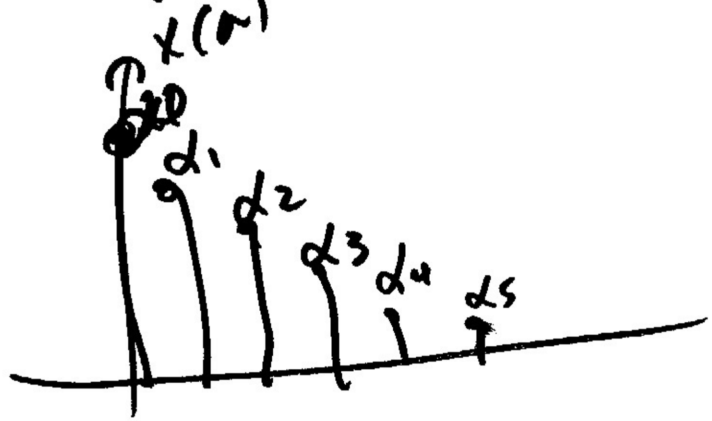


DFT doesnot have good compaction property

Sharp discontinuity artificially generated by replicating x(n) to get $\tilde{x}(n)$ before DFS is computed

nonzero High Freq. Component

Basic idea behind DCT is To replicate signal in a "better" way s.t. That there is no sharp discontinuities



By construction no sharp discontinuities

Take DFS of mirrored signal

4 Kinds of DCT

We'll focus on DCT 2

$$X^{C_2}(k) = 2 \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi k(2n+1)}{2N}\right)$$ analysis

Synthesis:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} \beta(k)\, X^{C_2}(k) \cos\left(\frac{\pi k(2n+1)}{2N}\right)$$

$$\beta(k) = \begin{cases} \frac{1}{2} & k = 0 \\ 1 & 1 \leq k < N \end{cases}$$

Relation Betwen DFT and DCT 2

Proposal ①:

① $x(n)$: N pt real seq.

② 2N pt DFT $\longrightarrow X(k)$

③ $2\,\mathrm{Re}\left\{ X(k)\, e^{-j\frac{2\pi k}{2N}} \right\} \longrightarrow$ DCT-2

Proposal ②

① start with N pt real seq. $x(n)$

② Pad it with N zeros $\rightarrow x_{2N}(n)$

③ Form a periodic seq.

$$\hat{x}_2(n) = \hat{x}_{2N}(n) + \hat{x}_{2N}(-n-1)$$

④ take 2N pt DFT of one period of $\hat{x}_2(n)$

$X_2(k)$

DCT and its relation to DFT.

Proposal 1

1. $x(n)$, N pt, assume it is real.
2. Take $2N$ pt DFT of $x(n) \longrightarrow X(k)$
3. $2\,\mathrm{Re}\left\{ X(k)\, e^{-j\frac{\pi k}{2N}} \right\} = X^{c_2}(k)$

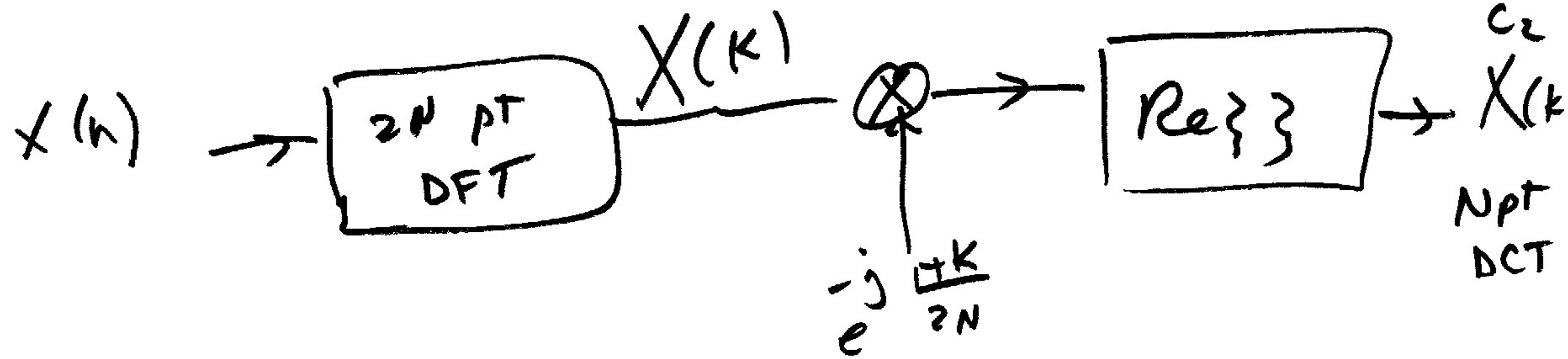
Proof:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n k}{2N}}$$

$$X(k)\, e^{-j\frac{\pi k}{2N}} = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{\pi k(2n+1)}{2N}}$$

$$2\,\mathrm{Re}\{\ \} = 2 \sum_{n=0}^{N-1} x(n) \cos \frac{\pi k(2n+1)}{2N} \overset{\Delta}{=} X^{c_2}(k)$$

QED

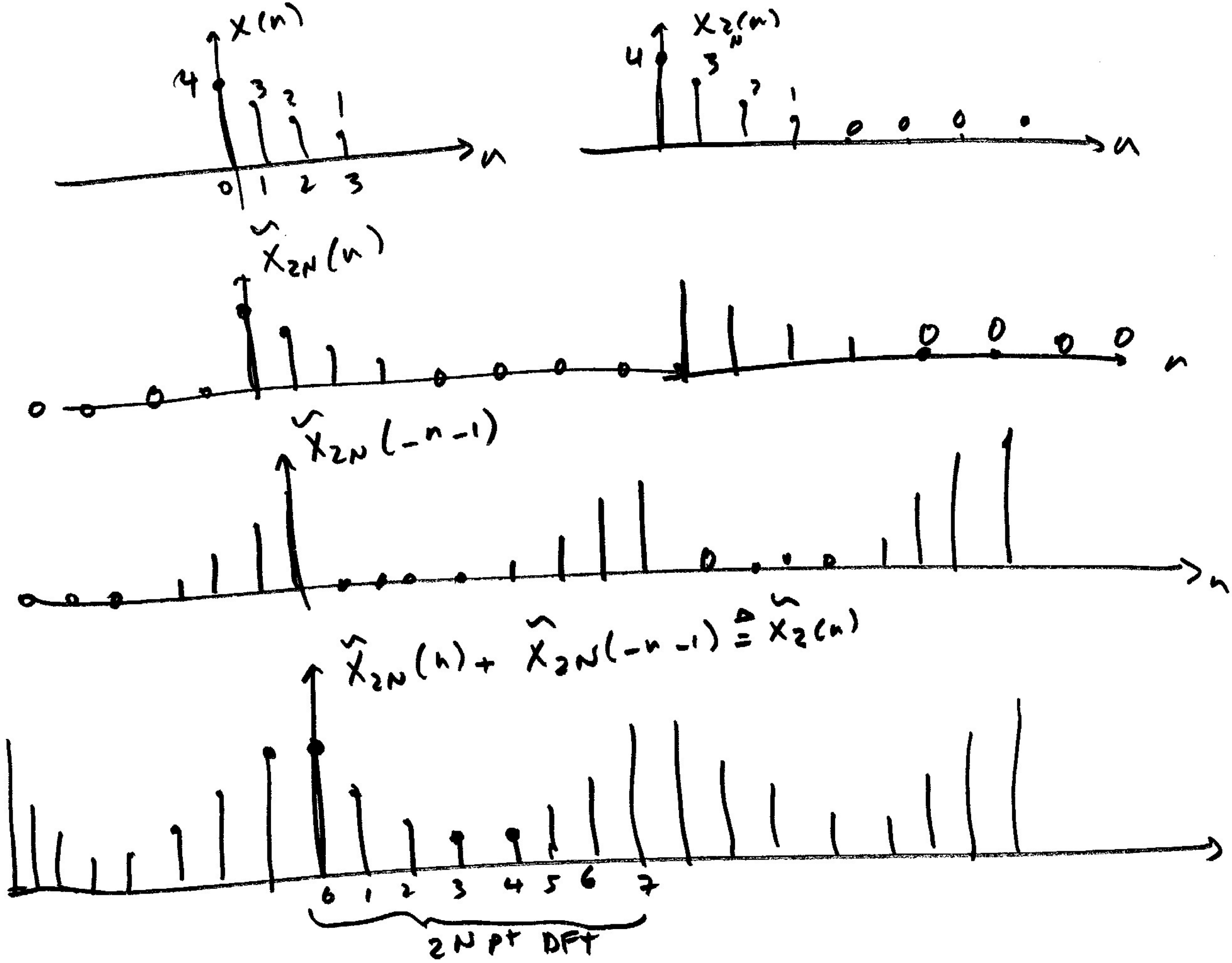


Proposal (2)

1. Start with N pt real seq $x(n)$
2. Pad it with N zeros $\longrightarrow$ $x_{2N}(n)$
3. Form a periodic seq.

$$\tilde{x}_2(n) = \tilde{x}_{2N}(n) + \tilde{x}_{2N}(-n-1)$$

4. take 2N pt DFT of one period of $\tilde{x}_2(n)$ $\longrightarrow$ $X_2(k)$
5. $X^{c_2}(k) = X_2(k)\, e^{-j\frac{\pi k}{2N}}$


$x(n)$
4
3
2
1
0
1
2
3
n
$x_{2N}(n)$
4
3
2
1
0
0
0
n
$\tilde{x}_{2N}(n)$
0
0
0
0
n
$\tilde{x}_{2N}(-n-1)$
n
$\tilde{x}_{2N}(n) + \tilde{x}_{2N}(-n-1) \triangleq \tilde{x}_2(n)$
0
1
2
3
4
5
6
7
2N pt DFT

How is $X_2(k)$ related to $X^{C2}(k)$

$x(n) \longrightarrow X(k) = 2N$ pt DFT of $x(n)$

$$X_2(k) = X(k) + e^{+j\frac{2\pi k}{2N}} X^*(k)$$

$$= e^{j\frac{\pi k}{2N}} \left[X(k) e^{-j\frac{\pi k}{2N}} + e^{j\frac{\pi k}{2N}} X^*(k) \right]$$

$$X_2(k) = e^{j\frac{\pi k}{2N}} \underbrace{2\,\mathrm{Re}\left\{ X(k) e^{-j\frac{\pi k}{2N}} \right\}}_{X^{C2}(k)}$$

$$X_2(k) = e^{j\frac{\pi k}{2N}} X^{C2}(k) \Longrightarrow \text{QED}$$

Fig 8.29 O &S.

Compaction prop of DCT.